

SOLUTION VECTORS ALGEBRA

EXERCISE-I (MHT CET LEVEL)

Q.1 (1)
Direction is not determined

Q.2 (4)
 $\overline{AB} = (6-2)\hat{i} + (-3+9)\hat{j} + (8+4)\hat{k} = 4\hat{i} + 6\hat{j} + 12\hat{k}$
 $|\overline{AB}| = \sqrt{16+36+144} = 14.$

Q.3 (1)
 $\vec{r} = \vec{a} + \vec{b} + \vec{c} = 4\hat{i} - \hat{j} - 3\hat{i} + 2\hat{j} - \hat{k}$
 $= \hat{i} + \hat{j} - \hat{k}$
 $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{1^2 + 1^2 + (-1)^2}} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

Q.4 (4)
The position vector of points D, E, F are respectively

$$\frac{\hat{i} + \hat{j}}{2} + \hat{k}, \hat{i} + \frac{\hat{k} + \hat{j}}{2} \text{ and } \hat{j}$$

So, position vector of centre of $\triangle DEF$

$$= \frac{1}{3} \left[\frac{\hat{i} + \hat{j}}{2} + \hat{k} + \hat{i} + \frac{\hat{k} + \hat{j}}{2} + \frac{\hat{i} + \hat{k}}{2} + \hat{j} \right]$$

$$= \frac{2}{3} [\hat{i} + \hat{j} + \hat{k}]$$

Q.5 (4)
for a unit cube unit vector along the diagonal

$$OP = \frac{1}{\sqrt{3}} (\hat{i} + \hat{j} + \hat{k})$$

unit vector along the diagonal

$$CD = \frac{1}{\sqrt{3}} (\hat{i} + \hat{j} - \hat{k})$$

$$\therefore \cos \theta = \frac{1}{3} (1+1-1) = \frac{1}{3} \quad \therefore \tan \theta = 2\sqrt{2}$$

Q.6 (1)

Q.7 (4)

Q.8 (2)

Q.9 (1)

Q.10 (4)
 $\overline{AB} = \lambda \overline{BC}$, (for collinearity)

Here $\overline{AB} = -2\hat{b}$, $\overline{BC} = (k+1)\hat{b}$

Hence $\forall \hat{k} \in \mathbb{R} \Rightarrow \overline{AB} = \lambda \overline{BC}$.

Q.11 (1)

$$\begin{vmatrix} 1 & 2 & 3 \\ \lambda & 4 & 7 \\ -3 & -2 & -5 \end{vmatrix} = 0 \Rightarrow \lambda = 3.$$

Q.12 (1)
Since \vec{a} and \vec{b} are collinear, we have for some scalar m.

$$\Rightarrow \hat{i} - \hat{j} = m (-2\hat{i} + \hat{k}) \Rightarrow \hat{i} - \hat{j} = -2m\hat{i} + m\hat{k}$$

$$\Rightarrow -2m = 1, m\hat{k} = -\hat{j}$$

$$\Rightarrow m = -\frac{1}{2}, \text{ So } \hat{k} = 2$$

Q.13 (1)
 $a = 4\hat{i} + 2\hat{j} - 4\hat{k} \Rightarrow |a| = \sqrt{16+16+4} = 6$
 $b = -3\hat{i} + 2\hat{j} + 12\hat{k} \Rightarrow |b| = \sqrt{144+4+9} = \sqrt{157}$
 $c = -\hat{i} - 4\hat{j} - 8\hat{k} \Rightarrow |c| = \sqrt{64+16+1} = 9$
Hence perimeter is $15 + \sqrt{157}$.

Q.14 (2)
Unit vector perpendicular to both the given vectors is,

$$\frac{(\hat{6i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 6\hat{j} - 2\hat{k})}{|(\hat{6i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 6\hat{j} - 2\hat{k})|} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}$$

Q.15 (2)

Q.16 (4)

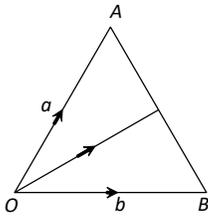
Q.17 (2)
 $v = b + c$ (i)
 $w = b + a$ (ii)

We have, $x = v + w = a + 2b + c$.

Q.18 (2)
It is obvious.

Q.19 (4)

Since given that $\overline{AC} = 3\overline{AB}$. It means that point divides externally. Thus $\overline{AC} : \overline{BC} = 3 : 2$



Hence $\overline{OC} = \frac{3 \cdot b - 2 \cdot a}{3 - 2} = 3b - 2a$.

Q.20 (2)

$$\frac{a+b}{2} = 2\hat{i} - \hat{j} + \frac{5}{2}\hat{k}$$

Q.21 (2)

Let position vector of D is $xi + yj + zk$,

then $\overline{AB} = \overline{DC}$

$$\Rightarrow -2\hat{j} - 4\hat{k} = (7-x)\hat{i} + (7-y)\hat{j} + (7-z)\hat{k}$$

Hence position vector of will be .

$$\Rightarrow x = 7, y = 9, z = 11.$$

Q.22 (3)

Since $\overline{OA} = a, \overline{OB} = b$ and $2AC = CO$

By section formula $\overline{OC} = \frac{2}{3}a$.

Therefore, $|\overline{CD}| = 3|\overline{OB}| \Rightarrow \overline{CD} = 3b$

$$\Rightarrow \overline{OD} = \overline{OC} + \overline{CD} = \frac{2}{3}a + 3b$$

Hence, $\overline{AD} = \overline{OD} - \overline{OA} = \frac{2}{3}a + 3b - a = 3b - \frac{1}{3}a$.

Q.23 (2)

Position vectors of the points which divides internally

is, $\frac{3(2a - 3b) + 2(3a - 2b)}{5} = \frac{12a - 13b}{5}$.

Q.24 (4)

$\overline{AB} = -2\hat{j}$, Here $\overline{BC} = (a-1)\hat{i} + (b+1)\hat{j} + c\hat{k}$

The points are collinear, then $\overline{AB} = k(\overline{BC})$

$$-2\hat{j} = k\{(a-1)\hat{i} + (b+1)\hat{j} + c\hat{k}\}$$

On comparing, $k(a-1) = 0, k(b+1) = -2, kc = 0$.

Hence $c = 0, a = 1$ and b is arbitrary scalar.

Q.25 (1)

$$\overline{PQ} = 3a + 3\sqrt{3}b \text{ and } \overline{RS} = 2a + 2\sqrt{3}b$$

Hence $\overline{PQ} \parallel \overline{RS}$.

Q.26 (2)

We have $|a+b|^2 + |a-b|^2 = 2(|a|^2 + |b|^2)$

$$\therefore 25 + |a-b|^2 = 2(9+16) \Rightarrow |a-b| = 5$$

Q.27

(1) Let $a = x\hat{i} + y\hat{j} + z\hat{k}$.

Then $(a \cdot \hat{i})\hat{i} + (a \cdot \hat{j})\hat{j} + (a \cdot \hat{k})\hat{k} = a$.

Q.28 (1)

According to question $a = -\hat{i} + \hat{j} + \hat{k}$ and

$$b = \hat{i} - \hat{j} + \hat{k}$$

Then,
$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}[1+1] - \hat{j}[-1-1] + \hat{k}[1-1] = 2(\hat{i} + \hat{j})$$

and $|a \times b| = \sqrt{4+4} = 2\sqrt{2}$

$$\therefore \text{Required unit vector} = \pm \frac{2(\hat{i} + \hat{j})}{2\sqrt{2}}$$

$$= \pm \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

Q.29 (2)

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

$$= \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$= \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

Squaring both the sides, we get

$$\vec{A}^2 + \vec{B}^2 + 2\vec{A}\vec{B} \cos \theta$$

$$= \vec{A}^2 + \vec{B}^2 - 2\vec{A}\vec{B} \cos \theta$$

$$\text{or } 4\vec{A} + \vec{B} \cos \theta = 0 \text{ or } \cos \theta = 0$$

(since the scale or dot product is zero).

Therefore angle between \vec{A} to \vec{B} is 90°

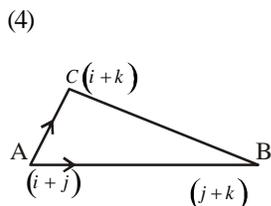
Q.30

(3)
 $a \cdot b = a \cdot c \Rightarrow a \cdot (b - c) = 0$
 $\Rightarrow a = 0$ or $b - c = 0$ or $a \perp (b - c)$
 $\Rightarrow a = 0$ or $b = c$ or $a \perp (b - c)$... (1)
 Also $a \times b = a \times c \Rightarrow a \times (b - c) = 0$
 $\Rightarrow a = 0$ or $b - c = 0$ or $a \parallel (b - c)$
 $\Rightarrow a = 0$ or $b = c$ or $a \parallel (b - c)$... (2)
 Observing to (1) and (2) we find that
 $a = 0$ or $b = c$

Q.31

(3)
 We have $\vec{a} + \vec{b} + \vec{c} = \vec{0}$
 $\therefore |\vec{a} + \vec{b} + \vec{c}| = 0 \Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 0$
 $\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$
 $+ 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$
 $\Rightarrow 1 + 1 + 1 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$
 $\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$

Q.32



Now, $\vec{AC} = \hat{k} - \hat{j}$ and $\vec{AB} = \hat{k} - \hat{i}$
 Let θ be the angle between \vec{AC} and \vec{AB} .

$$\cos \theta = \frac{1 - 0 - 0 + 0}{\sqrt{2}\sqrt{2}} = \frac{1}{2} \Rightarrow \theta = 60^\circ = \frac{\pi}{3}$$

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|-----------------|-----------------|-----------------|-----------------|-----------------|
| Q.33 (2) | Q.34 (3) | Q.35 (4) | Q.36 (3) | Q.37 (4) |
| Q.38 (3) | Q.39 (2) | Q.40 (4) | Q.41 (3) | Q.42 (3) |
| Q.43 (3) | Q.44 (2) | Q.45 (3) | Q.46 (2) | Q.47 (4) |
| Q.48 (2) | Q.49 (4) | Q.50 (3) | Q.51 (3) | Q.52 (2) |
| Q.53 (1) | Q.54 (3) | Q.55 (2) | Q.56 (1) | Q.57 (3) |
| Q.58 (3) | | | | |
| Q.59 (4) | | | | |

Let $r = x\hat{i} + y\hat{j} + z\hat{k}$.
 Since $r \cdot \hat{i} = r \cdot \hat{j} = r \cdot \hat{k} \Rightarrow x = y = z$ (i)
 Also $|r| = \sqrt{x^2 + y^2 + z^2} = 3 \Rightarrow x = \pm\sqrt{3}$, {By (i)}
 Hence the required vector $r = \pm\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$.
 Trick : As the vector $\pm\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$ satisfies both the conditions.

Q.60

(3)
 $a \cdot b = a \cdot c \Rightarrow a \cdot b - a \cdot c = 0 \Rightarrow a \cdot (b - c) = 0$
 \Rightarrow Either $b - c = 0$ or $a = 0 \Rightarrow b = c$
 or $a \perp (b - c)$.

Q.61

(1)
 Three mutually perpendicular unit vectors = $a, b,$ and c
 $a \cdot b = b \cdot c = c \cdot a = 0$.

Therefore $|a| = |b| = |c| = 1$ and $a \cdot b = b \cdot c = c \cdot a = 0$.
 We know that

$$|a + b + c|^2 = (a + b + c) \cdot (a + b + c) = |a|^2 + |b|^2 + |c|^2 + 2(a \cdot b + b \cdot c + c \cdot a) = 1 + 1 + 1 + 0 = 3$$

$$\text{or } |a + b + c| = \sqrt{3}.$$

Q.62

(4)
 Since $a \perp b \Rightarrow a \cdot b = 0$
 $|a - b|^2 = (a - b) \cdot (a - b) = a^2 + b^2 - 2a \cdot b = 25 + 25$
 $\Rightarrow |a - b| = 5\sqrt{2}$.

Q.63

(2)
 $(a + b) \cdot (a + b) = |a|^2 + |b|^2 + 2a \cdot b$

Q.64

(1)
 $|a + b| > |a - b|$
 Squaring both sides, we get
 $a^2 + b^2 + 2a \cdot b > a^2 + b^2 - 2a \cdot b$
 $\Rightarrow 4a \cdot b > 0 \Rightarrow \cos \theta > 0$ Hence $\theta < 90^\circ$, (acute).

Q.65

(4)
 Parallel vector = $(2 + b)\hat{i} + 6\hat{j} - 2\hat{k}$
 Unit vector = $\frac{(2 + b)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{b^2 + 4b + 44}}$

According to the condition, $1 = \frac{(2 + b) + 6 - 2}{\sqrt{b^2 + 4b + 44}}$
 $\Rightarrow b^2 + 4b + 44 = b^2 + 12b + 36 \Rightarrow 8b = 8 \Rightarrow b = 1$.

Q.66

(2)
 $|a + b| = |a - b|$;
 Squaring both sides, we get $4a \cdot b = 0$
 $\Rightarrow a$ is perpendicular to b .

Q.67

(3)
 $a \cdot b = (2 - 4 - \lambda) = 0 \Rightarrow \lambda = -2$.

Q.68

(2)
 $\vec{L} = \hat{i} + 4\hat{j}$
 Therefore, vector perpendicular to $\vec{L} = \lambda(4\hat{i} - \hat{j})$

∴ Unit vector is $\frac{4\hat{i} - \hat{j}}{\sqrt{17}}$. But it points towards origin

∴ Required vector = $-\frac{4\hat{i} + \hat{j}}{\sqrt{17}}$.

Q.69 (1)

$14 \cos 60^\circ, 14 \sin 60^\circ$ or $7, \frac{14\sqrt{3}}{2}$ or $7, 7\sqrt{3}$.

Q.70 (2)

Required value = $\frac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{b}|} \div \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} = \frac{|\mathbf{a}|}{|\mathbf{b}|} = \frac{7}{3}$.

Q.71 (2)

Projection of \mathbf{a} on $\mathbf{b} = |\mathbf{a}| \cos \theta = |\mathbf{a}| \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$

$$= \frac{4 + 8 + 7}{\sqrt{16 + 16 + 49}} = \frac{19}{\sqrt{81}} = \frac{19}{9}$$

Q.72 (2)

$$\begin{aligned} (\mathbf{a} \times \mathbf{b})^2 &= (|\mathbf{a}| |\mathbf{b}| \sin \theta)^2 \\ &= (4 \cdot 2 \sin 30^\circ)^2 = \left(8 \cdot \frac{1}{2}\right)^2 = 4^2 = 16 \end{aligned}$$

Q.73 (3)

$$\Delta = |\mathbf{a} \times \mathbf{b}| = \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix} \right| = |8\hat{i} + 8\hat{j} - 8\hat{k}| = 8\sqrt{3}$$

Q.74 (2)

Let $\mathbf{p} = 2\mathbf{a} - \mathbf{b}$ and $\mathbf{q} = 4\mathbf{a} - 5\mathbf{b}$.

Then $\mathbf{p} \times \mathbf{q} = (2\mathbf{a} - \mathbf{b}) \times (4\mathbf{a} - 5\mathbf{b}) = -6(\mathbf{a} \times \mathbf{b})$

$$= -6 |\mathbf{a}| |\mathbf{b}| \sin \frac{\pi}{4} \hat{\mathbf{n}} = -6 \times \frac{1}{\sqrt{2}} \hat{\mathbf{n}} = -3\sqrt{2} \hat{\mathbf{n}}$$

Hence the area of the given parallelogram

$$= \frac{1}{2} |\mathbf{p} \times \mathbf{q}| = \frac{3}{\sqrt{2}}$$

Q.75 (4)

Unit vectors perpendicular to the plane of \mathbf{a} and \mathbf{b}

$$= \pm \frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a} \times \mathbf{b}|}$$

∴ Required vector is $\pm \frac{(\hat{i} - \hat{j} + \hat{k}) \times (-\hat{i} + \hat{j} + \hat{k})}{|(\hat{i} - \hat{j} + \hat{k}) \times (-\hat{i} + \hat{j} + \hat{k})|}$

$$= \pm \frac{(-(\hat{i} + \hat{j}))}{\sqrt{2}} \text{ i.e., } -\frac{(\hat{i} + \hat{j})}{\sqrt{2}} \text{ and } \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

Q.76 (3)

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0 \text{ or } (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = 0$$

Q.77 (4)

$$\vec{\mathbf{a}} = \hat{i} - \hat{k}, \vec{\mathbf{b}} = x\hat{i} + \hat{j} + (1-x)\hat{k} \text{ and}$$

$$\vec{\mathbf{c}} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$$

$$[\vec{\mathbf{a}} \vec{\mathbf{b}} \vec{\mathbf{c}}] = \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$$= 1[1+x-y-x+x^2] - [x^2-y]$$

$$= 1-y+x^2-x^2+y$$

$$= 1$$

Hence $[\vec{\mathbf{a}} \vec{\mathbf{b}} \vec{\mathbf{c}}]$ is independent of x and y both.

Q.78 (2)

Q.79 (4)

Q.80 (1)

Q.81 (3)

Q.82 (1)

$$\text{Let } \mathbf{a} = 3\hat{i} - 2\hat{j} - \hat{k}, \mathbf{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}, \mathbf{c} = -\hat{i} + \hat{j} + 2\hat{k}$$

and $\mathbf{d} = 4\hat{i} + 5\hat{j} + \lambda\hat{k}$.

Since the points are coplanar,

So, $[\mathbf{d} \mathbf{b} \mathbf{c}] + [\mathbf{d} \mathbf{c} \mathbf{a}] + [\mathbf{d} \mathbf{a} \mathbf{b}] = [\mathbf{a} \mathbf{b} \mathbf{c}]$

$$\Rightarrow \begin{vmatrix} 4 & 5 & \lambda \\ 2 & 3 & -4 \\ -1 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 4 & 5 & \lambda \\ -1 & 1 & 2 \\ 3 & -2 & -1 \end{vmatrix} + \begin{vmatrix} 4 & 5 & \lambda \\ 3 & -2 & -1 \\ 2 & 3 & -4 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & -2 & -1 \\ 2 & 3 & -4 \\ -1 & 1 & 2 \end{vmatrix}$$

$$\Rightarrow 40 + 5\lambda + 37 - \lambda + 94 + 13\lambda = 25 \Rightarrow \lambda = \frac{-146}{17}$$

Q.83 (1)

$$\mathbf{p} + \mathbf{q} + \mathbf{r} = \frac{\mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a} + \mathbf{a} \times \mathbf{b}}{[\mathbf{a} \mathbf{b} \mathbf{c}]}$$

$$(\mathbf{a} + \mathbf{b} + \mathbf{c}) \cdot (\mathbf{p} + \mathbf{q} + \mathbf{r}) = \frac{[\mathbf{a} \mathbf{b} \mathbf{c}] + [\mathbf{b} \mathbf{c} \mathbf{a}] + [\mathbf{c} \mathbf{a} \mathbf{b}]}{[\mathbf{a} \mathbf{b} \mathbf{c}]} = 3$$

Q.84 (4)

Volume of cube = $[\mathbf{a} \mathbf{b} \mathbf{c}]$

$$= \begin{vmatrix} 12 & 4 & 3 \\ 8 & -12 & -9 \\ 33 & -4 & -24 \end{vmatrix} = 12 \begin{vmatrix} 12 & 1 & 1 \\ 8 & -3 & -3 \\ 33 & -1 & -8 \end{vmatrix} = 3696$$

Q.85 (1)

Since x is a non-zero vector, the given conditions will be satisfied, if either (i) at least one of the vectors a , b , c is zero or (ii) is perpendicular to all the vectors. In case (ii), are coplanar and so $a \cdot b \cdot c = 0$

Q.86 (3)

$$V = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -2 \\ 3 & -1 & 1 \end{vmatrix} = |-7| = 7 \text{ cubic unit.}$$

Q.87 (2)

$$\hat{i} \times \hat{j} \times \hat{k} = \hat{k} \times \hat{k} = 0.$$

Q.88 (4)

As we know, $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$ (i)

$$\therefore a \times (b \times c) = \frac{b}{2} \text{ (Given)}$$

From equation (i),

$$(a \cdot c)b - (a \cdot b)c = \frac{b}{2} \text{ or } \left(a \cdot c - \frac{1}{2}\right)b - (a \cdot b)c = c$$

Comparison on both sides of b and c

$$a \cdot c - \frac{1}{2} = 0, \quad a \cdot b = 0$$

$$\Rightarrow |a||c| \cos \theta = \frac{1}{2} \Rightarrow (1)(1) \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

$$\text{or } a \cdot b = 0 \therefore \theta = 90^\circ$$

So the angle between with b and c are and respectively.

Q.89 (4)

$$(\hat{i} \times \hat{i}) + (\hat{j} \times \hat{j}) + (\hat{k} \times \hat{k}) = 0.$$

Q.90 (2)

$$\therefore a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$$

$$\therefore a \cdot c = (\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{j} - \hat{k}) = 1 - 1 + 1 = 1$$

$$a \cdot b = (\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{j} + \hat{k}) = 1 - 1 - 1 = -1$$

$$\therefore a \times (b \times c) = (1)b - (-1)c$$

$$= b + c = \hat{i} - \hat{j} + \hat{k} + \hat{i} - \hat{j} - \hat{k} = 2\hat{i} - 2\hat{j}$$

Q.91 (1)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix} = \hat{i}(2+3) - \hat{j}(-1+6) + \hat{k}(1+4)$$

$$= 5\hat{i} - 5\hat{j} + 5\hat{k}$$

$$\text{Now } (\vec{A} \times \vec{B}) \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -5 & 5 \\ 1 & 3 & -2 \end{vmatrix}$$

$$= \hat{i}(10-15) - \hat{j}(-10-5) + \hat{k}(15+5) = -5\hat{i} + 15\hat{j} + 20\hat{k} = 5(-\hat{i} + 3\hat{j} + 4\hat{k})$$

Q.92 (1)

$$a \cdot c = 1 \text{ and } b \cdot c = 1$$

Given that $(a \times b) \times c = (c \cdot a)b - (c \cdot b)a = \mu b + \lambda a$

where $\mu = c \cdot a = 1, \lambda = -(c \cdot b) = -1$

$$\Rightarrow \mu + \lambda = 1 - 1 = 0$$

Q.93 (4)

$$a \times [a \times (a \times b)] = a \times \{(a \cdot b)a - (a \cdot a)b\}$$

$$= (a \cdot b)(a \times a) - (a \cdot a)(a \times b) = (a \cdot b)0 + (a \cdot a)(b \times a)$$

$$= (a \cdot a)(b \times a)$$

Q.94 (4)

Required distance

$$= \left| \frac{d - a \cdot n}{|n|} \right| = \left| \frac{5 - (2\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + 5\hat{j} + \hat{k})}{\sqrt{1+25+1}} \right|$$

$$= \left| \frac{5 - (2 - 10 + 3)}{\sqrt{27}} \right| = \frac{10}{3\sqrt{3}}$$

Q.95 (1)

Q.96 (1)

Since a , b and $a \times b$ are non-coplanar, hence $r = xa + yb + z(a \times b)$ for some scalars x , y and z .

$$\text{Now, } b = r \times a = \{xa + yb + z(a \times b)\} \times a$$

$$= y(b \times a) + z[(a \times b) \times a] = -y(a \times b) - z[a \times (a \times b)]$$

$$= -y(a \times b) - z[(a \cdot b)a - (a \cdot a)b]$$

$$= -y(a \times b) + z(a \cdot a)b \quad \{\because a \cdot b = 0\}$$

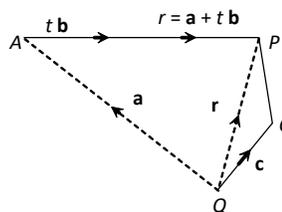
$$\Rightarrow y = 0 \text{ and } z = \frac{1}{(a \cdot a)} \Rightarrow r = xa + \frac{1}{a \cdot a}(a \times b).$$

Q.97 (2)

For point P on the line $r = a + tb$

$$\therefore \vec{PC} = (c - a) - tb, \therefore \vec{PC} \perp b$$

$$\therefore |(c - a) - tb| \cdot b = 0 \text{ or } t = \frac{(c - a) \cdot b}{b^2} \quad \dots(i)$$



Distance of c from line $|\vec{PC}| = d = |c - a - tb|$

$$d = \left| c - a - \frac{(c-a) \cdot bb}{b^2} \right| = \left| \frac{(c-a) \cdot b \cdot b - (c-a) \cdot bb}{b^2} \right|$$

$$d = \left| \frac{b \times (c-a) \times b}{b^2} \right| = \frac{|b \parallel (c-a) \times b| \sin 90^\circ}{|b|^2},$$

($\because b \perp (c-a) \times b$)

$$d = \frac{|(c-a) \times b|}{|b|}$$

Q.98 (2)

Here $d = 8$ and $n = 2\hat{i} + \hat{j} + 2\hat{k}$

$$\therefore \hat{n} = \frac{n}{|n|} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{4+1+4}} = \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

Hence, the required equation of the plane is

$$r \cdot \left(\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k} \right) = 8 \text{ or } r \cdot (2\hat{i} + \hat{j} + 2\hat{k}) = 24.$$

Q.99 (3)

The given lines are $r = a_1 + \lambda b_1, r = a_2 + \mu b_2$

where $a_1 = 3\hat{i} - 2\hat{j} - 2\hat{k}, b_1 = \hat{i}$

$$a_2 = \hat{i} - \hat{j} + 2\hat{k}, b_2 = \hat{j}$$

$$|b_1 \times b_2| = |\hat{i} \times \hat{j}| = |\hat{k}| = 1$$

Now, $[(a_2 - a_1) \cdot b_1 \cdot b_2] = (a_2 - a_1) \cdot (b_1 \times b_2)$

$$= (-2\hat{i} + \hat{j} + 4\hat{k}) \cdot (\hat{k}) = 4$$

\therefore Shortest distance

$$= \frac{[(a_2 - a_1) \cdot (b_1 \times b_2)]}{|b_1 \times b_2|} = \frac{4}{1} = 4.$$

Q.100 (1)

It is obvious.

Q.101 (2)

The equation of a plane parallel to the plane

$$r \cdot (4\hat{i} - 12\hat{j} - 3\hat{k}) - 7 = 0 \text{ is } r \cdot (4\hat{i} - 12\hat{j} - 3\hat{k}) + \lambda = 0$$

This passes through $2\hat{i} - \hat{j} - 4\hat{k}$

$$\text{Therefore, } (2\hat{i} - \hat{j} - 4\hat{k}) \cdot (4\hat{i} - 12\hat{j} - 3\hat{k}) + \lambda = 0$$

$$\Rightarrow 8 + 12 + 12 + \lambda = 0 \Rightarrow \lambda = -32$$

So, the required plane is $r \cdot (4\hat{i} - 12\hat{j} - 3\hat{k}) - 32 = 0$

Q.102 (2)

The equation of a line passing through the points

$A(\hat{i} - \hat{j} + 2\hat{k})$ and $B(3\hat{i} + \hat{j} + \hat{k})$ is

$$r = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + \hat{j} + \hat{k})$$

The position vector of any point P which is a variable point on the line, is

$$(\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + \hat{j} + \hat{k})$$

$$\vec{AP} = \lambda(3\hat{i} + \hat{j} + \hat{k}) \Rightarrow |\vec{AP}| = \lambda\sqrt{11}$$

Now, if $\lambda\sqrt{11} = 3\sqrt{11}$ i.e., $\lambda = 3$ then the position

vector of the point P is $10\hat{i} + 2\hat{j} + 5\hat{k}$.

If $\lambda\sqrt{11} = -3\sqrt{11}$, i.e., $\lambda = -3$ then the position

vector of the point P is $-8\hat{i} - 4\hat{j} - \hat{k}$.

Q.103 (1)

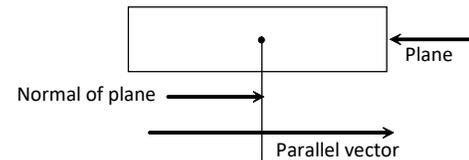
Q.104 (1)

As plane is parallel to a given vector \Rightarrow Normal of plane must be perpendicular to the given vectors. Given point to which plane passes through is $(2, -1, 3)$.

Let A, B, C are direction ratios of its normal.

\therefore Equation of plane is,

$$A(x - 2) + B(y + 1) + C(z - 3) = 0$$



Now normal to plane $A\hat{i} + B\hat{j} + C\hat{k}$ is perpendicular

to the given vectors $a = 3\hat{i} + 0\hat{j} - \hat{k}$ and

$$b = -3\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\therefore 3A + 0B - C = 0 \quad \dots(i)$$

$$-3A + 2B + 2C = 0 \quad \dots(ii)$$

Solving (i) and (ii) we get, $\frac{A}{2} = \frac{B}{-3} = \frac{C}{6}$

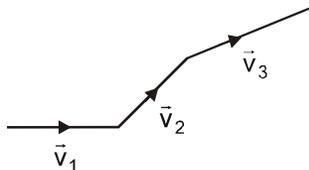
\therefore Equation of plane be

$$2(x - 2) - 3(y + 1) + 6(z - 3) = 0$$

$$\text{i.e., } 2x - 3y + 6z - 25 = 0$$

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (2)
Clearly triangle is not possible as $v_1 + v_2 + v_3 \neq 0$

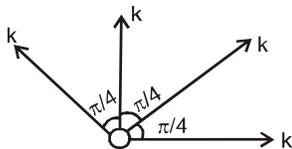


Since $\vec{v}_3 = \vec{v}_1 + \vec{v}_2$

Hence $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are coplaner

Q.2 (3)
These forces can be written in terms of vector as

$k\hat{i}, \frac{k}{\sqrt{2}}\hat{i} + \frac{k}{\sqrt{2}}\hat{j}, k\hat{j}$ and $-\frac{k}{\sqrt{2}}\hat{i} + \frac{k}{\sqrt{2}}\hat{j}$



Resultant = $k\hat{i} + (k + \sqrt{2}k)\hat{j}$

magnitude = $\sqrt{k^2 + (k + \sqrt{2}k)^2} = k\sqrt{4 + 2\sqrt{2}}$

Q.3 (4)
If given vectors are coplanar, then there exists two scalar quantities x and y such that
 $2\hat{i} - \hat{j} + \hat{k} = x(\hat{i} + 2\hat{j} - 3\hat{k}) + y(3\hat{i} + \hat{j} + 5\hat{k})$ (1)

Comparing coefficient of \hat{i}, \hat{j} and \hat{k} on both sides

of (1) we get $x+3y=2, 2x+ay=-1,$

$3x+5y=1$ (2)

Solving first and third equations, we get $x=1/2, y=1/2$

since the vectors are coplanar, therefore these values of x and y will satisfy the equation

$2x+ay=-1$

$2(1/2) + a(1/2) = -1 \Rightarrow a = -4$

Q.4 (2)
Before rotation $\vec{a} = 2p\hat{i} + \hat{j}$

after rotation $\vec{a} = (p+1)\hat{i}' + \hat{j}'$

Since length of vector remains unaltered

$\sqrt{4p^2 + 1} = \sqrt{(p+1)^2 + 1}$

$\Rightarrow 4p^2 = (p+1)^2$

$\Rightarrow p+1 = \pm 2p$

$\Rightarrow p=1$ or $-\frac{1}{3}$

Q.5 (3)
 $\vec{a} = (2\sqrt{2}, -1, 4) \quad |\vec{b}| = 10$

$\vec{b} = \lambda\vec{a}$

$|\vec{b}|^2 = \lambda^2 |\vec{a}|^2$

$100 = \lambda^2 (8 + 1 + 16)$

$\lambda^2 = 4 \Rightarrow \lambda = \pm 2$

$2\vec{a} \pm \vec{b} = 0$

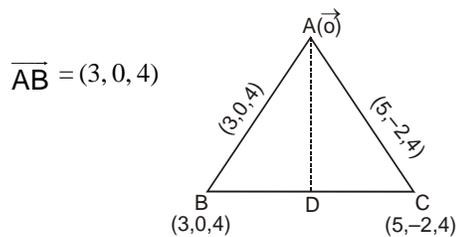
Q.6 (4)
 $\vec{AB} = -6\hat{i} - 10\hat{j} + 3\hat{k}$

$\vec{AD} = -2\hat{i} - 5\hat{j} - 2\hat{k}$

$\vec{AB} \cdot \vec{AD} \neq 0$

so not a square or rectangle $|\vec{AB}| \neq |\vec{AD}|$ so not a rhombus.

Q.7 (3)



$\vec{AB} = (3, 0, 4)$

$\vec{AC} = (5, -2, 4)$

Let \vec{A} be origin.

D is the mid

point of BC

$D(4, -1, 4)$

$\vec{AD} = (4, -1, 4)$

$|\vec{AD}| = \sqrt{16 + 1 + 16} = \sqrt{33}$

Q.8 (4)

$\vec{a} \cdot (\vec{b} + \vec{c}) = 0$... (i)

$\vec{b} \cdot (\vec{c} + \vec{a}) = 0$... (ii)

$$\vec{c} \cdot (\vec{a} + \vec{b}) = 0 \quad \dots(\text{iii})$$

Add all equation $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0$

$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{a^2 + b^2 + c^2 + 0}$$

$$= \sqrt{9 + 16 + 25} = 5\sqrt{2}$$

Q.9 (4)

$$\vec{A} (2, -1, -1), \vec{B} = (1, -3, -5),$$

$$\vec{C} = (a, -3, -1)$$

$$\vec{AC} \cdot \vec{CB} = 0 \Rightarrow (a-2, -2, 0) \cdot (a-1, 0, 4) = 0$$

$$(a-1)(a-2) = 0 \Rightarrow a = 1 \text{ and } 2$$

Q.10 (1)

$$(\vec{a} \times \vec{b})^2 (\vec{a} \cdot \vec{b})^2 = 676$$

$$(|\vec{a}| \cdot |\vec{b}| \sin \theta)^2 + (|\vec{a}| \cdot |\vec{b}| \cos \theta)^2 = 676$$

$$\Rightarrow a^2 b^2 \sin^2 \theta + a^2 b^2 \cos^2 \theta$$

$$= 676 [(\hat{n})^2 = 1]$$

$$a^2 b^2 (\sin^2 \theta + \cos^2 \theta) = 676$$

$$\Rightarrow a^2 = \frac{676}{b^2} = \frac{676}{4}$$

$$|\vec{a}| = \sqrt{\frac{676}{4}} \Rightarrow |\vec{a}| = \frac{26}{2} \Rightarrow |\vec{a}| = 13$$

Q.11 (1)

Let θ be the angle between the line and the normal to the plane. Converting the given equations into vector form, we have

$$\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \text{ and}$$

$$\vec{r} \cdot (10\hat{i} + 2\hat{j} - 11\hat{k}) = 3$$

$$\text{Her, } \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k} \text{ and}$$

$$\vec{n} = 10\hat{i} + 2\hat{j} - 11\hat{k}$$

$$\sin \phi = \frac{|(2\hat{i} + 3\hat{j} + 6\hat{k}) \cdot (10\hat{i} + 2\hat{j} - 11\hat{k})|}{\sqrt{2^2 + 3^2 + 6^2} \sqrt{10^2 + 2^2 + 11^2}}$$

$$= \frac{|-40|}{7 \times 15} = \frac{|-8|}{21} = \frac{8}{21} \text{ or } \phi = \sin^{-1} \left(\frac{8}{21} \right)$$

Q.12 (4)

Let θ be the angle between b and c .

$$\text{Given, } |b \times c| = \sqrt{15}$$

$$\Rightarrow |b||c| \sin \theta = \sqrt{15}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{15}}{|b||c|}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{15}}{\sqrt{4 \times 1}} = \frac{\sqrt{15}}{4}$$

$$\therefore \cos \theta = \sqrt{1 - \frac{15}{16}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$$

$$\text{Now given, } b - 2c = \lambda a \Rightarrow |b - 2c| = |\lambda a|$$

$$\Rightarrow |b|^2 + 4|c|^2 - 4(b \cdot c) = \lambda^2 |a|^2$$

$$\Rightarrow 16 + 4 - 4|b||c| \cos \theta = \lambda^2$$

$$\Rightarrow 20 - 4 = \lambda^2 \Rightarrow \lambda^2 = 16 \left(\because \cos \theta = -\frac{1}{4} \right)$$

$$\Rightarrow \lambda = \pm 4$$

Q.13

(1)

Let the required vector be

$$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}, \text{ then}$$

$$\vec{v} \cdot (\hat{i} + \hat{j} - 3\hat{k}) = 0 \Rightarrow x + y - 3z = 0$$

.....(i)

$$\vec{v} \cdot (\hat{i} + 3\hat{j} - 2\hat{k}) = 5 \Rightarrow x + 3y - 2z = 5 \quad \dots(\text{ii})$$

$$\vec{v} \cdot (2\hat{i} + \hat{j} + 4\hat{k}) = 8 \Rightarrow 2x + y - 4z = 8 \quad \dots(\text{iii})$$

Subtracting (ii) from (i), we have

$$-2y - z = -5 \dots 2y + z = 5 \quad \dots(\text{iv})$$

Multiply (ii) by 2 and subtracting (iii) from it, we obtain

$$5y - 8z = 2 \quad \dots(\text{v})$$

Multiply (iv) by 8 and adding (v) to it, we have

$$21y = 42 \Rightarrow y = 2 \quad \dots(\text{v})$$

Substituting $y = 2$ in (iv), we get

$$2 \times 2 + z = 5 \Rightarrow z = 5 - 4 = 1$$

Substituting these values in (i), we get

$$x + 2 - 3 = 0 \Rightarrow x = 3 - 2 = 1$$

Hence, the required vector is

$$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k} = \hat{i} + 2\hat{j} + \hat{k}$$

Q.14 (1)

Q.15 (1)

Q.16 (1)

$$\vec{F}_1 = (4, 1, -3) \quad \vec{F}_2 = (3, 1, -1)$$

$$d\vec{s} = (5, 4, 1) - (1, 2, 3) = (4, 2, -2)$$

$$\text{work done} = \vec{F}_1 \cdot d\vec{s} + \vec{F}_2 \cdot d\vec{s} = 24 + 16 = 40$$

Q.17 (2)

$$|\vec{a} - \vec{b}| = 8 \Rightarrow |a|^2 + |b|^2 - 2\vec{a} \cdot \vec{b} = 64 \quad \dots(i)$$

$$|\vec{a} + \vec{b}| = 10 \Rightarrow |a|^2 + |b|^2 + 2\vec{a} \cdot \vec{b} = 100 \quad \dots(ii)$$

Add (i) and (ii) equation

$$2|a|^2 + 2|b|^2 = 164$$

$$|b|^2 = 82 - 25$$

$$|b| = \sqrt{57}$$

Q.18 (1)

$$\text{Diagonals are } \vec{a} + \vec{b} = (3, 0, 0)$$

$$\text{and } \vec{a} - \vec{b} = (1, 2, 2)$$

$$\cos \theta = \frac{(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})}{|\vec{a} + \vec{b}| |\vec{a} - \vec{b}|} = \frac{3}{3 \cdot 3} = \frac{1}{3}$$

$$\theta = \cos^{-1} \left(\frac{1}{3} \right)$$

Q.19 (4)

$$|\vec{u}| = 1; |\vec{v}| = 1$$

$$|2\vec{u} \times 3\vec{v}| = 1$$

$$|\vec{u} \times \vec{v}| = \frac{1}{6}$$

$$|\vec{u}| |\vec{v}| \sin \theta = \frac{1}{6} \Rightarrow \boxed{\sin \theta = \frac{1}{6}}$$

As θ is acute angle than only one value possible

Q.20 (2)

$$\vec{u} = \vec{a} - \vec{b}, \vec{v} = \vec{a} + \vec{b}, |\vec{a}| = |\vec{b}| = 2$$

$$|\vec{u} \times \vec{v}| = |(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})|$$

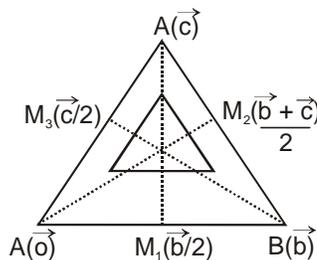
$$= |(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})|$$

$$= 2|\vec{a} \times \vec{b}| = 2|a||b| \sin \theta$$

$$= 2|a||b| \sqrt{\frac{|a|^2|b|^2 - (\vec{a} \cdot \vec{b})^2}{|a||b|}} = 2\sqrt{16 - (\vec{a} \cdot \vec{b})^2}$$

Q.21 (2)

$$C_1 \left(\frac{\vec{b}/2 + 3\vec{c}}{4} \right)$$



$$A_1 \left(\frac{\vec{b} + \vec{c}/2 + 0}{4} \right)$$

$$B_1 \left(\frac{\vec{c}/2 + 3\vec{b}}{4} \right)$$

$$\text{Area of } \Delta A_1 B_1 C_1 = \frac{1}{2} |\overline{A_1 B_1} \times \overline{A_1 C_1}|$$

$$\text{Area of } \Delta ABC = \frac{1}{2} |\vec{b} \times \vec{c}|$$

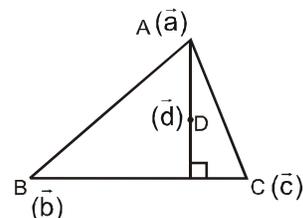
$$\text{Ratio} = \frac{\text{Area of } \Delta A_1 B_1 C_1}{\text{Area of } \Delta ABC} = \frac{25}{64}$$

Q.22 (3)

Given

$$(\vec{a} - \vec{d}) \cdot (\vec{b} - \vec{c}) = (\vec{b} - \vec{d}) \cdot (\vec{c} - \vec{a}) = 0$$

$$\Rightarrow \overline{DA} \cdot \overline{CB} = 0 \quad \& \quad \overline{DB} \cdot \overline{AC} = 0$$



$AD \perp BC$ & $BD \perp AC$
Hence D is orthocentre.

Q.23 (2)

$$\text{A vector normal to plane is } (\vec{a} - \vec{b}) \times (\vec{c} - \vec{b})$$

$$= \vec{a} \times \vec{c} - \vec{a} \times \vec{b} - \vec{b} \times \vec{c} = -(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$$

$$\text{unit vector} = \pm \frac{(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$$

Q.24 (1)

$$|\vec{e}_1 - \vec{e}_2|^2 < 1 \Rightarrow \vec{e}_1^2 + \vec{e}_2^2 - 2\vec{e}_1 \cdot \vec{e}_2 < 1$$

$$\Rightarrow 1 + 1 - 2\cos(2\theta) < 1$$

$$\Rightarrow 2\cos 2\theta > 1 \Rightarrow \cos 2\theta > \frac{1}{2}$$

$$2\theta \in \left[0, \frac{\pi}{3}\right] \Rightarrow \theta \in \left[0, \frac{\pi}{6}\right]$$

Q.25 (4)

$$\vec{a} = (1, x, 3) \cos \theta = \frac{11}{14}$$

$$\vec{b} = (4, 4x - 2, 2)$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \quad |\vec{b}| = 2 |\vec{a}|$$

$$\frac{11}{14} = \frac{4 + x(4x - 2) + 6}{2|\vec{a}|^2} \Rightarrow x = 2 \text{ and } x = -\frac{20}{17}$$

Q.26 (2)

$$\vec{a} = (-2, 1, 1), \vec{b} = (1, 5, 0), \vec{c} = (4, 4, -2)$$

$$\vec{d} = 3\vec{a} - 2\vec{b}$$

$$= 3(-2, 1, 1) - 2(1, 5, 0)$$

$$= (-6, 3, 3) - 2(2, 10, 0) = (-8, -7, 3)$$

$$\text{Projection} = |\vec{d}| \cos \theta$$

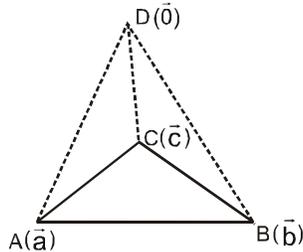
$$= |\vec{d}| \frac{\vec{d} \cdot \vec{c}}{|\vec{d}| |\vec{c}|} = \frac{\vec{d} \cdot \vec{c}}{|\vec{c}|} = \frac{-31 - 28 - 6}{\sqrt{16 + 16 + 4}}$$

$$= \frac{-66}{6} = -11.$$

Q.27 (1)

$$\vec{a}_1 = \vec{AC} \times \vec{AB} = (\vec{c} - \vec{a}) \times (\vec{b} - \vec{a})$$

$$\vec{a}_2 = \vec{DB} \times \vec{DC} = \vec{b} \times \vec{c}$$



$$\vec{a}_3 = \vec{DC} \times \vec{DA} = \vec{c} \times \vec{a}$$

$$\vec{a}_4 = \vec{DA} \times \vec{DB} = \vec{a} \times \vec{b}$$

Q.28 (4)

$$\vec{a} = (1, 1, 1) \vec{b} = (1, 1, 1)$$

$$\vec{c} = (2, -3, 0)$$

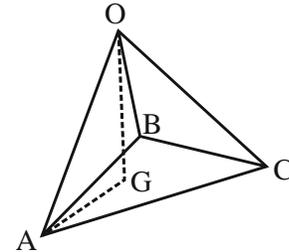
$$\vec{v} = \vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = (-7, 8, -1)$$

$$\vec{v} = \frac{(-7, 8, -1)}{\sqrt{114}}$$

$$\text{Reqd. Vector} = \frac{3}{\sqrt{114}} (-7\hat{i} + 8\hat{j} - \hat{k})$$

Q.29 (3)

$\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$ & $\vec{OC} = \vec{c}$ are unit vectors and equally inclined to each other at an acute angle θ .



\therefore ABC is an equilateral triangle

$$\text{and } AB = \sqrt{OA^2 + OB^2 - 2OA \cdot OB \cdot \cos \theta}$$

\therefore Area of ΔABC

$$= \frac{\sqrt{3}}{4} AB^2 = \frac{\sqrt{3}}{4} \cdot 2(1 - \cos \theta) = \frac{\sqrt{3}}{2} (1 - \cos \theta)$$

If G is the centroid of the ΔABC , then

$$OG = \frac{1}{3} |\vec{a} + \vec{b} + \vec{c}|$$

$$= \frac{1}{3} \sqrt{a^2 + b^2 + c^2 + 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a}}$$

$$= \frac{1}{\sqrt{3}} \sqrt{1 + 2\cos \theta}$$

$\therefore [\vec{a} \vec{b} \vec{c}] = \text{Volume of parallelepiped}$

$$= OG \times 2 \text{ ar}(\Delta ABC)$$

$$= 2 \cdot \frac{1}{\sqrt{3}} \sqrt{1 + 2\cos \theta} \times \frac{\sqrt{3}}{2} (1 - \cos \theta)$$

$$= (1 - \cos \theta) \sqrt{1 + 2\cos \theta}$$

Q.30 (1)

We know, scalar triple product

$$[\vec{a} \vec{b} \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c}) \equiv (\vec{a} \times \vec{b}) \cdot \vec{c}$$

consider $[\vec{a} + \vec{b} \vec{b} + \vec{c} \vec{c} + \vec{a}]$

$$\begin{aligned}
 &= (\vec{a} + \vec{b}) \cdot \{(\vec{b} + \vec{c}) \times (\vec{c} \times \vec{a})\} \\
 &= (\vec{a} + \vec{b}) \cdot \{(\vec{b} \times \vec{c}) + (\vec{b} \times \vec{a}) + (\vec{c} \times \vec{c}) + (\vec{c} \times \vec{a})\} \\
 &= (\vec{a} + \vec{b}) \cdot \{(\vec{b} \times \vec{c}) + (\vec{b} \times \vec{a}) + (\vec{c} \times \vec{a})\} \\
 (\because \vec{c} \times \vec{c} = 0) \\
 &\vec{a} \cdot \{(\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{c} \times \vec{a})\} \\
 &\vec{b} \cdot \{(\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{b} \times \vec{a}) + \vec{b} \cdot (\vec{c} \times \vec{a})\} \\
 &= [\vec{a}\vec{b}\vec{c}] + [\vec{a}\vec{b}\vec{a}] + [\vec{a}\vec{c}\vec{a}] + [\vec{b}\vec{b}\vec{c}] \\
 &= [\vec{b}\vec{b}\vec{a}] + [\vec{b}\vec{c}\vec{a}] \\
 (\text{By definition of scalar triple product}) \\
 &= [\vec{a}\vec{a}\vec{b}] = 0, [\vec{a}\vec{b}\vec{a}] = 0 \text{ and } [\vec{b}\vec{a}\vec{a}] = 0 \\
 &= [\vec{a}\vec{b}\vec{c}] + [\vec{b}\vec{c}\vec{a}] = 2[\vec{a}\vec{b}\vec{c}]
 \end{aligned}$$

- Q.31** (1)
Q.32 (3)
Q.33 (4)
Q.34 (3)
Q.35 (2)
Q.36 (4)

$$\begin{aligned}
 \vec{a} \times (2\vec{b} \times \vec{c}) = \vec{b} &\Rightarrow 2[(\vec{a}\vec{c})\vec{b} - (\vec{a}\vec{b})\vec{c}] = \vec{b} \\
 \Rightarrow \vec{a}\vec{c} = \frac{1}{2} &\quad \& \quad \vec{a}\vec{b} = 0
 \end{aligned}$$

- Q.37** (1)

$$\begin{aligned}
 \vec{a} \parallel (\vec{b} \times \vec{c}) &\Rightarrow \vec{a} = \lambda(\vec{b} \times \vec{c}) \\
 \text{also } (\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c}) &= \begin{vmatrix} \vec{a}\vec{a} & \vec{a}\vec{c} \\ \vec{b}\vec{a} & \vec{b}\vec{c} \end{vmatrix} = \begin{vmatrix} \vec{a}\vec{a} & 0 \\ 0 & \vec{b}\vec{c} \end{vmatrix} \\
 &= |\vec{a}|^2 (\vec{b}\vec{c})
 \end{aligned}$$

- Q.38** (2)

$$[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a} \vec{b} \vec{c}]^2 \neq 0$$

$$[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a} \vec{b} \vec{c}] \neq 0$$

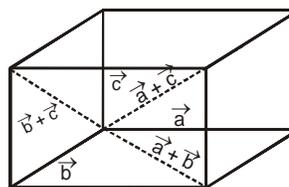
- Q.39** (3)

$$\begin{aligned}
 \vec{r} \cdot (\vec{a} + \vec{b} + \vec{c}) &= 0 \\
 \Rightarrow \ell[\vec{a} \vec{b} \vec{c}] + m[\vec{a} \vec{b} \vec{c}] + n[\vec{a} \vec{b} \vec{c}] &= 0 \\
 \text{or } (\ell + m + n) [\vec{a} \vec{b} \vec{c}] &= 0 \\
 \text{or } \ell + m + n &= 0
 \end{aligned}$$

- Q.40** (1)

$$V_{\text{old}} = [\vec{a} \vec{b} \vec{c}]$$

$$V_{\text{New}} = [\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = 2[\vec{a} \vec{b} \vec{c}]$$



so $m = 2$

- Q.41** (3)

$$\begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix} = [\vec{a} \vec{b} \vec{c}]^2 = 4^2 = 16$$

- Q.42** (3)

$$\begin{aligned}
 \vec{a}, \vec{b}, \vec{c} \text{ are non-coplanar} &\Rightarrow [\vec{a} \vec{b} \vec{c}] \neq 0 \\
 &[\vec{a} + 2\vec{b} + 3\vec{c} \quad \lambda\vec{b} + 4\vec{c} \quad (2\lambda - 1)\vec{c}] \\
 &= (\vec{a} + 2\vec{b} + 3\vec{c}) \cdot [(\lambda\vec{b} + 4\vec{c}) \times (2\lambda - 1)\vec{c}] \\
 &= \lambda(2\lambda - 1) (\vec{a} + 2\vec{b} + 3\vec{c}) \cdot (\vec{b} \times \vec{c}) \\
 &= \lambda(2\lambda - 1) [\vec{a} \vec{b} \vec{c}]
 \end{aligned}$$

- Q.43** (3)

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\text{Given } \vec{c}\vec{a} = 0 \quad \& \quad \vec{c}\vec{b} = 0 = \begin{vmatrix} \vec{a}\vec{a} & \vec{a}\vec{b} & \vec{a}\vec{c} \\ \vec{b}\vec{a} & \vec{b}\vec{b} & \vec{b}\vec{c} \\ \vec{c}\vec{a} & \vec{c}\vec{b} & \vec{c}\vec{c} \end{vmatrix}$$

$$= \begin{vmatrix} |a|^2 & |a||b| \cos \frac{\pi}{6} & 0 \\ |a||b| \cos \frac{\pi}{6} & |b|^2 & 0 \\ 0 & 0 & |c|^2 \end{vmatrix}$$

$$= |c|^2 [|a|^2 + |b|^2 - |a|^2 |b|^2 \cos^2 \frac{\pi}{6}]$$

$$= |c|^2 |a|^2 |b|^2 \left[1 - \frac{3}{4} \right] = \frac{1}{4} |c|^2 |a|^2 |b|^2$$

$$= \frac{1}{4} (a_1^2 + a_2^2 + a_3^2) (b_1^2 + b_2^2 + b_3^2)$$

Q.44 (1)

Assume $\vec{b} = \hat{i}$, $\vec{c} = \hat{j}$ and $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{a} \cdot \vec{b} = 1, \vec{a} \cdot \vec{c} = 1$$

$$\vec{b} + \vec{c} + \vec{k} = \hat{i} + \hat{j} + \hat{k} = \vec{a}.$$

Q.45 (1)

$$\begin{vmatrix} m & m+1 & m+8 \\ m+3 & m+4 & m+5 \\ m+6 & m+7 & m+8 \end{vmatrix}$$

$$R_1 \rightarrow R_2 - R_1$$

$$R_2 \rightarrow R_2 - R_3$$

$$= \begin{vmatrix} 3 & 3 & -3 \\ -3 & -3 & -3 \\ m+6 & m+7 & m+8 \end{vmatrix} = -$$

$$9 \begin{vmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ m+6 & m+7 & m+8 \end{vmatrix}$$

$$= -9[m+8-m-7] - 1[m+8-m-6] - 1[m+7-m-6]$$

$$= -9 - 2 - 1 = -12.$$

Q.46 (3)

$$\vec{a} = \hat{i} + \hat{j} \quad \vec{b} = 2\hat{i} - \hat{k}$$

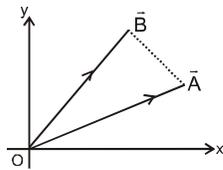
$$\vec{r} \times \vec{a} = \vec{b} \times \vec{a} \quad \dots(i)$$

$$\vec{r} \times \vec{b} = \vec{a} \times \vec{b} \quad \dots(ii)$$

Add (i) & (ii)

$$\vec{r} \times (\vec{a} \times \vec{b}) = 0 \Rightarrow \vec{r} = (\vec{a} + \vec{b}) = (3, 1, -1)$$

Q.47 (3)



$$\vec{OA} \times \vec{OB} = \text{a fixed vector}$$

$$\Rightarrow |\vec{OA} \times \vec{OB}| = \text{const. number}$$

$$\Rightarrow \Delta OAB = \text{const.}$$

$\Rightarrow B$ is on the line \parallel to base OA

Q.48 (3)

Let D is of c on line

$$AC = \sqrt{(1)^2 + (-2)^2 + (1)^2}$$

$AD = \text{proj. of } AC \text{ on } AD$

$$= \frac{1(6) + (-2)(-3) + 1(2)}{7}$$

$$AD = 2$$

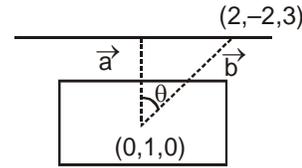
So shortest distance $(CD)^2 = (AC)^2 - (AD)^2 = 6 - 4 = 2$

$$CD = \sqrt{2}.$$

Q.49 (3)

$$\vec{r} = (2, -2, 3) + \lambda(1, -1, 4)$$

$$\vec{r} \cdot (1, 5, 1) = 5$$



$$\vec{L} \cdot \vec{n} = 1 - 5 + 4 = 0$$

So line and plane are parallel.

Let a point on the plane $(0, 1, 0)$

$$\text{distance} = |b| \cos \theta$$

$$= |b| \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{2 - 15 + 3}{\sqrt{27}} = \frac{10}{3\sqrt{3}}$$

Q.50 (2)

Q.51 (4)

Q.52 (3)

Q.53 (1)

Q.54 (2)

Q.55 (2)

Equation of Altitude or plane is

$$\vec{r} = i - 2\hat{i} + 2\hat{k} + \lambda(2\hat{i} + 3\hat{j})$$

Let a point of line

$$x = 1 + 2\lambda$$

$$y = -2 + 3\lambda$$

$$z = 2 - 2\lambda$$

Put there point in the of plane

$$((1 + 2\lambda)\hat{i} + (-2 + 3\lambda)\hat{j} + (2 - 2\lambda)\hat{k})$$

$$(2\hat{i} + 3\hat{j} - 2\hat{k}) + 312 = 0$$

and find l

Put the value of l and get two point

Be cause Intersection point is midpoint of Required point and given point

Q.56 (1)

$$(2\hat{i} + \hat{j} + 2\hat{k}) \cdot (3\hat{i} - 2\hat{j} - m\hat{k}) = 0$$

$$= 6 - 2 - 2m = 0 \text{ or } m = 2$$

Q.57 (1)

$$\text{Normal Vector } \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 5(\hat{i} - \hat{j} - \hat{k})$$

Let $\vec{A} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$. If θ is the angle between vector \vec{A} and plane then $90 - \theta$ will be the angle between normal and plane

$$\cos(90 - \theta) = \frac{5\alpha - 5\beta - 5\gamma}{5\sqrt{3}\sqrt{\alpha^2 + \beta^2 + \gamma^2}}$$

$$\sin^2\theta = \frac{(\alpha - \beta - \gamma)^2}{3(\alpha^2 + \beta^2 + \gamma^2)} \Rightarrow \boxed{\beta\gamma = \alpha(\beta + \gamma)}$$

Q.58 (2)

$$\vec{A} \cdot \vec{X} = C$$

$$\vec{A} \times \vec{X} = \vec{B}$$

take cross with \vec{A}

$$\vec{A} \times (\vec{A} \times \vec{X}) = \vec{A} \times \vec{B}$$

$$\vec{X} = \frac{C\vec{A} - \vec{A} \times \vec{B}}{|\vec{A}|^2}$$

Q.59 (3)

$$\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C} \text{ and } \vec{A} \times \vec{B} = \vec{A} \times \vec{C}$$

$$\vec{A} \times \vec{B} = \vec{A} \times \vec{C}$$

$$\vec{A} \times (\vec{A} \times \vec{B}) = \vec{A} \times (\vec{A} \times \vec{C})$$

$$(\vec{A} \cdot \vec{B})\vec{A} - |\vec{A}|^2 \vec{B} = (\vec{A} \cdot \vec{C})\vec{A} - |\vec{A}|^2 \vec{C}$$

$$\vec{B} = \vec{C}$$

EXERCISE-III

Q.1 0002

$$[\vec{r}_1 \vec{r}_2 \vec{r}_3] = 0$$

$$\Rightarrow \begin{vmatrix} \lambda x & 1 & 0 \\ 2 & 1 & 1 \\ 2 & 1 & x \end{vmatrix} = 0$$

$$\Rightarrow \lambda x(x-1) + 1(2-2x) = 0$$

$$\Rightarrow \lambda x^2 - (\lambda+2)x + 2 = 0$$

$$\therefore D = 0$$

$$(\lambda+2)^2 - 8\lambda = 0$$

$$\Rightarrow \lambda = 2$$

Q.2 0003

Given that $\vec{a} \vec{b} \vec{c}$ are non coplanar

$$\therefore [\vec{a} \vec{b} \vec{c}] \neq 0$$

$$\text{Also } \vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a} \vec{b} \vec{c}]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a} \vec{b} \vec{c}]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a} \vec{b} \vec{c}]}$$

.....(i)

$$\text{Now, } (\vec{a} + \vec{b}) \cdot \vec{p} + (\vec{b} + \vec{c}) \cdot \vec{q} + (\vec{c} + \vec{a}) \cdot \vec{r}$$

$$= \frac{[\vec{a} \vec{b} \vec{c}]}{[\vec{a} \vec{b} \vec{c}]} + \frac{[\vec{a} \vec{b} \vec{c}]}{[\vec{a} \vec{b} \vec{c}]} + \frac{[\vec{a} \vec{b} \vec{c}]}{[\vec{a} \vec{b} \vec{c}]}$$

$$= 1 + 1 + 1 = 3$$

Q.3 1.67

The adjacent sides are

$$\frac{1}{2}(4\hat{i} - 2\hat{j} + 2\hat{k}) = 2\hat{i} - \hat{j} + \hat{k}$$

$$\text{and } \frac{1}{2}(2\hat{i} + 4\hat{j} - 6\hat{k}) = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$\text{Area} = \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 2 & -3 \end{vmatrix} \right| = \hat{i} + 7\hat{j} + 5\hat{k} \text{ magnitude}$$

$$= \sqrt{1+49+25} = \sqrt{75} = 5\sqrt{3}$$

Q.4 14

$$\text{Projection of } \vec{a} \times \vec{b} \text{ of } \vec{c} = \frac{(\vec{a} \times \vec{b}) \cdot \vec{c}}{|\vec{c}|}$$

$$= \frac{|\vec{a} \cdot \vec{b} \cdot \vec{c}|}{\sqrt{9+16+144}}$$

$$= \frac{1}{13} [\vec{a} \cdot \vec{b} \cdot \vec{c}] = \frac{1}{13} \begin{vmatrix} 3 & -6 & -1 \\ 1 & 4 & -3 \\ 3 & -4 & 12 \end{vmatrix}$$

$$= \frac{-182}{13} = -14$$

Q.5

2

Since \vec{a} and \vec{b} are collinear

$$\therefore \vec{a} = m\vec{b}$$

$$\hat{i} - \hat{j} = m(-2\hat{i} + \hat{j})$$

$$\hat{i} - \hat{j} = -2m\hat{i} + m\hat{j}$$

$$\therefore -2m = 1 \text{ and } m = -1$$

$$m = \frac{-1}{2}$$

$$\therefore \frac{-1}{2}k = -1$$

$$\therefore k = 2$$

Q.6

0.67

Let $\vec{p} = \vec{a} + (x+1)\vec{b}$, $\vec{q} = (2x-3)\vec{b} - \vec{a}$

\vec{p} and \vec{q} are collinear

$$\vec{p} = k\vec{q}$$

$$\vec{a} + (x+1)\vec{b} = k[-\vec{a} + (2x-3)\vec{b}]$$

$$1 = -k \quad \text{and} \quad (x+1) = (2x-3)k$$

$$k = -1 \quad \text{and} \quad (x+1) = (2x-3)(-1)$$

$$3x = 2$$

$$\therefore x = 2/3$$

Q.7

2

Since the vector

$\vec{i} + 3\vec{j} - 2\vec{k}$, $2\vec{i} - \vec{j} + 4\vec{k}$ and $3\vec{i} + 2\vec{j} + x\vec{k}$ are coplanar.

$$\therefore \begin{vmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 3 & 2 & x \end{vmatrix} = 0$$

$$1(-x-8) - 3(2x-12) - 2(4+3) = 0$$

$$-x-8-6x+36-14=0$$

$$-7x+14=0$$

$$x=2$$

Q.8

0.167

$\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$ and $\vec{b} = 2\vec{i} + 2\vec{j} + \vec{k}$

$$\vec{a} = \sqrt{4+1+1} = \sqrt{6}$$

$$\vec{b} = \sqrt{1+4+1} = \sqrt{6}$$

$$\vec{a} \cdot \vec{b} = (2)(1) + (-1)(2) + (1)(1) = 1$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

$$\cos \theta = \frac{1}{6}$$

Q.9

103

$$\vec{c} = \vec{a} + 3\vec{b}$$

$$\vec{c} \cdot \vec{c} = (\vec{a} + 3\vec{b}) \cdot (\vec{a} + 3\vec{b})$$

$$|\vec{c}|^2 = \vec{a} \cdot \vec{a} + 3\vec{a} \cdot \vec{b} + 3\vec{a} \cdot \vec{b} + 9\vec{b} \cdot \vec{b}$$

$$|\vec{c}|^2 = \vec{a} \cdot \vec{a} + 6\vec{a} \cdot \vec{b} + 9|\vec{b}|^2$$

$$|\vec{c}|^2 = |\vec{a}|^2 + 6|\vec{a}||\vec{b}|\cos \frac{\pi}{3} + 9|\vec{b}|^2$$

$$|\vec{c}|^2 = 4 + 6(2)(3)\frac{1}{2} + 9(9)$$

$$|\vec{c}|^2 = 85 + 18$$

Q.10

74

$$(\alpha \times \beta) = -10\vec{i} + 9\vec{j} + 7\vec{k}$$

$$(\alpha \times \gamma) = 4\vec{i} - 3\vec{j} - \vec{k}$$

$$(\alpha \times \beta) \cdot (\alpha \times \gamma) = (-10)(4) + (9)(-3) + (7)(-1)$$

$$= -40 - 27 - 7$$

$$= -74$$

PREVIOUS YEAR'S

MHT CET

Q.1 (4)	Q.2 (4)	Q.3 (1)	Q.4 (3)	Q.5 (4)
Q.6 (4)	Q.7 (4)	Q.8 (2)	Q.9 (3)	Q.10 (3)
Q.11 (2)	Q.12 (2)	Q.13 (1)	Q.14 (3)	Q.15 (2)
Q.16 (3)	Q.17 (4)	Q.18 (1)	Q.19 (2)	Q.20 (3)
Q.21 (1)	Q.22 (4)	Q.23 (3)	Q.24 (1)	Q.25 (2)
Q.26 (1)	Q.27 (2)	Q.28 (3)	Q.29 (3)	Q.30 (1)
Q.31 (2)	Q.32 (1)	Q.33 (3)	Q.34 (3)	Q.35 (4)
Q.36 (1)	Q.37 (4)	Q.38 (2)	Q.39 (2)	Q.40 (1)
Q.41 (3)	Q.42 (2)	Q.43 (3)	Q.44 (1)	Q.45 (4)
Q.46 (1)	Q.47 (1)	Q.48 (2)	Q.49 (4)	Q.50 (3)
Q.51 (2)	Q.52 (2)	Q.53 (4)	Q.54 (1)	Q.55 (4)
Q.56 (2)	Q.57 (1)	Q.58 (4)	Q.59 (3)	Q.60 (2)
Q.61 (3)	Q.62 (4)			

Q.63

(1)
Let point A = (2,3,-1) and point B = (-2, -4, 3).
Now the position vector of line joining A and B,
AB = OB - OA

$$= -2\hat{i} - 4\hat{j} + 3\hat{k} - 2\hat{i} - 3\hat{j} + \hat{k}$$

$$= -4\hat{i} - 7\hat{j} + 4\hat{k}$$

Again let a = AB = -4\hat{i} - 7\hat{j} + 4\hat{k}

and b = 4\hat{i} - 3\hat{j} + \hat{k}

Then, a · b = (-4\hat{i} - 7\hat{j} + 4\hat{k}) · (4\hat{i} - 3\hat{j} + \hat{k})

$$= -16 + 21 + 4 = 9$$

$$|a| = \sqrt{(-4)^2 + (-7)^2 + (4)^2}$$

$$= \sqrt{16 + 49 + 16} = 9$$

Now projection of b on a

$$= \frac{(a \cdot b)}{|a|} = \frac{9}{9} = 1$$

Q.64

(2)

$$a = 3\hat{i} - 2\hat{j} + \hat{k} \quad \dots(i)$$

$$b = 2\hat{i} - 4\hat{j} - 3\hat{k} \quad \dots(ii)$$

Multiplying by 2 both sides, we get

$$2b = 4\hat{i} - 8\hat{j} - 6\hat{k} \quad \dots(iii)$$

Subtracting Eq. (iii) from Eq. (i), we get

$$a - 2b = (3\hat{i} - 2\hat{j} + \hat{k}) - (4\hat{i} - 8\hat{j} - 6\hat{k})$$

$$= -\hat{i} + 6\hat{j} + 7\hat{k}$$

$$= \sqrt{(-1)^2 + (6)^2 + (7)^2} = \sqrt{86}$$

Q.65

(2)

Let a = 2\hat{i} - \hat{j} + \hat{k}, b = \hat{i} + 2\hat{j} - 3\hat{k}, c = 3\hat{i} + \lambda\hat{j} + 5\hat{k}

Now given vectors a,b,c will be coplanar if

$$a \cdot (b \times c) = 0, \text{ i.e. } [a \ b \ c] = 0$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

$$\Rightarrow 2(10 + 3\lambda) + 1(5 + 9) + 1(1 - 6) = 0$$

$$\Rightarrow 7\lambda = -28 = 0 \Rightarrow \lambda = -4$$

Q.66

(2)

We know that area of triangle = \frac{1}{2} |OA \times OB|

$$\Delta^2 = \frac{1}{4} |a \times b| \quad \dots(i)$$

Now |a \times b|^2 + (a \cdot b)^2

$$= |a|^2 |b|^2 \sin^2 \theta + |a|^2 |b|^2 \cos^2 \theta$$

$$= |a|^2 |b|^2 (\sin^2 \theta + \cos^2 \theta)$$

$$= |a|^2 |b|^2 \times 1 = |a|^2 |b|^2$$

$$\Rightarrow |a \times b|^2 = |a|^2 |b|^2 - (a \cdot b)^2$$

From Eq. (i), we get

$$\Delta^2 = \frac{1}{4} [|a|^2 |b|^2 - (a \cdot b)^2]$$

$$\Delta^2 = \frac{1}{2} \sqrt{|a|^2 |b|^2 - (a \cdot b)^2}$$

Q.67

(1)

$$r = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \quad \dots(i)$$

$$r = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}) \quad \dots(ii)$$

Compare with vector equation r = a + \lambda b

$$a_1 = \hat{i} + \hat{j} = 2\hat{i} - \hat{j} + \hat{k}$$

$$a_2 = 2\hat{i} + \hat{j} - \hat{k}, b_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

$$d = \frac{|(b_1 \times b_2) \cdot (a_2 - a_1)|}{|b_1 \times b_2|} \quad \dots(iii)$$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$b_1 \times b_2 = \hat{i}(-2 + 5) - \hat{j}(4 - 3) + \hat{k}(-10 + 3)$$

$$= 3\hat{i} - \hat{j} - 7\hat{k}$$

$$|b_1 \times b_2| = \sqrt{(3)^2 + (-1)^2 + (-7)^2}$$

$$\text{Also } a_2 - a_1 = (2\hat{i} + \hat{j} - \hat{k}) - (\hat{i} + \hat{j}) = \hat{i} - \hat{k} \quad \dots(iv)$$

$$d = \frac{|(3\hat{i} - \hat{j} - 7\hat{k}) \cdot (\hat{i} - \hat{k})|}{\sqrt{59}} = \frac{|3 - 0 + 7|}{\sqrt{59}} = \frac{10}{\sqrt{59}}$$

Q.68

(1)

Let vector r be coplanar to a and b.

$$r = a + tb$$

$$\text{Given, } a = \hat{i} + 2\hat{j} + \hat{k}, b = \hat{i} - \hat{j} + \hat{k}$$

$$\text{and } c = \hat{i} + \hat{j} - \hat{k}$$

$$\Rightarrow r = (\hat{i} + 2\hat{j} + \hat{k}) + t(\hat{i} - \hat{j} + \hat{k})$$

$$= (1+t)\hat{i} + (2-t)\hat{j} + (1+t)\hat{k}$$

The projection of r along with c = 1/\sqrt{3}

(given)

$$\Rightarrow \frac{r \cdot c}{|c|} = \frac{1}{\sqrt{3}} \Rightarrow (2-t) = \pm 1 \Rightarrow t = 1 \text{ or } 3$$

When $t = 1$, we have $r = 2\hat{i} + \hat{j} + 2\hat{k}$

when $t = 3$, we have $r = 4\hat{i} + \hat{j} + 4\hat{k}$

Q.69

(3)

$$\therefore [u \ v \ w] = |u \ (v \times w)|$$

$$\therefore v \times w = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix}$$

$$= \hat{i}(3 \cdot 0) - \hat{j}(6 + 1) + \hat{k}(0 - 1) \\ = 3\hat{i} - 7\hat{j} - \hat{k}$$

$$\text{Now, } |u \cdot (3\hat{i} - 7\hat{j} - \hat{k})| = |u| \sqrt{59} \cos \theta$$

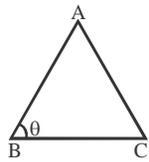
$$\therefore \text{Maximum } [u \ v \ w] = \sqrt{59} \quad [\because |u| = 1, \cos \theta \leq 1]$$

Q.70

(1)

Let position vectors of ΔABC are

$$A = 4\hat{i} - 2\hat{j}, B = \hat{i} + 4\hat{j} - 3\hat{k}, C = -\hat{i} + 5\hat{j} + \hat{k}$$



Let $\angle ABC = q$

$$\Rightarrow \text{BA} \cdot \text{BC} = |\text{BA}| |\text{BC}| \cos q$$

$$\Rightarrow \cos q = \frac{\text{BA} \cdot \text{BC}}{|\text{BA}| |\text{BC}|}$$

$$\text{BA} = \text{OA} - \text{OB} = (4\hat{i} - 2\hat{j}) - (\hat{i} + 4\hat{j} - 3\hat{k}) \\ = 3\hat{i} - 6\hat{j} + 3\hat{k}$$

$$\text{BC} = \text{OC} - \text{OB} = (-\hat{i} + 5\hat{j} + \hat{k}) - (\hat{i} + 4\hat{j} - 3\hat{k}) \\ = 2\hat{i} + \hat{j} + 4\hat{k}$$

$$\text{BA} \cdot \text{BC} = -6 - 6 + 12 = 0$$

$$\text{Now, } \cos q = \frac{\text{BA} \cdot \text{BC}}{|\text{BA}| |\text{BC}|} = 0 \quad [\because \text{BA} \cdot \text{BC} = 0]$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

Q.71

(3)

Given lines are

$$\frac{x}{2} = \frac{y}{-1} = \frac{z}{2} \Rightarrow \frac{x-0}{2} = \frac{y-0}{-1} = \frac{z-0}{2}$$

$$\text{or } r = 0\hat{i} + 0\hat{j} + 0\hat{k} + \lambda(2\hat{i} - \hat{j} + 2\hat{k}) \quad \dots(i)$$

$$\text{and } \frac{x-1}{2} = \frac{y-1}{-1} = \frac{z-1}{2}$$

$$\Rightarrow r = (\hat{i} + \hat{j} + \hat{k}) + (2\hat{i} - \hat{j} + 2\hat{k}) \quad \dots(ii)$$

Now, distance between lines (i) and (ii) is given by

$$d = \frac{|\mathbf{b} \times (\mathbf{a}_2 - \mathbf{a}_1)|}{|\mathbf{b}|} \quad \dots(iii)$$

$$\text{Now, } \mathbf{a}_2 - \mathbf{a}_1 = \hat{i} + \hat{j} + \hat{k}$$

$$\text{So, } \mathbf{b} \times (\mathbf{a}_2 - \mathbf{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 3\hat{i} + 3\hat{k}$$

$$\text{and } |\mathbf{b}| = \sqrt{4+1+4} = 3$$

$$\therefore \text{From Eq. (iii), } d = \frac{|-3\hat{i} + 3\hat{k}|}{3} = \frac{3\sqrt{2}}{3} = \sqrt{2}$$

JEE MAIN

Q.1

(4)

Area of parallelogram

$$|\vec{a} \times \vec{b}| = \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -1 \\ -2 & \alpha & 1 \end{vmatrix} \right| = \sqrt{15(\alpha^2 + 4)}$$

$$= \left\| (2 + \alpha)\hat{i} - (\alpha - 2)\hat{j} + (\alpha^2 + 4)\hat{k} \right\| = \sqrt{15(\alpha^2 + 4)}$$

$$= \sqrt{(\alpha + 2)^2 + (\alpha - 2)^2 + (\alpha^2 + 4)^2} = \sqrt{15(\alpha^2 + 4)}$$

$$= \alpha^4 + 10\alpha^2 + 24 = 15\alpha^2 + 60$$

$$= \alpha^4 - 5\alpha^2 - 36 = 0$$

$$= (\alpha^2 - 9)(\alpha^2 + 4) = 0$$

$$= \alpha^2 = 9$$

$$2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$$

$$|\vec{a}|^2 = \alpha^2 + 4 + 1 = \alpha^2 + 5$$

$$|\vec{b}|^2 = \alpha^2 + 4 + 1 = \alpha^2 + 5$$

$$\vec{a} \cdot \vec{b} = -2\alpha + 2\alpha - 1 = -1$$

$$\therefore 2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2 = 2(\alpha^2 + 5) - 1(\alpha^2 + 5) = \alpha^2 + 5 = 14$$

Q.2

(3)

$$(\vec{a} \times (2\hat{i} + \hat{k})) \times \left(3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k} \right)$$

$$= (2\hat{i} - 13\hat{j} - 4\hat{k}) \times \left(3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k} \right)$$

$$-(6+2)\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -13 & -4 \\ 3 & \frac{1}{2} & 2 \end{vmatrix}$$

$$\vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$$

Projection of \vec{a} on vector $2\vec{i} + 2\vec{j} + \vec{k}$ is

$$\vec{a} \cdot \frac{(2\hat{i} + 2\hat{j} + 2\hat{k})}{3} = \frac{5}{3}$$

Q.3

(1)

$$\frac{\vec{a} \cdot \vec{c}}{|\vec{c}|} = \frac{10}{3}$$

$$\Rightarrow \frac{\alpha + 6 + 2}{\sqrt{1+4+4}} = \frac{10}{3} \Rightarrow \alpha = 2$$

$$\text{and } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -\beta & 4 \\ 1 & 2 & -2 \end{vmatrix} = 6\hat{i} + \hat{j} + \hat{k}$$

$$\Rightarrow 2\beta - 8 = -6 \Rightarrow \beta = 1$$

$$\Rightarrow \alpha + \beta = 3$$

Q.4

(28)

$$P_1 : \vec{r} \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = 4$$

$$P_1 : 2x + y - 3z = 4$$

$$P_2 : \begin{vmatrix} x-2 & y+3 & z-2 \\ 0 & 1 & -5 \\ -1 & -1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow -5x + 5y + z + 23 = 0$$

16, α , β be the dr's of line of intersection

$$2x + y - 3z = 4$$

$$-5x + 5y + z = -23$$

for dr's of line intersection

$$\frac{x}{1+15} = \frac{-y}{2-15} = \frac{z}{10+5} = \lambda$$

$$\Rightarrow x = 16\lambda, y = 13\lambda, z = 15\lambda = \alpha = 13 \text{ \& } \beta = 15$$

$$\alpha + \beta = 13 + 15 = 28$$

Q.5

(1)

$$\vec{AB} \parallel \vec{AC} \text{ if } \frac{1}{2} = \frac{\alpha-4}{-6} = \frac{1}{2}$$

$$\Rightarrow \alpha = 1$$

$\vec{a}, \vec{b}, \vec{c}$ are non-collinear if $\alpha = 2$

Which is smallest positive integer

Given points B(2, 2, 4), C(3, -2, 5)

$$\text{mid point of BC} = \left(\frac{5}{2}, 0, \frac{9}{2} \right)$$

$$\text{length of median, AM} = \sqrt{\frac{9}{4} + 16 + \frac{9}{4}} = \sqrt{\frac{82}{2}}$$

Q.6

(0)

$$\hat{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\hat{b} = \hat{i} + \hat{j} + \hat{k}$$

$$\hat{a} \cdot \hat{b} = 1 - 2 + 3 = 2 \quad \dots \text{(i)}$$

$$\text{Given } \hat{a} + (\hat{b} \times \hat{c}) = 0$$

$$\hat{a} = -(\hat{b} \times \hat{c})$$

$$\Rightarrow \hat{a} \perp \hat{b}$$

$$\text{So, } \hat{a}, \hat{b} = 0$$

(ii)

Equation (i) and (ii) are contradicting

Q.7

(3)

$$\text{Given } |\vec{a}| = |\vec{b}| = 1 \text{ \& } \vec{c} \wedge \hat{a} \text{ is } \frac{\pi}{12}$$

$$\text{also } \vec{b} = \vec{c} + 2(\vec{c} \times \hat{a})$$

$$\vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{c} + 2\vec{c} \cdot (\vec{c} \times \vec{a})$$

$$|\vec{b} - \vec{c}|^2 = 4|\vec{c} \times \hat{a}|^2 \quad \vec{b} \cdot \vec{c} = |\vec{c}|^2 + 0$$

$$|\vec{b}|^2 + |\vec{c}|^2 - 2\vec{b} \cdot \vec{c} = 4\left\{ |\vec{c}|^2 |\vec{a}|^2 \sin^2 \frac{\pi}{12} \right\} \quad \vec{b} \cdot \vec{c} = |\vec{c}|^2$$

$$1 + |\vec{c}|^2 - 2(\vec{b} \cdot \vec{c}) = 4\left\{ |\vec{c}|^2 (1) \left(\frac{\sqrt{3}-1}{2\sqrt{2}} \right)^2 \right\}$$

$$1 + |\vec{c}|^2 - 2|\vec{c}|^2 = 4|\vec{c}|^2 \frac{(\sqrt{3}-1)^2}{8}$$

$$1 - |\vec{c}|^2 = \frac{|\vec{c}|^2 (\sqrt{3}-1)^2}{2}$$

$$2 - 2|\vec{c}|^2 = |\vec{c}|^2 (\sqrt{3}-1)^2$$

$$\left[(\sqrt{3}-1)^2 + 2 \right] |\vec{c}|^2 = 2$$

$$(3+1+2-2\sqrt{3})|\vec{c}|^2 = 2$$

$$|\vec{c}|^2 = \frac{1}{3-\sqrt{3}} \Rightarrow |6\vec{c}|^2 = 36 \left(\frac{1}{3-\sqrt{3}} \right)$$

$$= 36 \left(\frac{3 + \sqrt{3}}{9 - 3} \right) = 6(3 + \sqrt{3})$$

Q.8 (3)

$$(\hat{a} + \hat{b})^2 + 4|\hat{a} \times \hat{b}|^2 + 4(\hat{a} + \hat{b}) \cdot (\hat{a} \times \hat{b}) = 4$$

$$\begin{aligned} [1 + 1 + 2\hat{a} \cdot \hat{b}] + 4[\sin^2\theta] + 0 &= 4 \\ (2 + 2\cos\theta) + 4\sin^2\theta &= 4 \\ 4\sin^2\theta + 2\cos\theta - 2 &= 0 \\ 2(1 - \cos^2\theta) + \cos\theta - 1 &= 0 \\ 2\cos^2\theta - \cos\theta - 1 &= 0 \\ (2\cos\theta + 1)(\cos\theta - 1) &= 0 \end{aligned}$$

$$\cos\theta = \frac{-1}{2}$$

$$\therefore \theta = \frac{2\pi}{3}$$

Q.9 (3)

$$\begin{aligned} P_1 + \lambda P_2 &= 0 \\ (x + 3y - z - 5) + \lambda(2x - y + z - 3) &= 0 \\ \text{It passes through } (2, 1, -2) & \\ 2 + \lambda(-2) = 0 &\Rightarrow \lambda = 1 \\ \text{So, equation of required plane is} & \\ P: 3x + 2y - 8 = 0 & \end{aligned}$$

$$(A) \left. \begin{aligned} X &= (1, -2, 4) \\ X + Y &= (6, -3, 6) \end{aligned} \right\} P|_X \cdot P|_{X+Y} = (3 - 4 - 8)(18 - 6 - 8) < 0$$

\Rightarrow X & X + Y are on opposite side

$$(B) \left. \begin{aligned} Y &= (5, -1, 2) \\ Y - X &= (4, 1, -2) \end{aligned} \right\} P|_Y \cdot P|_{Y-X} = (15 - 2 - 8)(12 + 2 - 8) > 0$$

\Rightarrow Y & Y are on same side

$$(C) \left. \begin{aligned} X &= (1, -2, 4) \\ Y &= (5, -1, 2) \end{aligned} \right\} P|_X \cdot P|_Y = (3 - 4 - 8)(15 - 2 - 8) < 0$$

\Rightarrow X & Y are on opposite side

$$(D) \left. \begin{aligned} X + Y &= (6, -3, 6) \\ X - Y &= (-4, -1, 2) \end{aligned} \right\} \Rightarrow P|_{X+Y} \cdot P|_{X-Y} = (18 - 6 - 8)(-12 - 2 - 8) < 0$$

\Rightarrow X + Y & X - Y are on opposite side

Q.10 [14]

$$\vec{a} \times \vec{b} = 13\hat{i} - \hat{j} - 4\hat{k}$$

$$\vec{b} \times (\vec{a} \times \vec{b}) = \vec{b} \times (13\hat{i} - \hat{j} - 4\hat{k})$$

$$\vec{a} |\vec{b}|^2 - (\vec{a} \cdot \vec{b})\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & \lambda \\ 13 & -1 & -4 \end{vmatrix}$$

$$(\lambda^2 + 2)\vec{a} + 21\vec{b} = \hat{i}(\lambda - 4) - \hat{j}(-4 - 13\lambda) + \hat{k}(-14)$$

Now, let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{a} \times \vec{b} = 13\hat{i} - \hat{j} - 4\hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 1 & \lambda \end{vmatrix} = (\lambda y - z)\hat{i} - \hat{j}(\lambda x - z) + \hat{k}(x - y) = 13\hat{i} - \hat{j} - 4\hat{k}$$

$$\lambda y - z = 13, z - \lambda x = -1, x - y = -4$$

$$\lambda(y - x) = 12 \Rightarrow \lambda = 3$$

Put in (1)

$$11\vec{a} + 21\vec{b} = -\hat{i} + 43\hat{j} - 14\hat{k}$$

$$11\vec{a} + 21\hat{i} + 21\hat{j} + 21\lambda\hat{k} = -\hat{i} + 43\hat{j} - 14\hat{k}$$

$$11\vec{a} + 21\hat{i} + 21\hat{j} + 63\hat{k} = -\hat{i} + 43\hat{j} - 14\hat{k}$$

$$11\vec{a} + 21\hat{i} - 22\hat{j} + 77\hat{k} = 0$$

$$\vec{a} = -2\hat{i} + 2\hat{j} - 7\hat{k}$$

$$(\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{j}) + (\vec{a} + \vec{b}) \cdot (\hat{i} - \hat{k})$$

$$\begin{aligned} &= [(1, 1, 3) - (-2, 2, -7)] \cdot (\hat{k} - \hat{j}) + [(1, 1, 3) + (-2, 2, -7)] \cdot (\hat{i} - \hat{k}) \\ &= [(3\hat{i} - \hat{j} + 10\hat{k}) \cdot (\hat{k} - \hat{j})] + [(-\hat{i} + 3\hat{j} - 4\hat{k}) \cdot (\hat{i} - \hat{k})] \\ &= (1 + 10) + (-1 + 4) = 14 \end{aligned}$$

Q.11 (2)

$$L_1: \vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} - \hat{k})$$

$$L_2: \vec{r} = (\alpha\hat{i} - \hat{j}) + \mu(2\hat{i} - 3\hat{k})$$

Normal vector to both lines

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix}$$

$$= \hat{i}(-9) - \hat{j}(2) + \hat{k}(-6)$$

$$\vec{n} = -9\hat{i} - 2\hat{j} - 6\hat{k}$$

equation of plane with normal vector \vec{n} and containing point (1, -1, 1)

$$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow -9x - 2y - 6z = -9 + 2 - 6$$

$$\Rightarrow 9x + 2y + 6z - 13 = 0$$

Point $(\alpha, -1, 0)$ on plane

$$9\alpha - 2 - 13 = 0 \Rightarrow \alpha = \frac{5}{3}$$

$$\text{distance of plane from } \left(\frac{5}{3}, 0, 0 \right)$$

$$d = \frac{|9\left(\frac{5}{3}\right) - 13|}{\sqrt{81 + 4 + 36}} = \frac{2}{11}$$

Q.12 (4)

$$\vec{v} = x\vec{a} + y\vec{b}$$

$$\vec{v} = (x+2y)\hat{i} + (x-3y)\hat{j} + (2x+y)\hat{k}$$

$$\frac{\vec{v} \cdot \vec{c}}{|\vec{c}|} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \frac{(x+2y) \cdot 1 + (x-3y)(-1) + (2x+y) \cdot 1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow 2 = 2x + 6y$$

$$\Rightarrow x + 3y = 1$$

.....(i)

$$\vec{v} \cdot \hat{j} = 7 \Rightarrow x - 3y = 7 \quad \text{.....(ii)}$$

(i) - (ii)

$$6y = -6 \Rightarrow y = -1$$

and $x = 4$

$$\vec{v} = 2\hat{i} + 7\hat{j} + 7\hat{k}$$

$$\vec{v} \cdot (\hat{i} + \hat{k}) = 2 + 7 = 9$$

Q.13

(1)

$$\vec{a} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{b} \times \vec{c} = \vec{a}$$

$$|\vec{b}| \in \{1, 2, \dots, 10\}$$

$$\therefore \vec{b} \times \vec{c} = \vec{a}$$

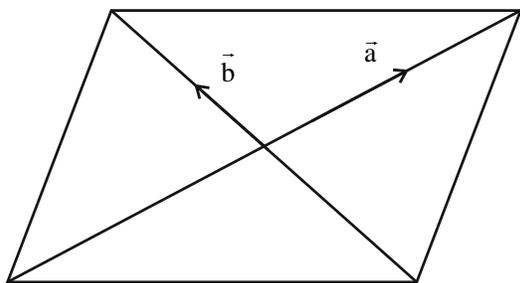
$\Rightarrow \vec{a}$ is perpendicular to \vec{b} as well as \vec{a} is perpendicular to \vec{c}

$$\text{Now } \vec{a} \cdot \vec{c} = 2 - 3 - 2 = -3 \neq 0$$

This $\vec{b} \times \vec{c} = \vec{a}$ is not possible.

No. of vectors $\vec{b} = 0$

Q.14 (4)



$$\text{Area} = \frac{1}{2} |\vec{a} \times \vec{b}| = 2\sqrt{2} \Rightarrow |\vec{a} \times \vec{b}| = 4\sqrt{2}$$

$$|\vec{a}| = 1 \text{ and } |\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$$

$$\Rightarrow \cos\theta = \sin\theta$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

$$\therefore |\vec{a} \times \vec{b}| = 4\sqrt{2} \Rightarrow |\vec{a}| |\vec{b}| \sin \frac{\pi}{4} = 4\sqrt{2}$$

$$\Rightarrow |\vec{b}| = 8$$

$$|\vec{c}| = \sqrt{(2\sqrt{2})^2 + |\vec{a} \times \vec{b}|^2 + (2|\vec{b}|)^2} = 16\sqrt{2}$$

$$\text{Now, } \vec{b} \cdot \vec{c} = -2|\vec{b}|^2$$

$$\Rightarrow 8 \times 16\sqrt{2} \times \cos\alpha = -2.64$$

$$\Rightarrow \cos\alpha = -\frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{3\pi}{4}$$

Q.15 (2)

$$\vec{a} = \lambda\hat{i} + \lambda\hat{j} + \lambda\hat{k}$$

$$\text{also } \frac{\vec{a} \cdot (3\hat{i} + 4\hat{j})}{5} = 7 \Rightarrow \frac{7\lambda}{5} = 7 \Rightarrow \lambda = 5$$

$$\therefore \vec{a} = 5\hat{i} + 5\hat{j} + 5\hat{k}$$

$$\text{Let } \vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{a} \cdot \vec{b} = 0 \Rightarrow x + y + z = 0 \quad \text{---(1)}$$

$$\text{also } \begin{vmatrix} x & y & z \\ 5 & 5 & 5 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow -y(-5) + z(-5) = 0$$

$$\Rightarrow y = z$$

From (1), $x = -2y$

$$x^2 + y^2 + z^2 = 25 \times 3$$

$$4y^2 + y^2 + y^2 = 25 \times 3$$

$$6y^2 = 25 \times 3$$

$$y = \frac{5}{\sqrt{2}} = z \Rightarrow x = \frac{-10}{\sqrt{2}}$$

$$\text{So, } \left| \frac{\vec{b} \cdot (3\hat{i} + 4\hat{j})}{5} \right| = \left| \frac{3x + 4y}{5} \right| = \left| \frac{-30 + -20}{\sqrt{2} + \sqrt{2}} \right| = \left| \frac{-10}{5\sqrt{2}} \right| = |-\sqrt{2}| = \sqrt{2}$$

Q.16 (576)

$$|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})|^2 + 4(\vec{a} \cdot \vec{b})^2$$

$$|-\vec{b} \times \vec{a} + \vec{a} \times \vec{b}|^2 + 4(\vec{a} \cdot \vec{b})^2$$

$$= 4a^2b^2 \sin^2\theta + 4a^2b^2 \cos^2\theta$$

$$= 4a^2b^2 = 4 \times 16 \times 9 = 576$$

Q.17 (1)

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = 3\vec{b} - \vec{c}$$

$$\vec{b} \times (\vec{c} \times \vec{a}) = (\vec{b} \cdot \vec{a})\vec{c} - (\vec{b} \cdot \vec{c})\vec{a} = \vec{c} - 2\vec{a}$$

$$\vec{c} \times (\vec{b} \times \vec{a}) = (\vec{c} \cdot \vec{a})\vec{b} - (\vec{c} \cdot \vec{b})\vec{a} = 3\vec{b} - 2\vec{a}$$

$$[3\vec{b} - \vec{c} \cdot \vec{c} - 2\vec{a}, 3\vec{b} - 2\vec{a}]$$

$$(3\vec{b} - \vec{c}) \cdot [3(\vec{c} \times \vec{b}) - 2(\vec{c} \times \vec{a}) - 6(\vec{a} \times \vec{b})]$$

$$-6(\vec{b} \cdot \vec{c} \cdot \vec{a}) + 6[\vec{c} \cdot \vec{a} \cdot \vec{b}]$$

Q.18 (150)

$$\vec{a} \cdot \vec{c} = 5 \Rightarrow 2c_1 + c_2 + 3c_3 = 5 \quad \dots (1)$$

$$\vec{b} \cdot \vec{c} = 0 \Rightarrow 3c_1 + 3c_2 + c_3 = 0 \quad \dots (2)$$

$$\text{and } [\vec{a} \ \vec{b} \ \vec{c}] = 0 \Rightarrow \begin{vmatrix} c_1 & c_2 & c_3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 8c_1 - 7c_2 - 3c_3 = 0 \quad \dots (3)$$

By solving (1), (2), (3) we get

$$c_1 = \frac{10}{122}, c_2 = \frac{-85}{122}, c_3 = \frac{225}{122}$$

$$\therefore 122(c_1 + c_2 + c_3) = 150$$

Q.19 [Bonus]

$\therefore \vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vector

$$|\vec{a} \times \vec{b}| = 4|\vec{c}| \Rightarrow |\vec{a}||\vec{b}| = 4|\vec{c}| \quad \dots (i)$$

$$\text{Similarly} \Rightarrow |\vec{b}||\vec{c}| = 9|\vec{a}| \quad \dots (ii)$$

$$\Rightarrow |\vec{c}| \|\vec{a}\| = \alpha \|\vec{b}\| \quad \dots (iii)$$

Multiply (i), (ii) & (iii) $|\vec{a}| |\vec{b}| |\vec{c}| = 36\alpha$

$$\therefore |\vec{c}| = 3\sqrt{\alpha} \quad |\vec{a}| = 2\sqrt{\alpha}, |\vec{b}| = 6$$

$$\therefore a + b + c = 36 \Rightarrow 5\sqrt{\alpha} + 6 = 36$$

$$5\sqrt{\alpha} = 30$$

$$\alpha = 36$$

Q.20 (1)

$$\vec{n}_1 = \hat{i} \times (\hat{i} + \hat{j}) = \hat{k}$$

$$\vec{n}_2 = (\hat{i} + \hat{k}) \times (\hat{i} - \hat{j}) = \hat{i} + \hat{j} - \hat{k}$$

Since line is parallel to both planes, then line is parallel

to $\vec{n}_1 \times \vec{n}_2$

$$\vec{n}_1 \times \vec{n}_2 = \hat{k} \times (\hat{i} + \hat{j} - \hat{k}) = -\hat{i} + \hat{j}$$

$$\text{D.R. of } \vec{a} = -\hat{i} + \hat{j}$$

$$\text{D.R. of } \vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{-3}{\sqrt{2} \times 3}$$

$$\theta = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$

Q.21 (2)

For angle to be acute

$$\vec{u} \cdot \vec{v} > 0$$

$$\Rightarrow a(\log_e b)^2 - 12 + 6a(\log_e b) > 0$$

$$\forall b > 1$$

let $\log_e b = t \Rightarrow t > 0$ as $b > 1$

$$y = at^2 + 6at - 12 \text{ and } y > 0, \forall t > 0$$

$$\Rightarrow a \in \phi$$

Q.22 (3)

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$-\vec{a} = \vec{b} + \vec{c}$$

$$|\vec{a}|^2 = |\vec{b} + \vec{c}|^2$$

$$a^2 = b^2 + c^2 + 2\vec{b} \cdot \vec{c}$$

$$c^2 = 72 - 12 - 24 = 36$$

$$|\vec{c}| = 6$$

$$\text{S-I: } \left| (\vec{a} \times \vec{b}) + (\vec{c} \times \vec{b}) \right| - |\vec{c}|$$

$$= |(\vec{a} + \vec{c}) \times \vec{b}| - |\vec{c}|$$

$$= |-\vec{b} \times \vec{b}| - |\vec{c}|$$

$$= -|\vec{c}|$$

$$= -6 \neq 6(2\sqrt{2} - 1) \quad \text{S - I False}$$

$$\text{S-II: } \vec{a} + \vec{b} + \vec{c} = 0$$

$$\vec{a} + \vec{b} = -\vec{c}$$

$$= |\vec{a} + \vec{b}|^2 - |\vec{c}|^2$$

$$= |72 + 12 + 24\sqrt{6} \cos(\pi - \theta)|$$

$$= -\cos \theta = \frac{-2}{\sqrt{6}} \Rightarrow \theta = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right) \quad \text{S-II true}$$

Q.23 (3)

$$|\vec{a}| |\vec{b}| |\vec{c}| = 14$$

$$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \wedge \vec{a} = \theta = \frac{2\pi}{3}$$

$$\text{So, } \vec{a} \cdot \vec{b} = -\frac{1}{2}ab, \vec{b} \cdot \vec{c} = -\frac{1}{2}bc, \vec{c} \cdot \vec{a} = -\frac{1}{2}ac$$

Let

$$(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{b})(\vec{b} \cdot \vec{c}) - (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{b})$$

$$= \frac{1}{4}ab^2c + \frac{1}{2}ab^2c = \frac{3}{4}ab^2c$$

$$\text{Similarly, } (\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) = \frac{3}{4}abc^2$$

$$(\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b}) = \frac{3}{4}a^2bc$$

$$168 = \frac{3}{4}abc(a+b+c)$$

$$\text{So, } (a+b+c) = 16$$

Q.24 (14)

$$|\vec{a} + \vec{b}|^2 = |a|^2 + 2|\vec{b}|^2; \vec{a} \cdot \vec{b} = 3$$

$$\text{As } |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a} + \vec{b}|^2$$

$$|\vec{b}|^2 = 2\vec{a} \cdot \vec{b} = 6$$

$$|\vec{a} \times \vec{b}|^2 = 75$$

$$|\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 = 75$$

$$6|\vec{a}|^2 - 9 = 75 \Rightarrow |\vec{a}|^2 = 14$$

Q.25 (1)

$$\vec{a} = 3\hat{i} + \hat{j}, \vec{b} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\text{As } \vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \lambda \vec{c}$$

$$\Rightarrow \vec{a} \cdot \vec{c}(\vec{b}) - (\vec{a} \cdot \vec{b})\vec{c} = \vec{b} + \lambda \vec{c}$$

$$\Rightarrow \vec{a} \cdot \vec{c} = 1, \vec{a} \cdot \vec{b} = -\lambda$$

$$\Rightarrow (3\hat{i} + \hat{j}) \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -\lambda$$

$$\Rightarrow \lambda = -5$$

Q.26 (1)

$$\hat{a} \wedge \hat{b} = \frac{\pi}{4} = \phi$$

$$\hat{a} \cdot \hat{b} = |\hat{a}| |\hat{b}| \cos \phi$$

$$\hat{a} \cdot \hat{b} = \cos \phi = \frac{1}{\sqrt{2}}$$

$$\cos \theta = \frac{(\hat{a} + \hat{b}) \cdot (\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b}))}{|\hat{a} + \hat{b}| |a + 2\hat{b} + 2(\hat{a} \times \hat{b})|}$$

$$|\hat{a} + \hat{b}|^2 = (\hat{a} + \hat{b}) \cdot (\hat{a} + \hat{b})$$

$$|\hat{a} + \hat{b}|^2 = 2 + 2\hat{a} \cdot \hat{b}$$

$$= 2 + \sqrt{2}$$

$$\hat{a} \times \hat{b} = \frac{\hat{n}}{\sqrt{2}} \text{ When } \hat{n} \text{ is vector } \perp \hat{a} \text{ and } \hat{b}$$

$$\text{Let } \vec{c} = \hat{a} \times \hat{b}$$

We know,

$$\vec{c} \cdot \vec{a} = 0$$

$$\vec{c} \cdot \vec{b} = 0$$

$$|\hat{a} + 2\hat{b} + 2\vec{c}|^2$$

$$= 1 + 4 + \frac{(4)}{2} + 4\hat{a} \cdot \hat{b} + 8\hat{b} \cdot \vec{c} + 4\vec{c} \cdot \hat{a}$$

$$= 7 + \frac{4}{\sqrt{2}} = 7 + 2\sqrt{2}$$

Now,

$$(\hat{a} + \hat{b}) \cdot (\hat{a} + 2\hat{b} + 2\vec{c})$$

$$= |\hat{a}|^2 + 2\hat{a} \cdot \hat{b} + 0 + \hat{b} \cdot \hat{a} + 2|\hat{b}|^2 + 0$$

$$= 1 + \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 2$$

$$= 3 + \frac{3}{\sqrt{2}}$$

$$\cos \theta = \frac{3 + \frac{3}{\sqrt{2}}}{\sqrt{2 + \sqrt{2}} \sqrt{7 + 2\sqrt{2}}}$$

$$\cos^2 \theta = \frac{9(\sqrt{2} + 1)^2}{2(2 + \sqrt{2})(7 + 2\sqrt{2})}$$

$$\cos^2 \theta = \left(\frac{9}{2\sqrt{2}} \right) \frac{(\sqrt{2} + 1)}{(7 + 2\sqrt{2})}$$

$$164 \cos^2 \theta = \frac{(82)(9)(\sqrt{2} + 1)(7 - 2\sqrt{2})}{\sqrt{2}(7 + 2\sqrt{2})(7 - 2\sqrt{2})}$$

$$= \frac{(82)(9)[7\sqrt{2} - 4 + 7 - 2\sqrt{2}]}{\sqrt{2}(41)}$$

$$= (9\sqrt{2})[5\sqrt{2} + 3]$$

$$= 90 + 27\sqrt{2}$$

Q.27 (3)

By its given condition

: $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors

$$\Rightarrow [\vec{a}, \vec{b}, \vec{c}] \neq 0 \quad \dots(i)$$

Now, $\vec{a}, \vec{b}, \vec{c}$

$$\begin{vmatrix} 1+t & 1-t & 1 \\ 1-t & 1+t & 2 \\ t & -t & 1 \end{vmatrix}$$

$$C_2 \rightarrow C_1 + C_2$$

$$= \begin{vmatrix} 1+t & 2 & 1 \\ 1-t & 2 & 2 \\ t & 0 & 1 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1+t & 1 & 1 \\ 1-t & 1 & 2 \\ t & 0 & 1 \end{vmatrix}$$

$$= 2 [(1+t) - (1-t) + t] \\ = 2[3t] = 6t$$

$$[\vec{a}\vec{b}\vec{c}] \neq 0 \Rightarrow t \neq 0$$

Q.28

(2)

$$|\vec{a}| = 9 \& (x\vec{a} + y\vec{b}) \cdot (6y\vec{a} - 18x\vec{b}) = 0$$

$$\Rightarrow 6xy|\vec{a}|^2 - 18x^2(\vec{a}\cdot\vec{b}) + 6y^2(\vec{a}\cdot\vec{b}) - 18xy|\vec{b}|^2 = 0$$

$$\Rightarrow 6xy(|\vec{a}|^2 - 3|\vec{b}|^2) + (\vec{a}\cdot\vec{b})(y^2 - 3x^2) = 0$$

This should hold $\forall x, y \in \mathbb{R} \times \mathbb{R}$

$$\therefore |\vec{a}| = 3|\vec{b}|^2 \& (\vec{a}\cdot\vec{b}) = 0$$

$$\text{Now } |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$$

$$= |\vec{a}|^2 \cdot \frac{|\vec{a}|^2}{3}$$

$$\therefore |\vec{a} \times \vec{b}| = \frac{|\vec{a}|^2}{3} = \frac{81}{\sqrt{3}} = 27\sqrt{3}$$

Q.29

(Bonus)

$$\text{Let } \vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$$

$$\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & \beta \\ 3 & -5 & 4 \end{vmatrix} = -\hat{i} + 9\hat{j} + 12\hat{k}$$

$$\Rightarrow (4 + 5\beta)\hat{i} + (3\beta - 4\alpha)\hat{j} + (-5\alpha - 3)\hat{k}$$

$$= -\hat{i} + 9\hat{j} + 12\hat{k}$$

$$\therefore 4 + 5\beta = -1, 3\beta - 4\alpha = 9, -5\alpha - 3 = 12$$

$$\beta = -1, \alpha = -3$$

$$\therefore \vec{a} = -3\hat{i} + \hat{j} - \hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$$

$$\therefore \vec{a} + \vec{b} = -4\hat{j} + 3\hat{k}$$

$$|\vec{a}|^2 = 11, |\vec{b}|^2 = 50$$

$$\vec{a}\cdot\vec{b} = -9 + (-5) - 4 = -18$$

\therefore Projection of $(\vec{b} - 2\vec{a})$ on $\vec{a} + \vec{b}$ is

$$\frac{(\vec{b} - 2\vec{a}) \cdot (\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|}$$

$$= \frac{|\vec{b}|^2 - 2|\vec{a}|^2 - (\vec{a}\cdot\vec{b})}{|\vec{a} + \vec{b}|} = \frac{50 - 22 - (-18)}{5} = \frac{46}{5}$$

Q.30

(2)

$$\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}, \vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$$

$$((\vec{a} \times \vec{b}) \times \hat{i}) \cdot \hat{k} = \frac{23}{2}, \text{ then } |\vec{b} \times 2\hat{j}| \text{ is}$$

$$((\vec{a}\cdot\hat{i})\vec{b} - (\vec{b}\cdot\hat{i})\vec{a}) \cdot \hat{k} = \frac{23}{2}$$

$$(\vec{a}\cdot\hat{i})(\vec{b}\cdot\hat{k}) - (\vec{b}\cdot\hat{i})(\vec{a}\cdot\hat{k}) = \frac{23}{2}$$

$$2 \times 2 - \alpha \times 5 = \frac{23}{2} \Rightarrow 5\alpha = 4 - \frac{23}{2} \Rightarrow \alpha = \frac{-3}{2}$$

$$\vec{b} \times 2\hat{j} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & \beta & 2 \\ 0 & 2 & 0 \end{vmatrix} = -4\hat{i} + 2\alpha\hat{k}$$

$$\therefore |\vec{b} \times 2\hat{j}| = \sqrt{16 + 4\alpha^2} = \sqrt{16 + 4 \times \frac{9}{4}} = 5$$

Q.31

(4)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & -1 \\ 2 & 1 & -\alpha \end{vmatrix} = (1-\alpha)\hat{i} + (\alpha^2 - 2)\hat{j} + (\alpha - 2)\hat{k}$$

$$\therefore \text{projection } (\vec{a} \times \vec{b}) \cdot \hat{c} = 30$$

$$\frac{-(1-\alpha) + 2(\alpha^2 - 2) - 2(\alpha - 2)}{3} = 30$$

$$\Rightarrow 2\alpha^2 - \alpha = 91$$

$$\text{If } \alpha = 7 \text{ then } 2\alpha^2 - \alpha = 91$$

Q.32

(1)

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$$

$$5 = 6|\vec{b}|^2 - 9$$

$$6|\vec{b}|^2 = 14 \Rightarrow |\vec{b}|^2 = \frac{7}{3}$$

$$\text{Projection of } \vec{b} \text{ on the vector } \vec{a} - \vec{b} = \frac{|\vec{b}\cdot(\vec{a} - \vec{b})|}{|\vec{a} - \vec{b}|}$$

$$= \frac{|\vec{a}\cdot\vec{b} - |\vec{b}|^2|}{\sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}}}$$

$$= \frac{\frac{2}{3}}{\sqrt{7}} = \frac{2}{\sqrt{3}\cdot\sqrt{7}} = \frac{2}{\sqrt{21}}$$

3-DIMENSIONAL GEOMETRY

EXERCISE-I (MHT CET LEVEL)

- Q.1** (4)
 $\frac{-4+1}{5+4} = \frac{2-3}{5-2} = \frac{-2-2}{\lambda+2}$ or $\lambda+2=12$ or $\lambda=10$
- Q.2** (3)
 Distance from origin $= \sqrt{1+4+9} = \sqrt{14}$ and from y-axis $= \sqrt{1+9} = \sqrt{10}$.
- Q.3** (4)
 It is obvious.
- Q.4** (3)
 $d = \sqrt{1+4+1} = \sqrt{6}$
- Q.5** (2)
 From x-axis $= \sqrt{y^2+z^2} = \sqrt{4+9} = \sqrt{13}$
 From y-axis $= \sqrt{1+9} = \sqrt{10}$
 From z-axis $= \sqrt{1+4} = \sqrt{5}$
- Q.6** (3)
 $0 = \frac{a-2+4}{3} \Rightarrow a = -2, 0 = \frac{1+b+7}{3} \Rightarrow b = -8$ and
 $0 = \frac{3-5+c}{3} \Rightarrow c = 2$
- Q.7** (3)
 Check option (3), $\frac{4-(-2)}{-3-4} \neq \frac{-3-4}{-2-(-3)}$
 Therefore, this set of points is non-collinear.
- Q.8** (2)
 Let $A = (1,1,1)$; $B = (-2,4,1)$; $C = (-1,5,5)$ &
 $D = (2,2,5)$ $AB = \sqrt{9+9+0} = 3\sqrt{2}$, $BC = \sqrt{1+1+16} = 3\sqrt{2}$
 and $CD = 3\sqrt{2}$ and $AD = 3\sqrt{2}$. Hence it is a square.
- Q.9** (1)
 $\cos^2 \alpha = 1 - \cos^2 60^\circ - \cos^2 60^\circ = 1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2}$
 $\Rightarrow \alpha = 45^\circ$.
- Q.10** (1)
 Let point C divides the line AB in the ratio $1 : \lambda$
 $\therefore 5 = \frac{9\lambda+3}{\lambda+1} \Rightarrow 4\lambda = 2 \Rightarrow \lambda = \frac{1}{2}$
 Hence required ratio is $2 : 1$.
- Q.11** (4)
 Find angle between the lines PQ and RS, we get that neither $PQ \parallel RS$ nor $PQ \perp RS$. Also $PQ \neq RS$.
- Q.12** (1)
 For D'ratio $(1, -3, 2)$, the direction cosine will be
 $\left(\frac{1}{\sqrt{1+9+4}}, \frac{-3}{\sqrt{1+9+4}}, \frac{2}{\sqrt{1+9+4}} \right)$
 $\Rightarrow \left(\frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}} \right)$
- Q.13** (c)
 Let, l, m and n be the direction cosines,
 Then, $l = \cos \theta, m = \cos \beta, n = \cos \theta$
 we have $l^2 + m^2 + n^2 = 1$
 $\Rightarrow \cos^2 \theta + \cos^2 \beta + \cos^2 \theta = 1$
 $\Rightarrow 2 \cos^2 \theta + 1 - \sin^2 \beta = 1$
 $\Rightarrow 2 \cos^2 \theta - \sin^2 \beta = 0$
 $\Rightarrow 2 \cos^2 \theta - 3 \sin^2 \beta = 0$
 $[\because \sin^2 \beta = 3 \sin^2 \theta (\text{given})]$
 $\Rightarrow \tan^2 \theta = 2/3$
 $\therefore \cos^2 \theta = \frac{1}{1 + \tan^2 \theta} = \frac{1}{1 + 2/3} = \frac{3}{5}$
- Q.14** (c)
Q.15 (a)
Q.16 (2)
 Distance from x-axis $= \sqrt{y^2+z^2} = \sqrt{(b^2+c^2)}$
- Q.17** (1)
 $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
 $\Rightarrow \cos \gamma = \sqrt{1 - \left(\frac{14}{15}\right)^2 - \left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9} - \left(\frac{196}{225}\right)} = \pm \frac{2}{15}$
- Q.18** (2)
 Direction ratios are $l = 4 - (-2) = 6, m = 3 - 1 = 2$ and
 $n = -5 + 8 = 3$
- Q.19** (4)
 It is obvious.
- Q.20** (2)
 Since $\alpha = \beta = \gamma \Rightarrow \cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$

$$\Rightarrow \alpha = \cos^{-1}\left(\pm \frac{1}{\sqrt{3}}\right)$$

So, there are four lines whose direction cosines are

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right),$$

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$$

Q.21 (b)

If (l_1, m_1, n_1) and (l_2, m_2, n_2) are the direction ratios then angle between the lines is

$$\cos\theta = \frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\sqrt{l_1^2 + m_1^2 + n_1^2} \sqrt{l_2^2 + m_2^2 + n_2^2}}$$

Here $l_1 = 1, m_1 = 1, n_1 = 1$ and $l_2 = 1, m_2 = -1, n_2 = n$ and $\theta = 60^\circ$

$$\therefore \cos 60^\circ = \frac{1 \times 1 + 1 \times (-1) + 1 \times n}{\sqrt{1^2 + 1^2 + 1^2} \times \sqrt{1^2 + 1^2 + n^2}}$$

$$\Rightarrow \frac{1}{2} = \frac{h^2}{\sqrt{3}\sqrt{2+n^2}} \Rightarrow 3(2+n^2) = 4n^2$$

$$\Rightarrow n^2 = 6 \Rightarrow n = 2\sqrt{6}$$

Q.22 (3)

Q.23 (d)

Q.24 (b)

Q.25 (1)

$$\theta = \cos^{-1}\left(\frac{3+0-5}{\sqrt{1+1}\sqrt{9+16+25}}\right)$$

$$= \cos^{-1}\left(\frac{-2}{\pm 10}\right) = \cos^{-1}\left(\frac{1}{5}\right)$$

Q.26 (1)

Trick : Both lines are satisfied by $(-1, -1, -1)$

Q.27 (1)

$$x - ay - b = 0 \text{ and } cy - z + d = 0$$

If l, m, n are direction cosines, then $\frac{l}{a} = \frac{m}{1} = \frac{n}{c}$.

Hence direction ratios are $(a, 1, c)$.

Q.28 (2)

Required distance

$$= \sqrt{(5-1)^2 + (4-0)^2 + (-1-0)^2} \left[\frac{(5-1)2 + (4-0)9 + (-1-0)5}{\sqrt{4+81+25}} \right]^2$$

$$\sqrt{33 - \frac{39 \times 39}{110}} = \sqrt{\frac{2109}{110}}$$

Q.29 (3)

Projection of $[(1, 2, 3) - (6, 7, 7)]$ along line

$$= \frac{-15-10+8}{\sqrt{17}} = \frac{-17}{\sqrt{17}}$$

$$\text{Distance} = \sqrt{(5^2 + 5^2 + 4^2) - 17} = \sqrt{49} = 7$$

Q.30 (2)

Eliminating n , we have $(2l+m)(l-m) = 0$

$$\text{When } 2l+m=0, \text{ then } \frac{l}{1} = \frac{m}{-2} = \frac{n}{1}$$

$$\text{When } l-m=0, \text{ then } \frac{l}{1} = \frac{m}{1} = \frac{n}{-2}$$

Direction ratios are $1, -2, 1$ and $1, 1, -2$.

$$\cos\theta = \frac{\sum a_1 a_2}{\sqrt{(\sum a_1^2)} \cdot \sqrt{(\sum a_2^2)}} = -\frac{1}{2} \Rightarrow \theta = 120^\circ = \frac{2\pi}{3}$$

Q.31 (3)

The perpendicular distance of $(2, 4, -1)$ from the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} \text{ is}$$

$$\{(2+5)^2 + (4+3)^2 + (-1-6)\}$$

$$\left[\frac{1(2+5) + 4(4+3) - 9(-1-6)}{\sqrt{1+16+81}} \right]^2 \}^{1/2} = \sqrt{147 - \left(\frac{98}{\sqrt{98}}\right)^2}$$

$$= \sqrt{147 - 98} = \sqrt{49} = 7$$

Q.32 (4)

$$\theta = \cos^{-1}\left(\frac{(2)(1) + (2)(2) + (-1)(2)}{\sqrt{2^2 + 2^2 + 1^2} \sqrt{1^2 + 2^2 + 2^2}}\right) = \cos^{-1} \frac{4}{9}$$

Q.33 (4)

$$\text{Since } 2(1) + 2(2) + (-2)(3) = 0.$$

Hence lines are intersecting at right angles.

Q.34 (3)

$$\cos\theta = \frac{1(\sqrt{3}-1) - 1(\sqrt{3}+1) + 2 \times 4}{\sqrt{6}\sqrt{24}} = \frac{6}{12} \Rightarrow \theta = 60^\circ$$

Q.35 (1)

If l, m, n are direction ratios of line, then by

$$Al + Bm + Cn = 0$$

$$\text{For } x - y + z - 5 = 0, l - m + n = 0 \dots(i)$$

$$\text{For } x - 3y - 6 = 0, l - 3m + 0n = 0 \dots(ii)$$

$$\text{or } \frac{l}{0+3} = \frac{m}{1-0} = \frac{n}{-3+1} \text{ or } \frac{l}{3} = \frac{m}{1} = \frac{n}{-2}$$

Direction ratios are $(3, 1, -2)$.

Note : Option (3), may also be an answer but best answer is because in (3) direction cosines are written.

Q.36 (4)
Change the given equation in standard form, we get,

$$\frac{x + \frac{1}{3}}{-1} = \frac{y + \frac{2}{3}}{2} = \frac{z}{-1}$$

So direction cosine are $\left(\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

Q.37 (1)
Line joining the points (3, 5, -7) and (-2, 1, 8) is,

$$\frac{x-3}{(-2)-(3)} = \frac{y-5}{(1)-(5)} = \frac{z-(-7)}{8-(-7)}$$

$$\frac{x-3}{-5} = \frac{y-5}{-4} = \frac{z+7}{15} = K, \text{ (Let) } \dots\dots(i)$$

$$\therefore x = -5K + 3, y = -4K + 5, z = 15K - 7$$

\therefore Line (i) meets the yz-plane

$$\therefore -5K + 3 = 0 \Rightarrow K = 3/5$$

Put the value of K in x, y, z

So the required point is (0, 13/5, 2).

Q.38 (1,4)
The direction cosines of the normal to the plane are

$$\frac{1}{\sqrt{1^2 + 2^2 + 3^2}}, \frac{2}{\sqrt{1^2 + 2^2 + 3^2}}, \frac{-3}{\sqrt{1^2 + 2^2 + 3^2}}$$

$$\text{i.e., } \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}$$

But $x + 2y - 3z + 4 = 0$ can be written as $-x - 2y + 3z - 4 = 0$.

Thus the direction cosines are

$$-\frac{1}{\sqrt{4}}, \frac{-2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$$

Q.39 (c)
the point (4,2,k) on the line also lies on the plane $2x - 4y + z - 7$

Q.40 (b)
If the given plane contains the given line then the normal to the plane, must be perpendicular to the line and the condition for the same is $a \ell + bm + cn = 0$

Q.41 (a)
The angle between two planes is the angle between their normals. From the equation of the planes, the normal vectors are

$$\vec{N}_1 = 2\hat{i} + \hat{j} - 2\hat{k} \text{ and } \vec{N}_2 = 3\hat{i} - 6\hat{j} - 2\hat{k}$$

$$\text{Therefore, } \cos \theta = \frac{|\vec{N}_1 \cdot \vec{N}_2|}{|\vec{N}_1| |\vec{N}_2|}$$

$$= \frac{|(2\hat{i} + \hat{j} - 2\hat{k}) \cdot (3\hat{i} - 6\hat{j} - 2\hat{k})|}{\sqrt{4+1+4}\sqrt{9+36+4}} = \left(\frac{4}{21}\right)$$

$$\text{Hence, } \theta = \cos^{-1}\left(\frac{4}{21}\right)$$

- Q.42** (d)
- Q.43** (c)
- Q.44** (d)
- Q.45** (c)
- Q.46** (b)
- Q.47** (2)

$$\theta = \cos^{-1}\left[\frac{6+4-10}{\sqrt{50}\sqrt{9}}\right] = \cos^{-1}(0) = \frac{\pi}{2}$$

Q.48 (1)

Obviously, $(x - 2) + 5(y + 3) - 6(z - 1) = 0$

$$\Rightarrow x + 5y - 6z + 19 = 0$$

Q.49 (3)

$$\text{Ratio} = -\left[\frac{-2-8+21-17}{3+10+24-17}\right] = -\left(\frac{-6}{20}\right) = \frac{3}{10}$$

Q.50 (1)

Obviously, $3 \times 4 + (-2) \times 3 + 2 \times (-k) = 0$

$$\Rightarrow 12 - 6 - 2k = 0 \Rightarrow k = 3.$$

Q.51 (2)

Equation of required plane is, $\frac{x}{1} + \frac{y}{1} + \frac{z}{1} = 1$

$$\Rightarrow x + y + z = 1$$

Q.52 (4)

The plane will be $x + 2y + 4z = 2 \times 1 + 3 \times 2 + 4 \times 4$

$$\text{or } x + 2y + 4z = 24.$$

Q.53 (1)

If plane $x - 3y + 5z = d$ passes through point (1, 2, 4).

$$\text{Then } 1 - 6 + 20 = d \Rightarrow d = 15$$

$$\therefore \text{Plane, } x - 3y + 5z = 15, \Rightarrow \frac{x}{15} + \frac{y}{-5} + \frac{z}{3} = 1$$

Hence length of intercept cut by it on the axes (x, y, z) are respectively (15, -5, 3).

Q.54 (2)

$$D = \left| \frac{2+6+10-9}{\sqrt{1+4+4}} \right| = 3$$

Q.55 (1)

The equation of the plane through the intersection of the plane $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is

$$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$$

$$\text{or } (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z + 4\lambda - 1 = 0$$

Since the plane parallel to x-axis,

$$\therefore 1 + 2\lambda = 0 \Rightarrow \lambda = -\frac{1}{2}$$

Hence, the required equation will be $y - 3z + 6 = 0$.

Q.56

(1)
The given line is perpendicular to z-axis. Hence parallel to xy-plane.

Q.57

(1)
 $4(2) - 2(3) - 1(2) = 0$

\therefore Line is parallel to the plane.

Q.58

(4)
 $a(x-4) + b(y-3) + c(z-2) = 0$

$\therefore a + b + 2c = 0$ and $a - 4b + 5c = 0$

$$\frac{a}{5+8} = \frac{b}{2-5} = \frac{c}{-4-1} = k$$

$$\frac{a}{13} = \frac{b}{-3} = \frac{c}{-5} = k$$

Therefore, the required equation of plane is
 $-13x + 3y + 5z + 33 = 0$

Q.59

(3)
According to question, $4x - 7y + 3z = k$ (i)

Also, plane (i) passes through $\left(\frac{-1+3}{2}, \frac{2-5}{2}, \frac{3+6}{2}\right)$,

then $4 + \frac{21}{2} + \frac{27}{2} = k \Rightarrow k = 28$

Therefore, required equation is $4x - 7y + 3z = 28$.

Q.60

(4)
Any point on the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = t$ is

$$(3t+2, 4t-1, 12t+2)$$

This lies on $x - y + z = 5$

$$\therefore 3t+2 - 4t+1 + 12t+2 = 5 \quad \text{i.e., } 11t = 0 \Rightarrow t = 0$$

\therefore Point is $(2, -1, 2)$. Its distance from $(-1, -5, -10)$ is,

$$= \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = \sqrt{9+16+144} = 13$$

Q.61

(1)
Required plane is, $7(x+10) - 3(y-5) - (z-4) = 0$ or
 $7x - 3y - z + 89 = 0$.

Q.62

(4)
Let the ratio be $k : 1$. Since on xy-plane $z = 0$

$$\Rightarrow \frac{-bk+c}{k+1} = 0 \Rightarrow k = \frac{c}{b}. \text{ Hence the ratio is } c : b.$$

Aliter : Using formula, required ratio is $-\frac{c}{-b} = c : b$.

Q.63

(2)
Angle between the plane and line is

$$\sin \theta = \frac{aa' + bb' + cc'}{\sqrt{a^2 + b^2 + c^2} \sqrt{a'^2 + b'^2 + c'^2}}$$

Here, $aa' + bb' + cc' = 2 \times 3 + 3 \times 2 - 4 \times 3 = 0$

$$\therefore \sin \theta = 0 \Rightarrow \theta = 0^\circ.$$

EXERCISE-III

Q.1

(2)
 $x^2 + y^2 + y^2 + z^2 + z^2 + x^2 = 36$
 $2(x^2 + y^2 + z^2) = 36$

$$\sqrt{x^2 + y^2 + z^2} = 3\sqrt{2}$$

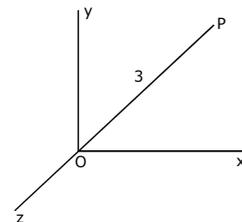
Q.2

(3)
 $PA^2 - PB^2 = 2k^2$
 $(x-3)^2 + (y-4)^2 + (z-5)^2 - (x+1)^2 - (y-3)^2 - (z+7)^2 = 2k^2$
 $\Rightarrow 8x + 2y + 24z + 9 + 2k^2 = 0$

Q.3

(2)
 $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
 $\alpha + \beta = 90^\circ$
 $\sin^2 \beta + \cos^2 \beta + \cos^2 \gamma = 1$
 $\alpha = 90^\circ - \beta$
 $\cos^2 \gamma = 0 \quad \cos \alpha = \sin \beta$
 $\therefore \gamma = 90^\circ$

Q.4

(1)


D.R. of OP = $(1, -2, -2)$

$$\text{D.C. of OP} = \left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right)$$

$$\text{Vector } \vec{OP} = |\vec{OP}| \left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right) = (1, -2, -2)$$

Q.5

(1)
Dr's of AB = $1, -3 - \alpha, 0$
Dr's of CD = $3 - \beta, 2, -2$
AB \perp CD
 $\therefore 1(3 - \beta) + (-3 - \alpha) \cdot 2 + 0 = 0$
 $3 - \beta - 6 - 2\alpha = 0$
 $2\alpha + \beta + 3 = 0$
 $\therefore \alpha = -1, \beta = -1$

- Q.6** (b) let the components of the line vector be a, b, c, then

$$a^2 + b^2 + c^2 = (63)^2 \dots(1)$$

$$\text{also } \frac{a}{3} = \frac{b}{-2} = \frac{c}{6} = \lambda(\text{say}), \text{ then } a = 3\lambda$$

$$b = 2\lambda \text{ and } c = 6\lambda \text{ and from (i) we have}$$

$$9\lambda^2 + 4\lambda^2 + 36\lambda^2 = (63)^2$$

$$\Rightarrow 49\lambda^2 = (63)^2$$

$$\Rightarrow \lambda = \pm \frac{63}{7} = \pm 9$$

Since $a = 3\lambda < 0$ as the line makes an obtuse

Angle with x-axis, $\lambda = -9$ and the required

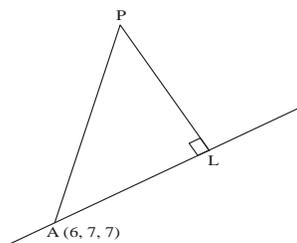
Components are -27, 18, -54.

- Q.7** (a)

The point A (6, 7, 7) is on the line. Let the perpendicular from P meet the line in L. Then

$$AP^2 = (6-1)^2 + (7-2)^2 + (7-3)^2 = 66$$

Also AL = projection of AP on line



$$\left(\text{actual d.c.'s } \frac{3}{\sqrt{17}}, \frac{2}{\sqrt{17}}, \frac{-2}{\sqrt{17}} \right)$$

$$\Rightarrow (6-1) \cdot \frac{3}{\sqrt{17}} + (7-2) \cdot \frac{2}{\sqrt{17}}$$

$$+ (7-3) \frac{-2}{\sqrt{17}} = \sqrt{17}$$

$\therefore \perp$ distance d of P from the line is given by

$$d^2 = AP^2 - AL^2 = 66 - 17 = 49$$

so that $d = 7$

- Q.8** (b)

The direction ratios of the line are $3-2, -4-(-3), -5-1$ i.e. 1, -1, -6

Hence equation of the line joining the given points

$$\text{is } \frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{-6} = r(\text{say})$$

Coordinates of any point on this line are $(r+2, -r-3, -6r+1)$

If this point lies on the given plane $2x+y+z=7$, then $2(r+2) + (-r-3) + (-6r+1) = 7$

$$\Rightarrow r = -1$$

Coordinates of any point on this line are $(-1+2, -(-1)-3, -6(-1)+1)$ i.e. (1, -2, 7)

- Q.9** (a)

Equation of the line through the given points is

$$\frac{x-3}{5-3} = \frac{y-4}{1-4} = \frac{z-1}{6-1}$$

$$\Rightarrow \frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$$

Any point on this line can be taken as

$$(3+2\lambda, 4-3\lambda, 1+5\lambda)$$

If this point lies on XY-plane then the z-coordinate is zero

$$\Rightarrow 1+5\lambda = 0 \Rightarrow \lambda = -\frac{1}{5}$$

Thus the required coordinates of the point are

$$\left(3 - \frac{2}{5}, 4 - 3\left(-\frac{1}{5}\right), 0 \right) \equiv \left(\frac{13}{5}, \frac{23}{5}, 0 \right)$$

- Q.10** (a)

The equations of given lines can be written as

$$L_1 : x-5 = \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2 : x-\alpha = \frac{y}{-1} = \frac{z}{2-\alpha}$$

Since, these lines are coplanar.

$$\text{Therefore, } \begin{vmatrix} 5-\alpha & 0-0 & 0-0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$$\Rightarrow (5-\alpha)(3-\alpha)(2-\alpha) - 2 = 0$$

$$\Rightarrow (5-\alpha)(6-3\alpha-2\alpha+\alpha^2-2)$$

$$\Rightarrow (5-\alpha)(\alpha^2-5\alpha+4) = 0$$

$$\Rightarrow (5-\alpha)(\alpha-1)(\alpha-4) = 0$$

$$\Rightarrow \alpha = 1, 4, 5$$

Q.11 (d)
Given points are A (k, 1, -1), B (2k, 0, 2) and C (2 + 2k, k, 1)

Let r_1 = length of line

$$AB = \sqrt{(2k - k)^2 + (0 - 1)^2 + (2 + 1)^2}$$

$$= \sqrt{k^2 + 10}$$

and r_2 = length of line

$$BC = \sqrt{(2)^2 + k^2 + (-1)^2}$$

$$= \sqrt{k^2 + 5}$$

Now, let ℓ_1, m_1, n_1 be direction-cosines of line AB and ℓ_2, m_2, n_2 be the direction cosines of BC.

Since AB is perpendicular to BC.

$$\therefore \ell_1 \ell_2 + m_1 m_2 + n_1 n_2 = 0$$

$$\text{Now, } \ell_1 = \frac{k}{\sqrt{k^2 + 10}}, m_1 = \frac{-1}{\sqrt{k^2 + 10}}$$

$$n_1 = \frac{3}{\sqrt{k^2 + 10}}$$

$$\text{and } \ell_2 = \frac{2}{\sqrt{k^2 + 5}}, m_2 = \frac{k}{\sqrt{k^2 + 5}}, n_2 = \frac{-1}{\sqrt{k^2 + 5}}$$

$$\text{So, } \ell_1 \ell_2 + m_1 m_2 + n_1 n_2 = 0$$

$$\Rightarrow \frac{2k}{\sqrt{k^2 + 10}\sqrt{k^2 + 5}} - \frac{k}{\sqrt{k^2 + 10}\sqrt{k^2 + 5}}$$

$$- \frac{3}{\sqrt{k^2 + 10}\sqrt{k^2 + 5}} = 0$$

$$\Rightarrow 2k - k - 3 = 0$$

$$\Rightarrow k = 3$$

For $k = 3$, AB is perpendicular to BC.

Q.12

(c)

Equation of the first line L_1 is

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and that of the second line}$$

$$L_2 \text{ is } \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} \text{ Clearly, these lines are}$$

not parallel (the ratios of D.R. are not equal).

Any point P on the first line is $(1 + 2\lambda, 2 + 3\lambda, 3 + 4\lambda)$ and any point Q on the second line is $(4 + 5\mu, 1 + 2\mu, \mu)$. If these two points P and Q are identical then.

$$1 + 2\lambda = 4 + 5\mu \quad \dots(1)$$

$$2 + 3\lambda = 1 + 2\mu \quad \dots(2)$$

$$3 + 4\lambda = \mu \quad \dots(3)$$

From (2) and (3) we get $\lambda = \mu = -1$, which also satisfies (1), Thus the two lines L_1 and L_2 intersect and the coordinates of the point of intersection are $(-1, -1, -1)$.

Q.13 (b)

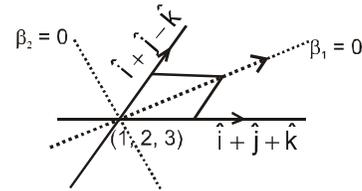
Q.14 (a)

Q.15 (1)

Dr's of bisector

$$\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} + \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}} = \lambda(\hat{i} + \hat{j})$$

Hence Dr's are $\lambda, \lambda, 0 (\lambda \in \mathbb{R})$



Equation of bisector

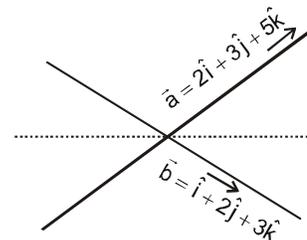
$$\frac{x-1}{\lambda} = \frac{y-2}{\lambda} = \frac{z-3}{0}$$

$$\frac{x-1}{2} = \frac{y-2}{2}; z-3=0$$

Q.16 (3)

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{5} \quad \dots(i)$$

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} \quad \dots(ii)$$



$$\hat{a} + \hat{b} = \frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{\sqrt{38}} + \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$$

\Rightarrow (1) and (2) will be incorrect

Let the dr's of line \perp to (1) and (2) be a, b, c

$$\Rightarrow 2a + 3b + 5c = 0 \quad \dots(iii)$$

$$\text{and } a + 2b + 3c = 0 \quad \dots(iv)$$

$$\therefore \frac{a}{9-10} = \frac{b}{5-6} = \frac{c}{4-3}$$

$$\Rightarrow \frac{a}{-1} = \frac{b}{-1} = \frac{c}{1}$$

$$\Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{-1}$$

\therefore equation of line passing through $(0, 0, 0)$ and is \perp to the lines (1) and (2) is

$$\frac{x}{1} = \frac{y}{1} = \frac{z}{-1}$$

Ans.

- Q.17** (1)
A (a, b, c) B(a', b', c')

$$\begin{aligned} \text{Line } \vec{AB} &= (a, b, c) + \lambda(a' - a, b' - b, c' - c) \\ &= (a + \lambda a', b + \lambda b', c + \lambda c') - \lambda(a, b, c) \end{aligned}$$

It will pass through origin when
 $a + \lambda a' = b + \lambda b' = c + \lambda c' = 0$

$$\Rightarrow \frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

- Q.18** (2)
 $x = y + a = z \dots(1)$
 $x + a = 2y = 2z \dots(2)$
we have option (2) & (3)
but if we look at option B
it will satisfy the given equation

- Q.19** (2)
 $\vec{a} = (2, 5, -3)$
 $\vec{b} = (-1, 8, 4)$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{-2 + 40 - 12}{\sqrt{38} \sqrt{81}} = \frac{26}{9\sqrt{38}} \Rightarrow \theta =$$

$$\cos^{-1} \left(\frac{26}{9\sqrt{38}} \right)$$

- Q.20** (2)

$$\text{dir}^n \text{ of line} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{vmatrix} = -2\hat{i} + \hat{k}$$

$$\text{DR} \& = (-2, 0, 1)$$

$$(\vec{n}_1 \times \vec{n}_2) \times \hat{k} = (-2\hat{i} + \hat{k}) \times \hat{k} = 2\hat{j}$$

$$\Rightarrow \text{distance} = 2$$

- Q.21** (1)
 $a(x-2) + b(y+3) + c(z-1) = 0$
Dr's of the line joining (3, 4, -1) & (2, -1, 5) are -1, -5, 6

normal of the plane and above line are parallel

$$\begin{aligned} \therefore \text{equation of the plane} \\ -1(x-2) - 5(y+3) + 6(z-1) = 0 \\ x + 5y - 6z + 19 = 0 \end{aligned}$$

- Q.22** (4)
 $(xy + yz) = 0$
 $x + z = 0$ and $y = 0$
Two perpendicular plane.

- Q.23** (b)
Since $3(1) + 2(-2) + (-1)(-1)$
 $= 3 - 4 + 1 = 0$

\therefore Given line is \perp to the normal to the plane i.e., given line is parallel to the given plane.

Also, (1, -1, 3) lies on the plane

$$x - 2y - z = 0 \text{ if}$$

$$1 - 2(-1) - 3 = 0 \text{ i.e., } 1 + 2 - 3 = 0$$

which is true \therefore L lies in plane π

- Q.24** (a)
We know that the length of the perpendicular from the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

and the co-ordinate (α, β, γ) of the foot of the \perp are given by

$$\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{\gamma - z_1}{c}$$

$$= \left(\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2} \right) \dots(1)$$

In the given ques., $x_1 = 7, y_1 = 14, z_1 = 5,$
 $a = 2, b = 4, c = -1, d = -2$

By putting these values in (1), we get

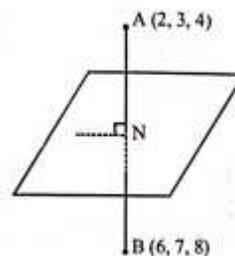
$$\frac{\alpha - 7}{2} = \frac{\beta - 14}{4} = \frac{\gamma - 5}{-1} = \frac{63}{-21}$$

$$\Rightarrow \alpha = 1, \beta = 2 \text{ and } \gamma = 8$$

Hence, foot of \perp is (1, 2, 8)

- Q.25** (b)
If the given points be A(2,3,4) and B(6,7,8), then their mid-point N(4,5,6) must lie on the plane.

The direction ratios of AB are 4, 4, 4, i.e. 1, 1, 1

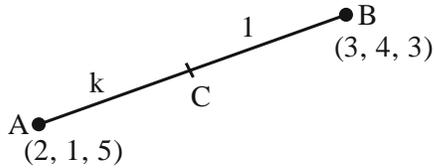


\therefore The required plane passes through N(4, 5, 6) and is normal to AB. Thus its equation is

$$1(x-4) + 1(y-5) + 1(z-6) = 0 \Rightarrow x + y + z = 15$$

Q.26 (b)

As given plane $x + y - z = \frac{1}{2}$ divides the line joining the points A(2, 1, 5) and B(3, 4, 3) at a point C in the ratio $k : 1$.



Then coordinates of C

$$\left(\frac{3k+2}{k+1}, \frac{4k+1}{k+1}, \frac{3k+5}{k+1} \right)$$

Point C lies on the plane,

⇒ Coordinates of C must satisfy the equation of plane.

$$\text{So, } \left(\frac{3k+2}{k+1} \right) + \left(\frac{4k+1}{k+1} \right) - \left(\frac{3k+5}{k+1} \right) = \frac{1}{2}$$

$$\Rightarrow 3k+2+4k+1-3k-5 = \frac{1}{2}(k+1)$$

$$\Rightarrow 4k-2 = \frac{1}{2}(k+1)$$

$$\Rightarrow 8k-4 = k+1 \Rightarrow 7k=5$$

$$\Rightarrow k = \frac{5}{7}$$

Ratio is 5 : 7.

Q.27 (d)

Q.28 (d)

Q.29 (d)

Q.30 (b)

Q.31 (d)

Q.32 (Bonus)

Q.33 (d)

Q.34 (1)

$$x+2y+2z=5 \quad \vec{n}_1 = (1, 2, 2)$$

$$3x+3y+2z=8 \quad \vec{n}_2 = (3, 3, 2)$$

$$\text{Normal vector of plane} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 3 & 2 \end{vmatrix} = -$$

$$2\hat{i} + 4\hat{j} + 3\hat{k}$$

Equation of plane $-2x + 4y - 3z = k$

passing through (1, -3, -2) $k = -8$

$$-2x + 4y - 3z = -8$$

Q.35

$$2x - 4y + 8z - 8 = 0$$

(2)

$$N(\alpha, \beta, \gamma)$$

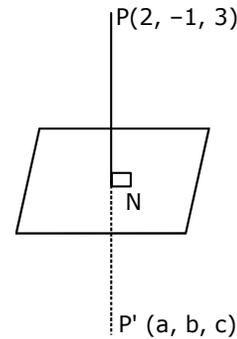
$$3x - 2y - z = 9$$

$$\frac{\alpha-2}{3} = \frac{\beta+1}{-2} = \frac{\gamma-3}{-1} = \lambda$$

$$\alpha = 3\lambda + 2, \beta = -2\lambda - 1, \gamma = -\lambda + 3$$

N point lies on the plane

$$3(3\lambda + 2) - 2(-2\lambda + 1) - (-\lambda + 3) = 9$$



$$\Rightarrow \lambda = \frac{2}{7}$$

$$N \left(\frac{20}{7}, \frac{-11}{7}, \frac{19}{7} \right)$$

$$N = \frac{P + P'}{2} \Rightarrow P' = 2N - P$$

$$\Rightarrow P' \left(\frac{26}{7}, \frac{-15}{7}, \frac{17}{7} \right)$$

Q.36 (4)

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$$

Use passes through P(2, -1, 2)

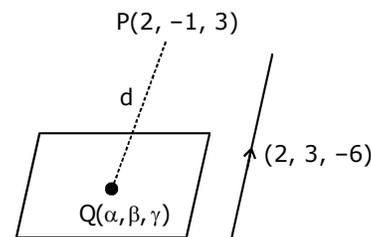
point P

So P_0I of line and plane is P(2, -1, 2)

(-1, -5, -10) so PQ = 13

Q.37 (1)

$$\frac{\alpha-1}{2} = \frac{\beta+2}{3} = \frac{\gamma-3}{-6} = \lambda$$



$$\alpha = 2\lambda + 1, \beta = 3\lambda - 2, \gamma = -6\lambda + 3$$

(α, β, γ) lie on the plane $x + y + z = 5$

$$\Rightarrow \lambda = \frac{1}{7}$$

$$Q\left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$$

$$d = PQ = 1$$

Q.38

(1)

 Let the Eqⁿ of plane

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$$

passes through (a, b, c)

$$\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma} = 1$$

 common point will be (α, β, γ)

so locus

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$$

Q.39

(1)

Let the equation of planes

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \text{ \& } \frac{x}{a_1} + \frac{y}{b_1} + \frac{z}{c_1} = 1$$

perpendicular distance from origin will be same

$$P_1 = P_2$$

$$\left| \frac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \right| = \left| \frac{-1}{\sqrt{\frac{1}{a_1^2} + \frac{1}{b_1^2} + \frac{1}{c_1^2}}} \right|$$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a_1^2} + \frac{1}{b_1^2} + \frac{1}{c_1^2}$$

Q.40

(4)

$$2x - y + z = 6 \quad \vec{n}_1 = (2, -1, 1)$$

normal vector of other plane

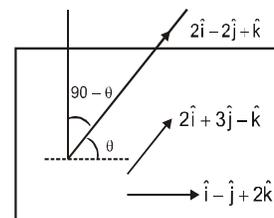
$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 1 & -1 & 0 \end{vmatrix} = 2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\text{angle } \cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1| |\vec{n}_2|} = 0 \Rightarrow \theta = \frac{\pi}{2}$$

Q.41 (2)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}(6-1) - \hat{j}(4+1) + \hat{k}(-2-3) =$$

$$5\hat{i} - 5\hat{j} - 5\hat{k}$$



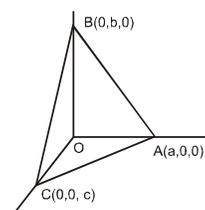
$$\cos(90 - \theta) = \left| \frac{10 + 10 - 5}{5\sqrt{3} \cdot 3} \right|$$

$$\sin \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Q.42 (1)

$$\text{Area of } \triangle ABC = \frac{1}{2} |\vec{AB} \times \vec{AC}|$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -a & 0 & c \\ -a & b & 0 \end{vmatrix}$$



$$\vec{AC} = -a\hat{i} + c\hat{k}$$

$$\vec{AB} = -a\hat{i} + b\hat{j}$$

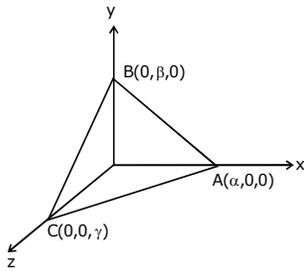
$$= -\hat{i}(bc) - \hat{j}(ac) + \hat{k}(-ab)$$

$$= -(bc)\hat{i} - (ac)\hat{j} - (ab)\hat{k}$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{b^2c^2 + a^2c^2 + a^2b^2}$$

$$\therefore \text{Area} = \frac{1}{2} \sqrt{b^2c^2 + a^2c^2 + a^2b^2}$$

Q.43 (3)



Let the equation of plane :

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1 \quad \dots(1)$$

$$\frac{\alpha}{3} = a \Rightarrow \alpha = 3a$$

$$\frac{\beta}{3} = b \Rightarrow \beta = 3b$$

$$\frac{\gamma}{3} = c \Rightarrow \gamma = 3c$$

$$\frac{x}{3a} + \frac{y}{3b} + \frac{z}{3c} = 1$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$$

Q.44

(1) Let the equation of plane

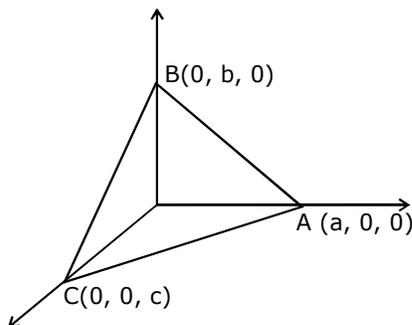
$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

given that $p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$

$$\text{or } \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2} \quad \dots(1)$$

Let centroid (u, v, w)

$$u = \frac{1}{4} \Rightarrow a = 4u$$



$$v = \frac{b}{4} \Rightarrow b = 4v$$

$$w = \frac{c}{4} \Rightarrow c = 4w$$

$$\frac{1}{16u^2} + \frac{1}{16v^2} + \frac{1}{16w^2} = \frac{1}{p^2}$$

$$\frac{1}{u^2} + \frac{1}{v^2} + \frac{1}{w^2} = \frac{16}{p^2}$$

$$u^{-2} + v^{-2} + z^{-2} = 16p^{-2}$$

Q.45 (2)

Let Point P (α, β, γ)

Given that

$$(\alpha - 1)^2 + (\alpha + 1)^2 + (\beta - 1)^2 + (\beta + 1)^2 + (\gamma - 1)^2 + (\gamma + 1)^2 = 10$$

$$2\alpha^2 + 2\beta^2 + 2\gamma^2 + 6 = 0$$

$$\alpha^2 + \beta^2 + \gamma^2 = 2 \Rightarrow x^2 + y^2 + z^2 = 2$$

Q.46 (1)

A(2-x, 2, 2) B(2, 2-y, 2) C(2, 2, 2-z) D(1, 1, 1)

$$\vec{AB} = (x, -y, 0), \vec{AC} = (x, 0, -2),$$

$$\vec{AD} = (x-1, -1, -1)$$

If A, B, C, D are coplanar points then

$$[\vec{AB} \ \vec{AC} \ \vec{AD}] = 0$$

$$\begin{vmatrix} x & -y & 0 \\ x & 0 & -2 \\ x-1 & -1 & -1 \end{vmatrix} = 0 \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

Q.47 (2)

$$a(x-1) + b(y-0) + c(z-0) = 0 \quad (1, 0, 0)$$

$$\Rightarrow ax + by + cz - a = 0 \quad (0, 1, 0)$$

$$0 + b + 0 - a = 0$$

$$\boxed{b = a}$$

∠ between planes = ∠ between normal
 $x + y = 3, \quad (1, 1, 0) \quad (a, b, c)$

$$\cos \frac{\pi}{4} = \frac{1(a) + 1(b) + 0(c)}{\sqrt{1^2 + 1^2 + 0^2} \sqrt{a^2 + b^2 + c^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{a + b}{\sqrt{2} \sqrt{a^2 + b^2 + c^2}} \quad \boxed{b = a}$$

$$\sqrt{2a^2 + c^2} = 2a$$

$$2a^2 + c^2 = 4a^2$$

$$2a^2 = c^2$$

$$\boxed{c = \sqrt{2}a}$$

$$b = a; c = \sqrt{2}a$$

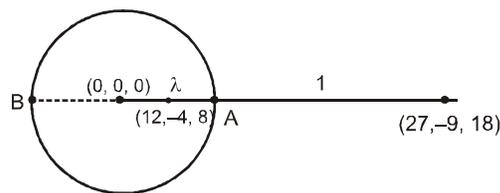
$$a : b : c = (1:1:\sqrt{2})$$

Q.48 (1)

$$A\left(\frac{27\lambda+12}{\lambda+1}, \frac{-9\lambda-4}{\lambda+1}, \frac{18\lambda+8}{\lambda+1}\right)$$

Which lies on the sphere

$$\therefore \left(\frac{27\lambda+12}{\lambda+1}\right)^2 + \left(\frac{-9\lambda-4}{\lambda+1}\right)^2 + \left(\frac{18\lambda+8}{\lambda+1}\right)^2 = 504$$



$$\text{Solving above we get } 9\lambda^2 = 4 \quad \lambda = \pm \frac{2}{3}$$

Q.49 (3)

$$\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1} = \lambda$$

$$(3\lambda+2, 2\lambda-1, 1-\lambda)$$

$$z=0 \Rightarrow \lambda=1$$

$$xy = c^2$$

$$(3\lambda+2)(2\lambda-1) = c^2$$

$$\text{put } \lambda=1 \Rightarrow c^2=5 \Rightarrow c = \pm \sqrt{5}$$

EXERCISE-III

NUMERICAL VALUE BASED

Q.1 (0000)

The given lines are coplanar if

$$0 = \begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 1-k \\ k & k+2 & 1+k \end{vmatrix}$$

$$\text{or if } 2(1+k) - (k+2)(1-k) = 0$$

$$\text{or if } k^2 + 3k = 0$$

$$\text{or if } k = 0, -3.$$

Q.2 0001

$$\frac{x-3}{2} = \frac{y+5}{k} = \frac{z+1}{2k} \text{ is parallel } 6x + 8y + 2z - 4 = 0$$

$$12 + 8k + 4k = 0$$

$$k = -1.$$

Q.3 0600

Equation of the given plane can be written as

$$\frac{x}{20} + \frac{y}{15} + \frac{z}{-12} = 1$$

which meets the coordinates axes in points A(20, 0, 0), B(0, 15, 0) and C(0, 0, -12) and the coordinates of the origin are (0, 0, 0).

\therefore The volume of the tetrahedron OABC is

$$\left| \frac{1}{6} \begin{vmatrix} 20 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & -12 \end{vmatrix} \right| = \left| \frac{1}{6} \times 20 \times 15 \times (-12) \right| = 600$$

Q.4 (1.33)

Let the D.C's of given line be (l, m, n). Diagonals of cube are OP, AL, BM, CN. The DC's of the diagonals are

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$$

$$\therefore \cos \alpha = \frac{1+m+n}{\sqrt{3}}, \cos \beta = \frac{-1+m+n}{\sqrt{3}},$$

$$\cos \gamma = \frac{1-m+n}{\sqrt{3}}, \cos \delta = \frac{1+m-n}{\sqrt{3}}$$

$$\therefore \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}(1^2 + m^2 + n^2)$$

$$= \frac{4}{3}$$

\therefore (b)

Q.5 4.5

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = r \text{ (say)}$$

$$\Rightarrow x = 2r + 1, y = 3r - 1, z = 4r + 1$$

As the two given lines intersect,

$$\frac{2r+1-3}{1} = \frac{3r-1-k}{2} = \frac{4r+1}{1}$$

For some value of r

$$\text{we have } 2r - 2 = 4r + 1 \Rightarrow r = -\frac{3}{2}$$

$$\text{Also, } 3r - 1 - k = 8r + 2$$

$$\Rightarrow k = -5r - 3 = \frac{15}{2} - 3 = \frac{9}{2}$$

\therefore (c)

Q.6 1.5

Let the line make the angle α with the axes, then we have $3\cos^2 \alpha = 1$ [\because sum of the square's of DC's = 1]

$$\cos \alpha = \frac{1}{\sqrt{3}} \text{ [}\because \text{ DC's are positive, given]}$$

$$\frac{x-2}{1/\sqrt{3}} = \frac{y+1}{1/\sqrt{3}} = \frac{z-2}{1/\sqrt{3}} = r \text{ \{where } r = PQ\}$$

$$x = \frac{r}{\sqrt{3}} + 2, y = \frac{r}{\sqrt{3}} - 1, z = \frac{r}{\sqrt{3}} + 2,$$

$$\frac{2r}{\sqrt{3}} + 4 + \frac{r}{\sqrt{3}} - 1 + \frac{r}{\sqrt{3}} + 2 = 9 \Rightarrow \frac{4r}{\sqrt{3}} = 4 \Rightarrow r = \sqrt{3}$$

Q.7 0007

The point (4, 2, k); through which the the line passes must also lie on the given plane

$$\therefore 2 \times 4 - 4 \times 2 + k = 7$$

$$\Rightarrow k = 7$$

Q.8 0009

Let the equation of variable plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ which meets the axes at A(a, 0, 0), B(0, b, 0) and C(0, 0, c).

Centroid of ΔABC is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

and it satisfied the relation

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$$

$$\Rightarrow \frac{9}{a^2} + \frac{9}{b^2} + \frac{9}{c^2} = k$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{k}{9} \quad \dots(i)$$

Also given that the distance of plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ from (0, 0, 0) is 1 unit.

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = 1 \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = 1 \quad \dots(ii)$$

from (i) and (ii) we get $\frac{k}{9} = 1$ i.e., $k = 9$

Q.9 0003

$$1 = \frac{a+1+2+0}{4}, 2 = \frac{2+b+1+0}{4}, 3 = \frac{3+2+c+0}{4}$$

$$\Rightarrow a = 1, b = 5, c = 7$$

\Rightarrow distance of centroid from origin is

$$\sqrt{1^2 + 25 + 49} = \sqrt{75} = 5\sqrt{3} \Rightarrow \lambda = 3$$

Q.10 0008

plane must pass through $\left(\frac{1-3}{2}, \frac{5+1}{2}, \frac{7-1}{2}\right)$

or (-1, 3, 3)

$$\Rightarrow -1 + 3 + 2 \times 3 = \lambda \Rightarrow \lambda = 8$$

PREVIOUS YEAR'S

MHT CET

- | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Q.1 (c) | Q.2 (c) | Q.3 (a) | Q.4 (a) | Q.5 (d) |
| Q.6 (a) | Q.7 (a) | Q.8 (b) | Q.9 (a) | Q.10 (a) |
| Q.11 (a) | Q.12 (d) | Q.13 (b) | Q.14 (d) | Q.15 (b) |
| Q.16 (c) | Q.17 (a) | Q.18 (b) | Q.19 (d) | Q.20 (b) |
| Q.21 (c) | Q.22 (d) | Q.23 (d) | Q.24 (a) | Q.25 (c) |
| Q.26 (d) | Q.27 (d) | Q.28 (2) | Q.29 (d) | Q.30 (c) |
| Q.31 (d) | Q.32 (b) | Q.33 (a) | Q.34 (a) | Q.35 (d) |
| Q.36 (b) | Q.37 (c) | Q.38 (b) | Q.39 (d) | Q.40 (c) |

Q.41 (1)

Given equation of line

$$\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2} \quad \dots(i)$$

The direction ratios of the normal are (1, 3, - α).

The direction ratios of the line are (3, -5, 2)

and equation of given plane

$$x + 3y - \alpha z + \beta = 0 \quad \dots(ii)$$

Four lines are perpendicular

$$\Rightarrow a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\Rightarrow 3 - 15 + 2\alpha = 0$$

$$\Rightarrow 2\alpha = -12 \Rightarrow \alpha = -6$$

(2, 1, -2) lies on the plane, so

$$2 + 3 + 6(-2) + \beta = 0 \Rightarrow \beta = 7$$

$$\alpha \cdot \beta = -6 \times 7 = -42$$

Q.42 (1)

The given equations are $3I + m + 5n = 0 \quad \dots(i)$

and $6mn - 2nl + 5lm = -9$

$\dots(ii)$

Now, from Eq. (i), we get

$$m = -3I - 5n$$

On substituting $m = -3I - 5n$ in Eq. (ii), we get

$$6(-3I - 5n)n - 2nI + 5I(-3I - 5n) = 0$$

$$\Rightarrow 30n^2 + 45In + 15I^2 = 0$$

$$\Rightarrow 2n^2 + 3In + I^2 = 0$$

$$\Rightarrow 2n^2 + 2nl + nl + I^2 = 0$$

$$\Rightarrow 2n(n+I) + I(n+I) = 0$$

$$\Rightarrow (n+I)(2n+I) = 0$$

$$\Rightarrow \text{Either } I = -n \text{ or } I = -2n$$

If $I = -n$, then $m = -2n$ and if $I = -2n$, then $m = n$

Thus, the direction ratios of two lines are proportional

(-n, -2n, n) and (-2n, n, n) i.e.

(-1, -2, 1) and (-2, 1, 1), respectively.

Now, let q be the acute angle between the lines,

$$\text{then, } \cos q = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{|-2 - 2 + 1|}{\sqrt{1+4+1} \sqrt{4+1+1}} = \frac{1}{6} \Rightarrow q = \cos^{-1}\left(\frac{1}{6}\right)$$

Q.43 (2)

If straight lines are coplanar, then

$$\Rightarrow \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Since, $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar.

$$\Rightarrow \begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0 \Rightarrow k^2 = 4 \Rightarrow k = \pm 2$$

$$\therefore n_1 = b_1 \times d_1 = 6\hat{j} - 6\hat{k} \text{ for } k = 2$$

$$\therefore n_2 = b_2 \times d_2 = 14\hat{j} + 14\hat{k} \text{ for } k = -2$$

So, equation of planes are $(r - a) \cdot n_1 = 0$

$$\Rightarrow y - z = -1 \text{ and } (r - a) \cdot n_2 = 0$$

$$\Rightarrow y + z = -1$$

Q.44

(2)

Given, $a + 3b$ is collinear with c

$$\text{So, } a + 3b = \lambda c \quad \dots(i)$$

where λ is some real number.

$$\text{Similarly, } b + 2c = \mu a \quad \dots(ii)$$

Adding Eqs. (i) and (ii), we get

$$a + 4b + 2c = \lambda c + \mu a$$

$$\Rightarrow (1 - \mu)a + 4b + (2 - \lambda)c = 0$$

Multiply by $3/4$, we get

$$\frac{3}{4}(1 - \mu)a + 3b + \frac{3}{4}(2 - \lambda)c = 0$$

$$\therefore \frac{3}{4}(1 - \mu) = 1 \text{ and } \frac{3}{4}(2 - \lambda) = 6$$

$$\Rightarrow 3 - 3\mu = 4 \text{ and } 2 - \lambda = 8 \Rightarrow \mu = -\frac{1}{3}$$

$$\text{and } \lambda = -6$$

$$\Rightarrow a + 3b + 6c = 0$$

Q.45

(4)

Given points are

$$(x_1, y_1, z_1) \equiv (1, 2, 3),$$

$$(x_2, y_2, z_2) \equiv (-1, 4, 2)$$

$$\text{and } (x_3, y_3, z_3) \equiv (3, 1, 1)$$

Now, required equation of the plane is given by

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 1 & y - 2 & z - 3 \\ -2 & 2 & -1 \\ 2 & -1 & -2 \end{vmatrix} = 0$$

$$\Rightarrow (x - 1)(-4 - 1) - (y - 2)(4 + 2) + (z - 3)(2 - 4) = 0$$

$$\Rightarrow -5(x - 1) - 6(y - 2) - 2(z - 3) = 0$$

$$\Rightarrow 5x + 6y + 2z - 23 = 0$$

JEE MAIN

PREVIOUS YEAR'S

Q.1 (4)

Direction ratio of normal of plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 18\hat{i} - \hat{j} + 7\hat{k}$$

so equation of plane

$$18x - y + 7z = d$$

It passes through $(2, 3, -5)$

$$36 - 3 - 35 = d \therefore d = -2$$

Eqⁿ of plane

$$18x - y + 7z = -2$$

$$-18x + y - 7z = 2$$

$$\therefore a = -18, b = 1, c = -7, d = 2$$

$$a + 7b + c + 20d = -18 + 7 - 7 + 40 = 22$$

(125)

Q.2

Let M be the mid point of PQ

$$\therefore M = (3\lambda + 6, 2\lambda + 1, 3\lambda + 2)$$

$$\text{Now, } \overline{PM} = (3\lambda + 5)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 1)\hat{k}$$

$$\therefore \overline{PM} \perp (3\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\therefore 3(3\lambda + 5) + 2(2\lambda - 1) + 3(3\lambda - 1) = 0$$

$$\lambda = \frac{-5}{11}$$

$$\therefore M \left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11} \right)$$

since R is mid-point of PM

$$22(\alpha + \beta + \gamma) = 125$$

Q.3

(3)

$$\frac{a-2}{3} = \frac{b-4}{-1} = \frac{c-7}{4} = \frac{-2(6-4+28-2)}{3^2+1^2+4^2}$$

$$\Rightarrow a = \frac{-84}{13} + 2, b = \frac{28}{13} + 4, c = \frac{-112}{13} + 7$$

$$2a + b + 2c = -6$$

Q.4 (26)

Points P (1, 2, -1) and Q (2, -1, 3) lie on same side of the plane. Perpendicular distance of point P from plane

$$\text{is } \frac{-1+2-1-1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}$$

Perpendicular distance of point Q from plane is

$$\left| \frac{-2-1+3-1}{\sqrt{1^2+1^2+1^2}} \right| = \frac{1}{\sqrt{3}}$$

PQ is parallel to given plane. So, distance between P and Q = distance between their foot of perpendiculars.

$$\begin{aligned} \Rightarrow |\overline{PQ}| &= \sqrt{(1-2)^2 + (2+1)^2 + (-1-3)^2} \\ &= \sqrt{26} \\ d^2 &= 26 \end{aligned}$$

Q.5 (2)

$$L: \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z+3}{-1}$$

Plane : $px - qy + z = 5$

Line is satisfying the plane

So (2, -1, -3) satisfying $px - qy + z = 5$

$$P(2) - q(-1) + (-3) = 5 \Rightarrow 2p + q = 8 \dots (i)$$

The line is parallel to plane

$$\therefore 3P + 2q - 1 = 0$$

(ii)

From (i) and (ii)

$$P = 15, q = -22$$

$$\text{Eq. of plane is } 15x - 22y + z - 5 = 0$$

Distance from origin

$$= \frac{5}{\sqrt{15^2 + (-22)^2 + (1)^2}} = \frac{5}{\sqrt{710}}$$

$$= \sqrt{\frac{25}{710}} = \sqrt{\frac{5}{142}}$$

Q.6 (2)

Image of P(1, 2, 1) in $x + 2y + 2z - 16 = 0$

Is given by Q(3, 6, 5)

Equation of plane T

$$\begin{vmatrix} x & y & z+1 \\ 3 & 6 & 6 \\ 1 & 1 & 2 \end{vmatrix} = 0$$

$$2x - z = 1$$

By options, (1, 2, 1) lies on plane T

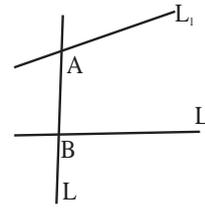
Q.7 [84]

$$L_1 \rightarrow \frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$$

so point A(3λ+7, -λ+1, λ-2)

$$L_2 \rightarrow \frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$$

so point B(2μ, 3μ, 7μ)



∴ D.R. of AB is

$$3\lambda+7-2\mu, -\lambda+1-3\mu-7, \lambda-2-\mu$$

$$3\lambda-2\mu+7, -\lambda-3\mu-6, \lambda-\mu-2$$

given DR's of AB are 1, -4, 2

$$\therefore \frac{3\lambda+7-2\mu}{1} = \frac{-\lambda-3\mu-6}{-4} = \frac{\lambda-\mu-2}{2}$$

$$\frac{3\lambda+7-2\mu}{1} = \frac{-\lambda-3\mu-6}{-4}$$

$$\frac{-\lambda-3\mu-6}{-4} = \frac{\lambda-\mu-2}{2}$$

$$-12\lambda-28+8\mu = -\lambda-3\mu-6 \quad -2\lambda-6\mu-12 = -4\lambda+4\mu+8$$

$$11\lambda-11\mu+22=0$$

$$2\lambda-10\mu-20=0$$

$$\boxed{\lambda-\mu+2=0} \quad \dots(1)$$

$$\boxed{\lambda-5\mu-10=0} \quad \dots(2)$$

∴ Solving Eq. (1) & (2)

λ = -5 & μ = -3

$$\therefore A(-8, 6, -7) \quad \& \quad B(-6, -2, -3)$$

$$AB = \sqrt{(-8+6)^2 + (6+2)^2 + (-7+3)^2}$$

$$AB = \sqrt{4+64+16}$$

$$(AB)^2 = 84$$

Q.8 (2)

$$L_1: \rightarrow \vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(\hat{i} - \hat{a}\hat{j})$$

$$\Rightarrow \frac{x+1}{1} = \frac{y-0}{-a} = \frac{z-3}{0} \quad \dots\dots(1)$$

$$L_2: \rightarrow \vec{r} = (-\hat{j} + 2\hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$$

$$\frac{x-0}{1} = \frac{y+1}{-1} = \frac{z-2}{1} \quad \dots\dots(2)$$

Given shortest distance between L_1 & L_2 is $= \sqrt{\frac{2}{3}}$

$$\frac{\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{|\vec{b}_1 \times \vec{b}_2|} = \sqrt{\frac{2}{3}}$$

$$\left| \begin{array}{ccc} -1-0 & 0+1 & 3-2 \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{array} \right| = \sqrt{\frac{2}{3}}$$

$$\frac{\sqrt{(-a)^2 + (1-0)^2 + (-1+a)^2}}{\sqrt{(-a)^2 + (1-0)^2 + (-1+a)^2}} = \sqrt{\frac{2}{3}}$$

$$\Rightarrow \frac{-1(-a) - 1(1) + 1(-1+a)}{\sqrt{a^2 + 1 + a^2 + 1 - 2a}} = \sqrt{\frac{2}{3}}$$

$$\left| \frac{2a-2}{\sqrt{2a^2-2a+2}} \right| = \sqrt{\frac{2}{3}}$$

$$\frac{(2(a-1))^2}{2(a^2-a+1)} = \frac{2}{3}$$

$$\begin{aligned} \Rightarrow 3(a-1)^2 &= a^2 - a + 1 \\ \Rightarrow 3a^2 - 6a + 3 &= a^2 - a + 1 \\ \Rightarrow 2a^2 - 5a + 2 &= 0 \\ 2a^2 - 4a - a + 2 &= 0 \\ 2a(a-2) - 1(a-2) &= 0 \\ a &= 1/2 \text{ \& } [a=2] \end{aligned}$$

Ans. 2

Q.9 (1)

$$\text{S.D.} = \frac{[(\bar{a}_2 - \bar{a}_1)(\bar{b}_1 \bar{b}_2)]}{|\bar{b}_1 \times \bar{b}_2|}$$

$$\begin{aligned} & \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & \lambda \\ 1 & 4 & 5 \end{vmatrix} \\ & \hat{i} \hat{j} \hat{k} = \frac{(15-4\lambda) - 2(10-\lambda) + 2(5)}{|(15-4\lambda)\hat{i} - \hat{j}(10-\lambda) + \hat{k}(5)|} \\ & \begin{vmatrix} 2 & 3 & \lambda \\ 1 & 4 & 5 \end{vmatrix} \end{aligned}$$

$$\begin{aligned} & \frac{1}{\sqrt{3}} = \frac{5-2\lambda}{\sqrt{(15-4\lambda)^2 + (10-\lambda)^2 + 5^2}} \\ & = \frac{(15-4\lambda)^2 + (10-\lambda)^2 + 25}{3(5-2\lambda)^2} \\ & = \frac{225 + 16\lambda^2 - 120\lambda + 100 + \lambda^2 - 20\lambda + 25}{75 + 12\lambda^2 - 60\lambda} \\ & 5\lambda^2 - 80\lambda + 175 = 0 \\ & \lambda^2 - 16\lambda + 35 = 0 \end{aligned}$$

$$\boxed{\lambda_1 + \lambda_2 = 16}$$

Q.10 (3)

$$\begin{aligned} & (r - (-1, 0, 2)) \cdot [6, -4, 2] = 0 \\ & 6(x+1) - 4(y) + 2(z-2) = 0 \\ & \Rightarrow 3x - 2y + z + 1 = 0 \quad \dots P_1 \\ & 2x + y + 3z = 1 \quad \dots P_2 \end{aligned}$$

$$\begin{aligned} \cos\theta &= \frac{2 \times 3 + 1(-2) + 3(1)}{\sqrt{14} \cdot \sqrt{14}} \\ &= \frac{6-2+3}{14} = \frac{1}{2} \\ \therefore \cos\theta &= \frac{1}{2} \quad \Rightarrow \theta = \frac{\pi}{3} \end{aligned}$$

Q.11 [51]

$$l_1 : 8x + 4\sqrt{2}y = 1$$

$$l_2 : -8 - 6\sqrt{3}z = 1$$

$$l_1 : a_1 = \left(\frac{1}{8}, 0, 0\right) \text{ and } a_2 = \left(0, \frac{1}{4\sqrt{2}}, 0\right)$$

$$P_1 : a_2 - a_1 = \left\langle \frac{-1}{8}, \frac{1}{4\sqrt{2}}, 0 \right\rangle$$

$$l_2 : b_1 = \left(-\frac{1}{8}, 0, 0\right) \text{ \& } b_2 = \left(0, 0, \frac{-1}{6\sqrt{3}}\right)$$

$$P_2 : b_2 - b_1 = \left\langle \frac{1}{8}, 0, \frac{-1}{6\sqrt{3}} \right\rangle$$

$$\bar{n} = P_1 \times P_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\frac{1}{8} & \frac{1}{4\sqrt{2}} & 0 \\ \frac{1}{8} & 0 & \frac{-1}{6\sqrt{3}} \end{vmatrix}$$

$$\bar{n} = \hat{i} \left(\frac{-1}{24\sqrt{6}}\right) - \hat{j} \left(\frac{1}{48\sqrt{3}}\right) - \hat{k} \left(\frac{1}{32\sqrt{2}}\right)$$

$$\bar{n} = \frac{-1}{96\sqrt{6}} \langle 4, 2\sqrt{2}, 3\sqrt{3} \rangle$$

$$\bar{c} = (a_1 - b_1) = \left(\frac{1}{4}, 0, 0\right)$$

$$d = \left| \frac{\bar{c} \cdot \bar{n}}{|\bar{n}|} \right|$$

$$= \left| \frac{1}{\sqrt{16+8+27}} \right| = \frac{1}{\sqrt{51}}$$

$$\Rightarrow \frac{1}{d^2} = 51$$

Q.12 (3)

Equation of family of planes passing through line of intersection of two planes $2x + y - 5z = 0$ and $3x - y + 4z - 7 = 0$

is $(3x - y + 4z - 7) + \lambda(2x + y - 5z) = 0$
 $(3 + 2\lambda)x + (-1 + \lambda)y + (4 - 5\lambda)z - 7 = 0$
(1) is perpendicular to plane
 $2x + y - 5z = 9$
 $(3 + 2\lambda) \cdot 2 + (-1 + \lambda) \cdot 1 + (4 - 5\lambda) \cdot (-5) = 0$
 $\Rightarrow 6 + 4\lambda - 1 + \lambda - 20 + 25\lambda = 0$
 $\Rightarrow 30\lambda - 15 = 0$

$\Rightarrow \lambda = \frac{15}{30} = \frac{1}{2}$

Put λ in equation (i)

$\left(3 + 2 \times \frac{1}{2}\right)x + \left(-1 + \frac{1}{2}\right)y + \left(4 - \frac{5}{2}\right)z - 7 = 0$

$\Rightarrow 4x - \frac{y}{2} + \frac{3}{2}z - 7 = 0, \Rightarrow 8x - y + 3z - 14 = 0$
 \Rightarrow Point $(1, 0, 2)$ satisfy equation.

Q.13

[33]
 6 digit numbers formed by digits 1 and 8 only
 for a number to be multiple of 21 it must be divisible by both 3 and 7
 To make divisible by '3'
 sum must be divisible by 3
 possible cases
 (i) All digits are 1's $\rightarrow 1$
 (ii) All digits are 8's $\rightarrow 1$

(iii) 3 1's and 3 8's $\rightarrow \frac{6!}{3!3!} = 20$

To make divisible by 7
 $|2(\text{last digit}) - (\text{remaining number})| = 7k, k \in Z$
 total possibilities = 2^6
 total numbers divisible by 21 = $20 + 1 + 1 = 22$

$p = \frac{22}{2^6} \Rightarrow 96 p = \frac{22}{64} \times 96 = 33$

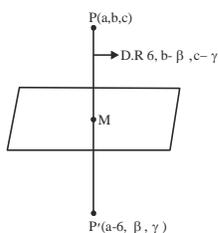
Q.14

(1)
 $1 + m - n = 0$
 $3l^2 + m^2 + cl(1 + m) = 0$
 $3l^2 + m^2 + cl^2 + clm = 0$
 $(3 + c)l^2 + clm + m^2 = 0$
 $(3 + c)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + 1 = 0$... (1)

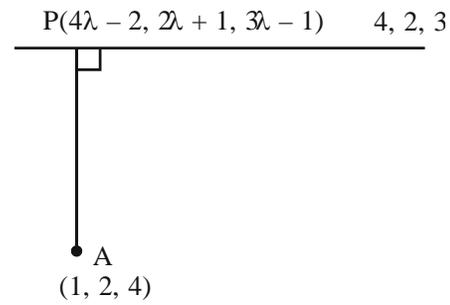
\therefore lines are parallel
 Roots of (1) must be equal
 $\Rightarrow D = 0$
 $C^2 - 4(3 + c) = 0$
 $C^2 - 4c - 12 = 0$
 $(c - 6)(c + 2) = 0$
 $c = 6$ or $c = -2$
 +ve value of $c = 6$

Q.15

(137)



Q.16 (1)



$\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{4} = \lambda$

$(x, y, z) = (4\lambda - 2, 2\lambda + 1, 3\lambda - 1)$

$\vec{AP} = (4\lambda - 3)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 5)\hat{k}$

$\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{AP} \cdot \vec{b} = 0$

$4(4\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 5) = 0$

$29\lambda = 12 + 2 + 15 = 29$

$\lambda = 1$

$P = (2, 3, 2)$

$3x + 4y + 12z + 23 = 0$

$d = \frac{|6 + 12 + 24 + 23|}{\sqrt{9 + 16 + 144}}$

$d = \frac{|65|}{13} = 5$

Q.17 (1)

$\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$

$\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$

$A = (3, 2, 1)$ $B = (-3, 6, 5)$

$\vec{n}_1 = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{n}_2 = 2\hat{i} + \hat{j} + 3\hat{k}$

$\vec{BA} = 6\hat{i} - 4\hat{j} - 4\hat{k}$

SHORTEST DISTANCE = $\frac{|\vec{BA} \cdot \vec{n}_1|}{|\vec{n}_1|}$

$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix}$

$= 10\hat{i} - 8\hat{j} - 4\hat{k}$

$|\vec{BA} \cdot \vec{n}_1| = 60 + 32 + 16 = 108$

$$[\vec{n}_1 \times \vec{n}_2] = \sqrt{100+64+16} = \sqrt{180}$$

$$S.D = \frac{108}{\sqrt{180}} = \frac{108}{6\sqrt{5}} = \frac{18}{\sqrt{5}}$$

Q.18

(2)

Equation of L

$$\frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{1} = \lambda$$

$$x = \lambda + 1, y = \lambda - 1, z = \lambda - 1$$

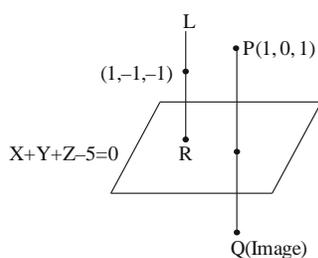
Putting in equation of plane

$$\lambda + 1 + \lambda - 1 + \lambda - 1 = 5$$

$$3\lambda = 6$$

$$\lambda = 2$$

$$\therefore R(3, 1, 1)$$



$$QR^2 = PR^2 = 4 + 1 + 0 = 5$$

Q.19

(5)

$$L_1: \vec{r} = (0, 0, 0) + \lambda(1, 2, 3)$$

$$L_2: \vec{r} = (1, 3, 1) + \mu(1, 1, 5)$$

$$\text{direction of } \vec{n} \text{ of plane} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & 1 & 5 \end{vmatrix} = 7\hat{i} - 2\hat{j} - \hat{k}$$

 for point of intersection of L_1 and L_2 :

$$\lambda = 1 + \mu \quad \dots(1)$$

$$2\lambda = 3 + \mu \quad \dots(2)$$

$$3\lambda = 1 + 5\mu \quad \dots(3)$$

Solving (1), (2) and (3),

$$\lambda = 2, \mu = 1$$

$$\therefore S(2, 4, 6)$$

$$\text{Equation of plane } 7(x-2) - 2(y-4) - (z-6) = 0$$

$$7x - 2y - z - 14 + 8 + 6 = -0$$

$$7x - 2y - z = 0$$

$$a=7, b=-2, d=0$$

$$a+b+d=5$$

Q.20

(2)

$$I_1: \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$$

$$I_2 = \frac{x-1}{1} = \frac{y+\frac{3}{2}}{\frac{\alpha}{2}} = \frac{z+5}{2}$$

$$I_3 = \frac{x-1}{-3} = \frac{y-\frac{1}{2}}{-2} = \frac{z+0}{4}$$

$$I_1 \perp I_2 \Rightarrow \frac{|3-\alpha+0|}{\sqrt{3}\sqrt{1+\frac{\alpha^2}{4}+4}} = 0 \Rightarrow \alpha = 3$$

 angle between I_2 & I_3

$$\cos\theta = \frac{|1 \times (-3) + (-2)(\alpha/2) + 2 \times 4|}{\sqrt{1+4+\frac{\alpha^2}{4}} \sqrt{9+16+4}}$$

$$\cos\theta = \frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}} \sqrt{29}}$$

 put $\alpha=3$

$$\cos\theta = \frac{2}{\sqrt{\frac{29}{4}} \sqrt{29}} = \frac{4}{29}$$

$$\theta = \cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right)$$

Q.21 (1)

Let equation of rotated plane be:

$$(2x+3y+z+20) + \lambda(x-3y+5z-8) = 0$$

$$(2+\lambda)x + (3-3\lambda)y + (1+5\lambda)z + 20 - 8\lambda = 0$$

 Above plane is perpendicular to $2x+3y+z-20=0$

$$\text{So, } (2+\lambda).2 + (3-3\lambda).3 + (1+5\lambda).1 = 0 \Rightarrow \lambda = 7$$

$$\Rightarrow \text{Equation of rotated plane: } x-2y+4z-4=0$$

 Mirror image of $A\left(2, \frac{-1}{2}, 2\right)$ in rotated plane is

 $B(a, b, c)$

$$\text{Equation of AB: } \frac{x-2}{1} = \frac{y+1/2}{-2} = \frac{z-2}{4} = k$$

 Let coordinate of B be $\left(2+k, \frac{-1}{2}-2k, 2+4k\right)$

 Mid point of AB is $\left(2+\frac{k}{2}, \frac{-1}{2}-k, 2+2k\right)$

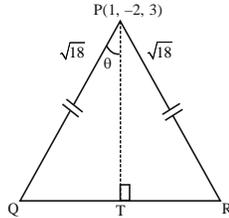
 which will lie on the plane $x-2y+4z-4=0$

Hence $k = \frac{-2}{3}$

Therefore B is $\left(\frac{4}{3}, \frac{5}{6}, \frac{-2}{3}\right) \equiv \left(\frac{8}{6}, \frac{5}{6}, \frac{-4}{6}\right)$

So, $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$

Q.22 (2)



$-x + 2y - z = 0$
 $3x - 5y + 2z = 0$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -1 \\ 3 & -5 & 2 \end{vmatrix}$$

$= \hat{i}(-1) - \hat{j}(1) + \hat{k}(-1)$

$\vec{n} = -\hat{i} - \hat{j} - \hat{k}$

Equation of LOI is $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$

dr's of PT $\rightarrow (\alpha - 1, \alpha + 2, \alpha - 3)$

dr's of QR $\rightarrow (1, 1, 1)$

$\Rightarrow (\alpha - 1) \times 1 + (\alpha + 2) \times 1 + (\alpha - 3) \times 1 = 0$

$3\alpha = 2$

$\alpha = \frac{2}{3}$

$PT^2 = \frac{1}{9} + \frac{64}{9} + \frac{49}{9}$

$PT^2 = \frac{114}{9}$

$PT = \frac{\sqrt{114}}{3}$

$\cos \theta = \frac{\sqrt{114}}{3} \times \frac{1}{3\sqrt{2}} = \frac{\sqrt{57}}{9} = \frac{\sqrt{19 \times 3}}{3 \times 3} = \frac{\sqrt{19}}{3\sqrt{3}}$

$\cos 2\theta = \frac{2 \times 19}{27} - 1 = \frac{11}{27}$

$\sin 2\theta = \sqrt{1 - \left(\frac{11}{27}\right)^2} = \frac{\sqrt{38}\sqrt{16}}{27}$

$= \frac{4}{27}\sqrt{38}$

Area $= \frac{1}{2} \times \sqrt{18}\sqrt{18} \times \frac{4}{27}\sqrt{38}$

$= \frac{18}{2} \times \frac{4}{27}\sqrt{38} = \frac{36}{27}\sqrt{38} = \frac{4}{3}\sqrt{38}$

Q.23

(1)

Equation of plane passing through the intersection of planes

$5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ is

$5x + 8y + 13z - 29 + \lambda(8x - 7y + z - 20) = 0$ and if it is

passing through $(2, 1, 3)$ then $\lambda = \frac{7}{2}$

P_1 : Equation of plane through intersection of $5x + 8y + 13z - 29 = 0$ and

$8x - 7y + z - 20 = 0$ and the point $(2, 1, 3)$ is

$5x + 8y + 13z - 29 + \frac{7}{2}(8x - 7y + z - 20) = 0$

$\Rightarrow 2x - y + z = 6$

Similarly P_2 : Equation of plane through intersection of $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the

point $(0, 1, 2)$ is $x + y + 2z = 5$

Angle between planes $= \theta = \cos^{-1} \left(\frac{3}{\sqrt{6}\sqrt{6}} \right) = \frac{\pi}{3}$

Q.24

(2)

Equation of plane passing through line of intersection of planes $P_1: \vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 6$ and

$P_2: \vec{r} \cdot (-6\hat{i} + 5\hat{j} - \hat{k}) = 7$ is

$P_1 + \lambda P_2 = 0$

$(\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) - 6) + \lambda(\vec{r} \cdot (-6\hat{i} + 5\hat{j} - \hat{k}) - 7) = 0$

and it passes through point $\left(2, 3, \frac{1}{2}\right)$

$\Rightarrow \left(2 + 9 - \frac{1}{2} - 6\right) + \lambda \left(-12 + 15 - \frac{1}{2} - 7\right) = 0$

$\Rightarrow \lambda = 1$

Equation of plane is $\vec{r} \cdot (-5\hat{i} + 8\hat{j} - 2\hat{k}) = 13$

$|\vec{a}|^2 = 25 + 64 + 4 = 93; d = 13$

Value of $\frac{|13\vec{a}|^2}{d^2} = 93$

Q.25 [2]

$$\frac{X+1}{2} = \frac{4-3}{3} = \frac{Z-1}{-1} = \lambda$$

$$\theta(2\lambda - 1, 3 + 3, -\lambda + 1)$$

$$\text{DR of PQ } 2\lambda - 1 - a, 3\lambda - 1, \lambda - 1$$

Now PQ and line is 1r

$$\text{So, } 2(2\lambda - 1 - a) + 3(3\lambda - 1) - (\lambda - 1) = 0$$

$$\Rightarrow \lambda = \frac{a+2}{7}$$

$$Q\left(\frac{2a-3}{7}, \frac{3a+27}{7}, \frac{5-a}{7}\right)$$

Distance between PQ

$$\sqrt{\left(\frac{2a-3}{7} - a\right)^2 + \left(\frac{3a+27}{7} - u\right)^2 + \left(\frac{5-a}{7} - 2\right)^2}$$

$$= \sqrt{6} \text{ (given)}$$

$$35a^2 + 42a + 91 = 14^2 \times 6$$

$$3\sqrt{a^2} + 42a - 1085 = 0$$

$$5a^2 + 6a - 155 = 0$$

$$a = 5, a = -\frac{62}{10} (\because a > 0)$$

$$\text{so } a = 5$$

$$\lambda = \frac{5+2}{7} = 1$$

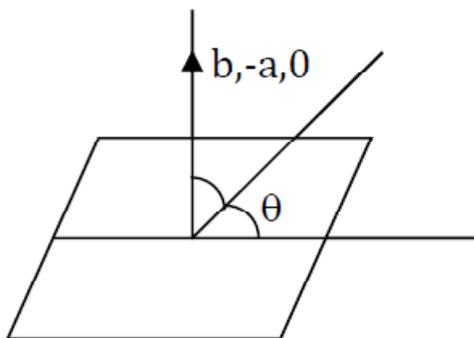
Point Q (1,6,0) is mid point P and R, so

$$1 = \frac{\alpha_1 + \alpha}{2} = \frac{\alpha_2 + 5}{2} \Rightarrow \alpha_1 = -3$$

$$\text{Similarly } \alpha_2 = 8, \alpha_3 = -2$$

$$a + \frac{3}{2}\alpha_1 = \alpha + \alpha_1 + \alpha_2 + \alpha_3 = 5 - 3 + 8 - 2 = 8$$

Q.26 (1 and 2)



$$\text{D.R of line is } = \begin{vmatrix} i & j & k \\ a & b & 0 \\ a & b & c \end{vmatrix} = i(bc) + j(ac) + 0k$$

D.R. of line (b, a 0)

$$0x + y - z + 2 = 0$$

$$(0, 1, -1)$$

$$\sin \theta = \frac{a}{\sqrt{2}\sqrt{b^2 + a^2}} = \frac{1}{2}$$

$$\sqrt{2}a = \sqrt{a^2 + b^2}$$

$$a^2 = b^2 \Rightarrow a = b \text{ or } a = -b$$

$$(1, -1, 0) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$$

Q.27 (4)

$$\text{Given: } L_1: \frac{x-1}{\lambda} = \frac{y-2}{1} = \frac{z-3}{2}$$

$$\text{and } L_2: \frac{x+26}{-2} = \frac{y+18}{3} = \frac{z+28}{\lambda}$$

are coplanar

$$\Rightarrow \begin{vmatrix} 27 & 20 & 31 \\ \lambda & 1 & 2 \\ -2 & 3 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda = 3$$

Now, normal of plane P, which contains L_1, L_2

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ -2 & 3 & 3 \end{bmatrix}$$

$$= -3\hat{i} - 13\hat{j} + 11\hat{k}$$

 \Rightarrow Equation of required plane P :

$$3x + 13y - 11z + 4 = 0$$

(0, 4, 5) does not lie on plane P.

Q.28 (2)

Normal of plane P :

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -3 \\ -1 & 2 & -2 \end{bmatrix} = 4\hat{i} - \hat{j} - 3\hat{k}$$

Equation of plane P which passes through (2, 2, -2)

$$\text{is } 4x - y - 3z - 12 = 0$$

Now, A(3, 0, 0), B(0, -12, 0), C(0, 0, -4)

$$\Rightarrow \alpha = 3, \beta = -12, \gamma = -4$$

$$\Rightarrow p = \alpha + \beta + \gamma = -13$$

Now, volume of tetrahedron OABC

$$V = \left| \frac{1}{6} \overline{OA} \times \overline{OC} \right| = 24$$

$$(V, p) = (24, -13)$$

Q.29 BONUS

DR's of line of shortest distance

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 2 & 2 & 1 \end{vmatrix} = -\hat{i} + 2\hat{j} - 2\hat{k}$$

Angle between line and plane is $\cos^{-1} \sqrt{\frac{2}{27}} = \alpha$

$$\cos \alpha = \sqrt{\frac{2}{27}}, \sin \alpha = \frac{5}{3\sqrt{3}}$$

DR's normal to plane $(1, -1, -1)$

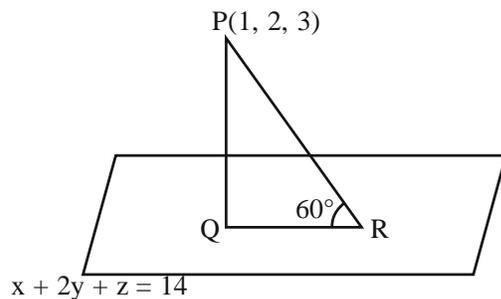
$$\sin \alpha = \left| \frac{-a - 2 + 2}{\sqrt{4 + 4 + 1} \sqrt{a^2 + 1 + 1}} \right| = \frac{5}{3\sqrt{3}}$$

$$\sqrt{3}|a| = 5\sqrt{a^2 + 2}$$

$$3a^2 = 25a^2 + 50$$

Ans. No value of (a) [Bonus]

Q.30 (2)



Length of perpendicular

$$PQ = \left| \frac{1 + 4 + 3 - 14}{\sqrt{6}} \right| = \sqrt{6}$$

$$QR = (PQ) \cot 60^\circ = \sqrt{2}$$

$$\therefore \text{Area of } \Delta PQR = \frac{1}{2} (PQ)(QR) = \sqrt{3}$$

Q.31 (4)

$A(2, 3, 9); B(5, 2, 1); C(1, \lambda, 8); D(\lambda, 2, 3)$

$$[\overline{AB} \ \overline{AC} \ \overline{AD}] = 0$$

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda - 3 & -1 \\ \lambda - 2 & -1 & -6 \end{vmatrix} = 0$$

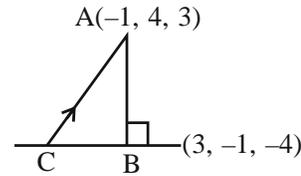
$$\Rightarrow [-6(\lambda - 3) - 1] - 8(1 - (\lambda - 3)(\lambda - 2)) + (6 + (\lambda - 2)) = 0$$

$$3(-6\lambda + 17) - 8(-\lambda^2 + 5\lambda - 5) + (\lambda + 4) = 8$$

$$8\lambda^2 - 57\lambda + 95 = 0$$

$$\lambda_1 \lambda_2 = \frac{95}{8}$$

Q.32 (2)



Let B be foot of \perp coordinates of $B = \left(-2, \frac{7}{2}, \frac{3}{2}\right)$

Direction ratio of line AB is $\langle 2, 1, 3 \rangle$ so $m = 1, n = 3$

So, equation of AC is $\frac{x+1}{3} = \frac{y-4}{-1} = \frac{z-3}{-4} = \lambda$

So point C is $(3\lambda - 1, -\lambda + 4, -4\lambda + 3)$. But C lies on the plane, so

$$6\lambda - 2 - \lambda + 4 - 12\lambda + 9 = 4$$

$$\Rightarrow \lambda = 1 \Rightarrow C(2, 3, -1)$$

$$\Rightarrow AC = \sqrt{26}$$

Q.33 (10)

$(a, -4a, -7) \perp$ to $(3, -1, 2b)$

$$a = 2b \quad \dots (1)$$

$(a, -4a, -7) \perp$ to $(b, a, -2)$

$$3a + 4a - 14b = 0$$

$$ab - 4a^2 + 14 = 0 \quad \dots (2)$$

From equations (1) and (2)

$$2b^2 - 16b^2 + 14 = 0$$

$$b^2 = 1$$

$$a^2 = 4b^2 = 4$$

$$\frac{x+1}{5} = \frac{y-2}{3} = \frac{z}{1} = k$$

$$\alpha = 5k - 1, \beta = 3k + 2, \gamma = k$$

As (α, β, γ) satisfies $x - y + z = 0$

$$5k - 1 - (3k + 2) + k = 0$$

$$k = 1$$

$$\therefore \alpha + \beta + \gamma = 9k + 1 = 10$$

Q.34 (450)

DR's of RS $\equiv (\alpha, -1, \beta)$

$$\text{DR of PQ} \equiv \left(\frac{56}{17} + 2, \frac{43}{17} + 1, \frac{111}{17} - 1 \right)$$

$$\equiv \left(\frac{90}{17}, \frac{60}{17}, \frac{94}{17} \right)$$

$$\frac{90}{17}\alpha + \frac{60}{17}(-1) + \frac{94}{17}\beta = 0$$

$$90\alpha + 94\beta = 60$$

$$\beta = \frac{60 - 90\alpha}{94}$$

$$\beta = \frac{30(2 - 3\alpha)}{94}$$

$$\beta = -30 \frac{(3\alpha - 2)}{94}$$

$$\beta = \frac{-15}{47}(3\alpha - 2)$$

$$\Rightarrow \frac{\beta}{-15} = \frac{3\alpha - 2}{47}$$

$$\Rightarrow \beta = -15, \alpha = -15$$

$$\alpha^2 + \beta^2 = 225 + 225$$

$$= 450$$

Q.35

(4)

2x + ky - 5z = 1 and 3kx - ky + z = 5, k < 3

are mutually perpendicular then

$$2(3k) + k(-k) + (-5)(1) = 0$$

$$-k^2 + 6k - 5 = 0$$

$$\Rightarrow k^2 - 6k + 5 = 0$$

$$\Rightarrow k^2 - 5k - k + 5 = 0$$

$$\Rightarrow k(k-5) - (k-5) = 0$$

$$\Rightarrow k = 1, 5$$

$$\Rightarrow k = 1 \quad \because k < 3$$

\(\therefore\) given planes are

$$2x + y - 5z = 1 \quad \dots(1)$$

$$\text{and } 3x - y + z = 5 \quad \dots(2)$$

Now eqⁿ of plane passing through intersection of (1) and (2) is

$$(2x + y - 5z - 1) + \lambda(3x - y + z - 5) = 0 \quad \dots(3)$$

Now (3) made intercept of unit length on x-axis, i.e., it passes through (1, 0, 0)

$$\Rightarrow (2 - 1) + \lambda(3 - 5) = 0$$

$$\Rightarrow 1 - 2\lambda = 0$$

$$\Rightarrow \lambda = \frac{1}{2}$$

$$\text{At } \lambda = \frac{1}{2} \text{ in eq}^n(3)$$

$$\left(2 + \frac{3}{2}\right)x + \left(1 - \frac{1}{2}\right)y + \left(-5 + \frac{1}{2}\right)z + \left(-1 - \frac{5}{2}\right) = 0$$

$$\Rightarrow 7x + y - 9z - 7 = 0 \quad \dots(4)$$

for finding intercept on y-axis; (y, 0, 0) satisfies (4),

$$y = 7$$

therefore, correct answer is D.

Q.36

[12]

$$\text{Equation of plane } 4ax - y + 5z - 7a + \lambda(2x - 5y - z - 3) = 0$$

$$= 0$$

this satisfies (4, -1, 0)

$$16a + 1 - 7a + \lambda(8 + 5 - 3) = 0$$

$$9a + 1 + 10\lambda = 0$$

Normal vector of the plane A is (4a + 2\lambda, -1 - 5\lambda, 5 - \lambda)

vector along the line which is contained in the plane A

is i - 2j + k

$$\therefore 4a + 2\lambda + 2 + 10\lambda + 5 - \lambda = 0$$

$$11\lambda + 4a + 7 = 0 \dots(2)$$

Solve (1) and (2) to get a = 1, \lambda = -1

Now equation of plane

$$x + 2y + 3z - 2 = 0$$

Let the point in the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4} = t$ is (7t

+ 3, -t + 2, -4t + 3) Satisfy the equation of plane A

$$\therefore 7t + 3 - 2t + 4 + 9 - 12t - 2 = 0$$

$$t = 2$$

$$\text{So } \alpha + \beta + \gamma = 2t + 8 = 12$$

$$[2]$$

Q.37

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + a\hat{j} - \hat{k})$$

$$\vec{r} = (\hat{i} + \hat{j}) + M(-\hat{i} + \hat{j} - a\hat{k})$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & a & -1 \\ -1 & 1 & -a \end{vmatrix}$$

$$= \hat{i}(-a^2 + 1) - \hat{j}(-a - 1) + \hat{k}(1 + a)$$

$$\langle -a^2 + 1, a + 1, a + 1 \rangle$$

$$(-a^2 + 1)(a - 1) + (a + 1)(y - 1) + (a + 1)z = 0$$

$$\left| \frac{(-a^2 + 1) + 4(a + 1)}{\sqrt{(-a^2 + 1)^2 + (a + 1)^2 + (a + 1)^2}} \right| = \sqrt{3}$$

$$\Rightarrow (-a^2 + 4a + 5)^2 = 3((-a^2 + 1)^2 + (a + 1)^2 + (a + 1)^2)$$

$$\Rightarrow a^4 + 16a^2 + 25 - 8a^3 + 409 - 109^2$$

$$\Rightarrow 3(a^4 + 1 - 2a^2 + 2a^2 + 2 + 49)$$

$$\Rightarrow a^4 + 16a^2 + 25 - 8a^3 + 409 - 109^2$$

$$\Rightarrow 3a^4 + 3 + 6 + 12a$$

$$\Rightarrow 2a^4 + 8a^3 - 6a^2 - 28a - 16 = 0$$

$$\Rightarrow a^4 + 4a^3 - 3a^2 - 14a - 8 = 0$$

$$\Rightarrow (a + 1)^2(a - 2)(a + 4) = 0$$

$$\Rightarrow a = -1, 2, -4$$

Largest value of a = 2

[125]

Q.38

$$\vec{n}_1 = \ell\hat{i} - \hat{j} + 3(1 - \ell)\hat{k}$$

$$\vec{n}_2 = \hat{i} + 2\hat{j} - \hat{k}$$

$$\text{Direction ratio of line} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \ell & -1 & 3(1 - \ell) \\ 1 & 2 & -1 \end{vmatrix}$$

$$= (6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$$

3x - 8y + 7z = 4 will contain the line

$$(6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$$

Normal of 3x - 8y + 7z = 4 will be perpendicular to the line

$$= 3(6\ell - 5) + (3 - 2\ell)(-8) + 7(2\ell + 1) = 0$$

$$\Rightarrow \ell = \frac{2}{3}$$

$$\therefore \text{direction ratio of line } \left(-1, \frac{5}{3}, \frac{7}{3}\right)$$

Angle with axis

$$\cos \theta = \frac{5/3}{\sqrt{1 + \frac{25}{9} + \frac{49}{9}}}$$

$$\therefore 415 \cos^2 \theta = \frac{25}{83} \times 415 = 125$$

Q.39 (1)

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix}$$

$$(\hat{i} - \hat{j}(4) + \hat{k}(-3))$$

$$= \hat{i} - 4\hat{j} - 3\hat{k}$$

Equation of line

$$\frac{x-1}{1} = \frac{y-2}{-4} = \frac{z-4}{-3} = \lambda$$

$$\vec{PM} \cdot (1, -4, -3) = 0$$

$$\Rightarrow (\lambda, -4\lambda + 4, -3\lambda - 1) \cdot (1, -4, -3) = 0$$

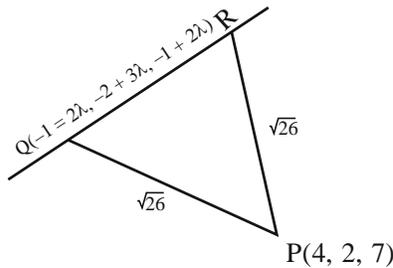
$$\Rightarrow \lambda + 16\lambda - 16 + 9\lambda + 3 = 0$$

$$\Rightarrow 26\lambda = 13$$

$$\lambda = \frac{1}{2}$$

$$PM = \sqrt{\frac{1}{4} + 4 + \frac{25}{4}} = \sqrt{\frac{13}{2} + 4} = \sqrt{\frac{21}{2}}$$

Q.40 [153]



$$PQ^2 = 26$$

$$(4 + 1 - 2\lambda)^2 + (2 + 2 - 3\lambda)^2 + (7 - 1 - 2\lambda)^2 = 26$$

$$(5 - 2\lambda)^2 + (4 - 3\lambda)^2 + (6 - 2\lambda)^2 = 26$$

$$\Rightarrow 25 + 4\lambda^2 - 20\lambda + 16 + 9\lambda^2 - 24\lambda + 36 + 4\lambda^2 - 24\lambda = 26$$

$$\Rightarrow 17\lambda^2 - 68\lambda + 77 = 26$$

$$\Rightarrow 17\lambda^2 - 68\lambda + 51 = 0$$

$$\Rightarrow \lambda^2 - 4\lambda + 3 = 0$$

$$\Rightarrow (\lambda - 3)(\lambda - 1) = 0$$

$$\Rightarrow \lambda = 1, 3$$

$$\therefore Q(1, 1, 3)$$

$$R(5, 7, 7)$$

$$P(4, 2, 3)$$

$$\vec{PQ} = 3\hat{i} + \hat{j} + 4\hat{k}$$

$$\vec{PR} = \hat{i} + 5\hat{j} + 0\hat{k}$$

$$A = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & 5 & 0 \end{vmatrix} = \frac{1}{2} |\hat{i}(0 - 20) - \hat{j}(0 - 4) + \hat{k}(15 - 1)|$$

$$= \frac{1}{2} |-20\hat{i} + 4\hat{j} + 14\hat{k}|$$

$$= \frac{1}{2} \sqrt{400 + 16 + 196}$$

$$= \frac{1}{2} \sqrt{612}$$

$$\therefore A^2 = \frac{1}{4} (612) = 153$$

Q.41 (3)

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= \hat{i}(-3) - \hat{j}(3) + \hat{k}(0)$$

$$\therefore \vec{n} = -\hat{i} - \hat{j}$$

$$\therefore \text{Plane is } -1(x - 1) - 1(y + 1) = 0$$

$$P: -x - y = 0 \Rightarrow x + y = 0$$

Distance from Q(a, a, 2)

$$\Rightarrow \left| \frac{a+a}{\sqrt{2}} \right| = 3\sqrt{2}$$

$$\therefore 2|a| = 6$$

$$a = 3 \text{ or } a = -3$$

$$P(1, -1, 1) \text{ \& } Q(3, 3, 2)$$

$$\text{or } Q(-3, -3, 2)$$

$$PQ^2 = 4 + 16 + 1 = 21$$

Q.42 (1)

$$\frac{x+7}{-6} = \frac{y-6}{7} = \frac{z-0}{1} \Rightarrow \vec{n}_1 = (-6, 7, 1)$$

$$\frac{x-7}{-2} = \frac{y-2}{1} = \frac{z-6}{1} \Rightarrow \vec{n}_2 = (-2, 1, 1)$$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 7 & 1 \\ -2 & 1 & 1 \end{vmatrix} = \hat{i}(6) - \hat{j}(-4) + \hat{k}(8) = (3, 2, 4)$$

$$\vec{a} = (-7, 6, 0), \vec{b} = (7, 2, 6)$$

$$\begin{aligned} \therefore S_d &= \left| \frac{(\vec{b} - \vec{a}) \cdot (\vec{n}_1 \times \vec{n}_2)}{|\vec{n}_1 \times \vec{n}_2|} \right| \\ &= \left| \frac{(14, -4, 6) \cdot (3, 2, 4)}{\sqrt{9+4+16}} \right| \\ &= \left| \frac{42 - 8 + 24}{\sqrt{29}} \right| = \frac{58}{\sqrt{29}} = 2\sqrt{29} \end{aligned}$$

LINEAR PROGRAMMING PROBLEMS

EXERCISE-I (MHT CET LEVEL)

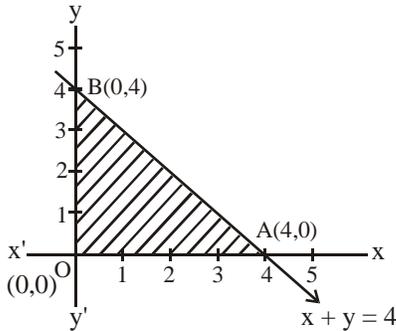
Q.1 (1)

$s x + y \leq 4, x \geq 0, y \geq 0$, therefore we shall shade the other inequalities in the first quadrant only. Now consider $x + y \geq 4$.

$$\text{Let } x + y = 4 \Rightarrow \frac{x}{4} + \frac{y}{4} = 1$$

Thus the line has 4 and 4 as intercepts along the axes. The corner points of the feasible are $O(0, 0), A(4, 0)$ and $B(0, 4)$.

The values of z at these points are



Corner point	$z = 3x + 4y$
$O(0, 0)$	0
$A(4, 0)$	12
$B(0, 4)$	$16 \rightarrow \text{maximum}$

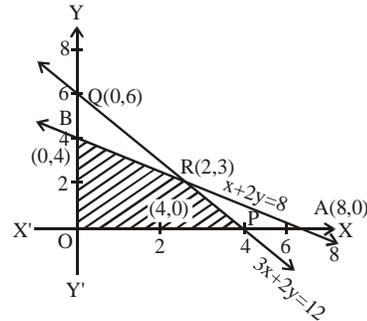
Therefore, the maximum value of z is 16 at the point $B(0, 4)$.

Q.2 (2)

Objective function $Z = -x + 2y \leq 8$,
 $3x + 2y \leq 12, x \geq 0, y \geq 0$

(i) Consider the line $x + 2y = 8$. It passes through $A(8, 0)$ and $B(0, 4)$, putting $x = 0, y = 0$ in $x + 2y \leq 8, 0 \leq 8$ which is true.

\Rightarrow region $x + 2y \leq 8$ lies on and below AB .



(ii) The line $3x + 2y = 12$ passes through $P(4, 0), Q(0, 6)$ putting $x = 0, y = 0$ in $3x + 2y \leq 12$

$$\Rightarrow 0 \leq 12, \text{ which is true}$$

\therefore Region $3x + 2y \leq 12$ lies on and below PQ .

(iii) $x \geq 0$ the region lies on and to the right of y -axis.

(iv) $y \geq 0$ lies on and above x -axis.

(v) Solving the equations $x + 2y = 8$ and $3x + 2y = 12$ we get $x = 2, y = 3 \Rightarrow R(2, 3)$ where AB and PQ intersect. The shaded region $OPRB$ is the feasible region.

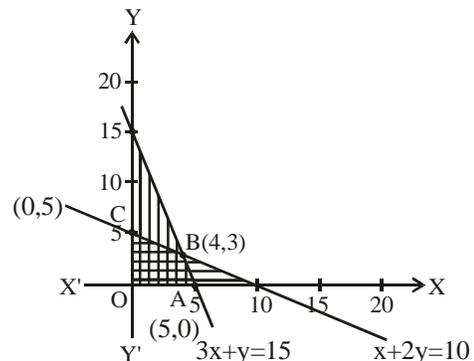
Corner point	$z = -3x + 4y$
$O(0, 0)$	0
$P(4, 0)$	$-12 \rightarrow \text{minimum}$
$R(2, 3)$	6
$B(0, 4)$	16

Thus minimum value of Z is -12 at $P(4, 0)$

Q.3

(1)

The feasible region determined by the constraints $x + 2y \leq 10, 3x + y \leq 15, x \geq 0$ and $y \geq 0$ is as follows.



The corner points of the feasible region are $O(0, 0), A(5, 0), B(4, 3), C(0, 5)$

A(5, 0), B(4, 3) and C(0, 5).

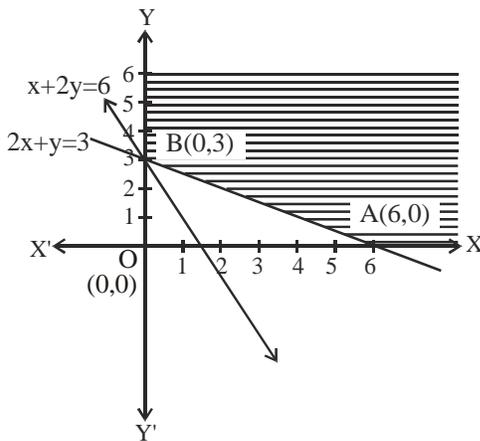
The value of Z at these corner points are as follows.

Corner point	$z = 3x + 2y$
A(5, 0)	15
B(4, 3)	18 → Max.
C(0, 5)	10

Therefore, the maximum value of z is 18 at the point (4, 3).

Q.4

(4) The feasible region determined by the constraints $2x + y \geq 3, x + 2y \geq 6, x \geq 0$ and $y \geq 0$ is as follows.



(0, 0) is not contained in the required half plane as (0, 0) does not satisfy the inequation $2x + y \geq 3$.

The corner points of the feasible region are A(6, 0) and B(0, 3).

The values of z at these corner points are as follows :

Corner point	$z = x + 2y$
A(6, 0)	6
B(0, 3)	6

Here also (0, 0) does not contain the required half plane. The double -shaded region XABY' is the solution set. Its corners are A(6, 0) and B(0, 3).

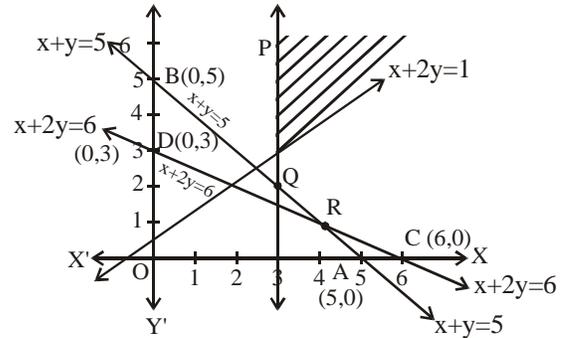
It can be seen that the value of z at points A and B is same. If we take any other point such as (2, 2) online $x + 2y = 6$, then $z = 6$.

Thus, the minimum value of z occur for more than 2 point.

Therefore, the value of z is minimum at every point on the line $x + 2y = 6$.

Q.5

(4) The feasible region determined by constraints $x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0$ is as follows



It can be seen that the feasible region is unbounded. The value of z at corner points A(6, 0), B(4, 1) and C(3, 2) as follows :

The value of z at corner points A(6, 0), B(4, 1) and C(3, 2) as follows :

It can be seen that the feasible region is unbounded. The value of z at corner points A(3, 2), R(4, 1) and C(6, 0) are as follows :

At Q (3, 2) $Z = -x + 2y = -3 + 4 = 1 \rightarrow \max^m$

At R (4, 1) $Z = -4 + 2 = -2$

At C (6, 0) $Z = -6 + 0 = -6 \rightarrow \min^m$

The maximum value of Z is 1 but the feasible region is unbounded. Consider the inequation $-x + 2y > 1$.

The line $-x + 2y = 1$ passes through (-1, 0) and (0, 1/2) putting $x = 0, y = 0$ in $-x + 2y > 1$,

We get $0 > 1$ which is not true.

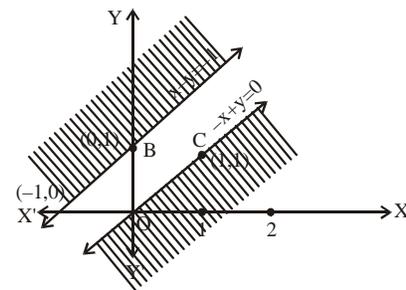
$\Rightarrow -x + 2y > 1$ lies above the line $-x + 2y = 1$ feasible region of $-x + 2y > 1$ have many points in common.

Therefore, there is no maximum value.

Q.6

(4) Objective function $Z = x + y$, constraints $x - y \leq -1, -x + y \leq 0, x, y \geq 0$

The region determined by the constraints is as follows :

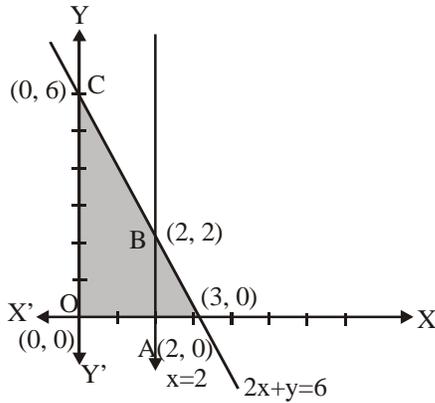


There is no feasible region and thus, Z has no maximum value.

Q.7

(1) We have maximise $Z = 11x + 7y$ (i)
Subject to the constraints(ii)
 $2x + y \leq 6 \quad x \leq 2$ (iii)
 $x \geq 0, y \geq 0$ (iv)

We see that, the feasible region as shaded determined by the system of constraints (ii) to (iv) is OABC and is bounded. So. now we shall use corner point method to determine the maximum value of Z.



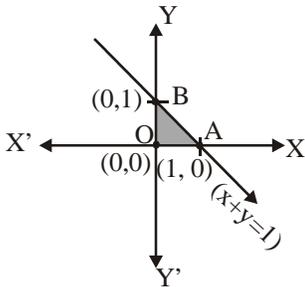
Corner points	Corresponding value of Z
(0,0)	0
(2,0)	22
(2,2)	36
(0,6)	42 ← maximum

Hence, the maximum value of Z is 42 at (0,6).

Q.8

(2) Maximise $Z = 3x + 4y$. Subject to the constraints $x + y \leq 1, x \geq 0, y \geq 0$.

The shaded region shown in the figure as OAB is bounded and the coordinates of corner points O, A and B are (0,0) (1,0) and (0,1), respectively.

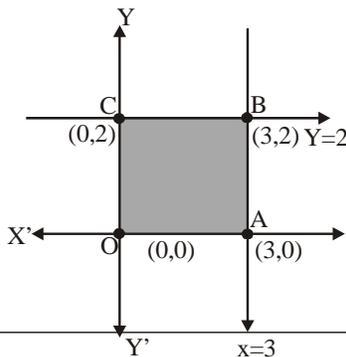


Corner points	Corresponding value of Z
(0,0)	0
(1,0)	3
(0,1)	4 ← Maximum

Hence, the maximum value of Z is 4 at (0,1).

Q.9

(3) Maximise $Z = 11x + 7y$, subject to the constraints $x \leq 3, y \leq 2, x \geq 0, y \geq 0$.



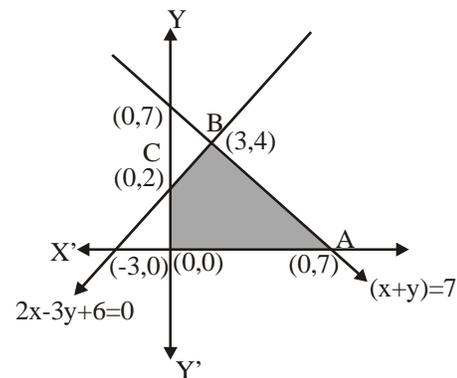
The shaded region as shown in the figure as OABC is bounded and the coordinates of corner points are (0,0) (3,0), (3,2) and (0,2), respectively.

Corner points	Corresponding value of Z
(0,0)	0
(3,0)	33
(3,2)	47 ← Maximum
(0,2)	14

Hence, Z is maximum at (3,2) and its maximum value is 47.

Q.10

(1) Minimise $Z = 13x - 15y$ subject to the constraints $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$.



Shaded region shown as OABC is bounded and coordinates of its corner points are (0,0) (7,0) (3,4), and (0,2), respectively.

Corner points	Corresponding value of Z
(0,0)	0
(7,0)	91
(3,4)	-21
(0,2)	-30 ← Minimum

Hence, the minimum value of Z is (-30) at (0, 2).

Q.11

(1) The shaded region is bounded and has coordinates of corner points as (0,0) , (7,0) , (3,4), and (0,2), Also, $Z = 5x + 7y$.

Corner points	Corresponding value of Z
(0,0)	0
(7,0)	35
(3,4)	43 ← Maximum
(0,2)	14

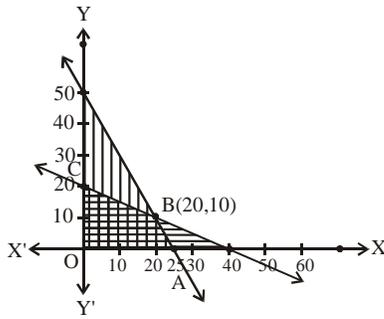
Hence, the maximum value of Z is 43 at (3,4).

Q.12

(1) Let number of cakes made of first kind are x and that of second kind is y.
 \therefore maximize $Z = x + y$
 Subject to $200x + 100y \leq 5000$ and $25x + 50y \leq 1000$
 $x \geq 0, y \geq 0$.

The feasible region determined by the system of

constraints is as follows.



The corner points are $O(0, 0)$, $A(25, 0)$, $B(20, 10)$, $C(0, 20)$.

The value of Z at these corner points are as follows :

Consider $Z = x + y$

Now at $O(0, 0)$, $Z = 0 + 0 = 0$

At $A(25, 0)$, $Z = 25 + 0 = 25$

At $B(20, 10)$, $Z = 20 + 10 = 30$

At $C(0, 20)$, $Z = 0 + 20 = 20$

Maximum number of cakes $Z = 30$ when $x = 20, y = 10$

Q.13 (1)

Let x tennis rackets and y cricket bats are produced in one day in the factory.

Item	Number	Machine Hours	Craftman Hours	Profit
Tennis	x	1.5	3	Rs.20 per item
Cricket Bats	y	3	1	Rs. 10 per item
Total time Available		42	24	

Total Machine hours = $1.5x + 3y$,

Maximum time available = 42 hours

$1.5x + 3y \leq 42$ or $x + 2y \leq 28$ (i)

Craftman's hours = $3x + y$

Maximum time available = 24

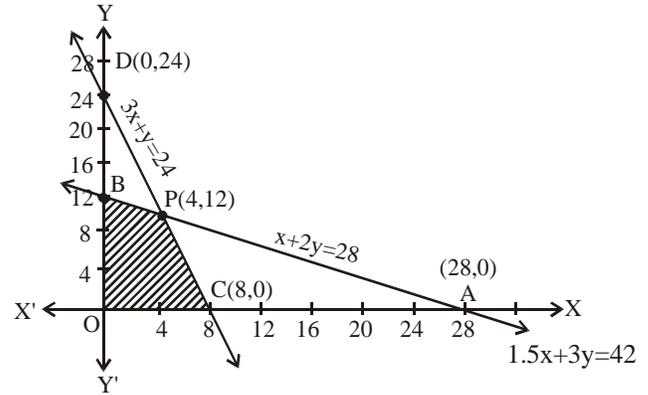
$3x + y \leq 24$ (ii)

Also $y \geq 0$

(i) $z = x + y$ constraints are $x + 2y \leq 28$,

$3x + y \leq 24, x, y \geq 0$

The feasible region determine by the system of constraint is as follows.



The corner points are $B(0, 14)$, $P(4, 12)$, $C(8, 0)$ and $D(0, 0)$.

The values of z at these corner points are as follow :

Profit function $z = 20x + 10y$

At $B(0, 14)$ $z = 0 + 10 \times 14 = 140$

At $P(4, 12)$ $z = 80 + 120 = 200 \rightarrow \text{max}^m$. Profit

At $C(8, 0)$ $z = 20 \times 8 + 0 = 160$

At $D(0, 0)$ $z = 0$

Thus maximum profit is Rs. 200, when 4 tennis racket and 12 cricket bats are produced.

Q.14 (2)

Let x nuts and y bolts are produced.

Item	Number	Machine A	Machine B	Profit
Nuts	x	1 hours	3 hours	17.50
Bolts	y	3 hours	1 hours	7.00
Max. Time available		12 hours	12 hours	

Machine A is used for $x \times 1 + y \times 3$ hours,

Maximum time available = 12 hours,

$\therefore x + 3y \leq 12$

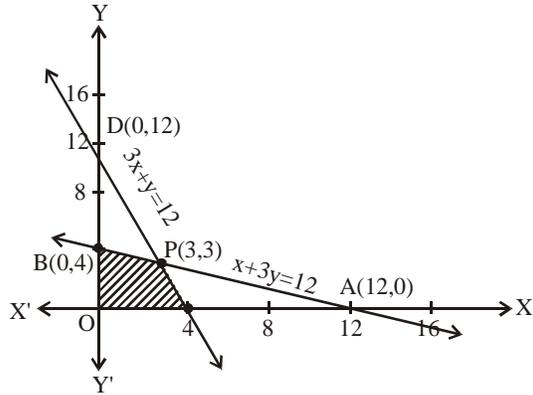
Machine B is used for $3 \times x + 1 \times y$ hours.

Maximum time available = 12 hours.

$\Rightarrow 3x + y \leq 12$

Profit function $z = (17.50) \times x + (7.00)y$. Thus the objective function $z = 17.5x + 7y$, subject to constraints are $x + 3y \leq 12, 3x + y \leq 12$ and $x, y \geq 0$

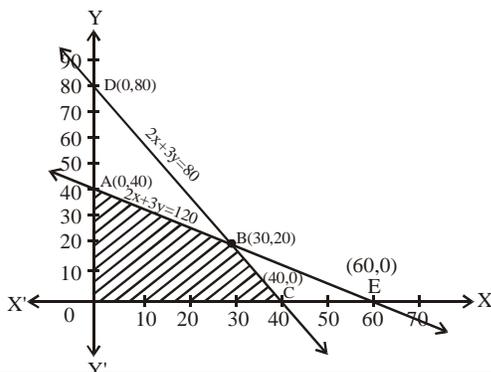
The feasible region determined by the system of constraints is as follows.



The corner points are $B(0, 4)$, $P(3, 3)$, $C(4, 0)$, $O(0, 0)$.
 The values of z at these corner points are as follows.
 At $B(0, 4)$ $z = 0 + 7 \times 4 = 28$
 At $P(3, 3)$ $z = 17.5 \times 3 + 7 \times 3 = 73.5$
 At $C(4, 0)$ $z = 17.5 \times 4 + 0 = 70$
 At $O(0, 0)$ $z = 0$
 Maximum profit is Rs. 73.50 when 3 nuts and 3 bolts packages are produced.

Q.15

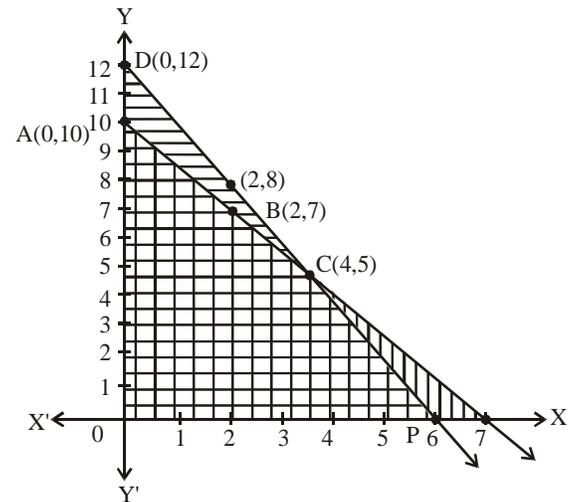
(3)
 Let the manufacturer produces x packages of screws A and y packages of screw B, then time taken by x packages of screw A and y packages of screw B on automatic machine = $(4x + 6y)$ minutes. And hand operated machine = $(6x + 3y)$ minutes
 As each machine is available for at the most 4 hours i.e. $4 \times 60 = 240$ minutes. So,
 we have $4x + 6y \leq 240$ i.e. $2x + 3y \leq 120$ and $6x + 3y \leq 240$ i.e. $2x + y \leq 80$
 Profit on selling x packages of screws A and y packages of screws B is $z = 7x + 10y$.
 So, to find x and y such that $z = 7x + 10y$ is maximum subject to $2x + 3y \leq 120$, $2x + y \leq 80$, $x \geq 0$ and $y \geq 0$. The feasible portion of the graph satisfying the inequalities $2x + 3y \leq 120$ and $2x + y \leq 80$ is OABC which is shaded in the figure. Coordinates of O, A, B and C $(0, 0)$, $(0, 40)$, $(30, 20)$ and $(40, 0)$ respectively.
 At A $(0, 40)$ $z = 7x + 10y = 0 + 10 \times 40 = 400$
 At B $(30, 20)$ $z = 7 \times 30 + 10 \times 20 = 410$ max^m
 At C $(40, 0)$ $z = 7 \times 40 + 0 = 280$
 At O $(0, 0)$ $z = 0$



Hence, the maximum profit Rs. 410 when 30 screws of type A and 20 screws of type B are produced.

Q.16

(3)
 Let the manufacturer produces x pedestal lamps and y wooden shades; then the time taken by x pedestal lamps and y wooden shades on grinding/ cutting machines = $(2x + y)$ hours and time taken by x pedestal lamps and y shades on the sprayer = $(3x + 2y)$ hours. Since grinding/cutting machine is available for at the most 12 hours, $2x + y \leq 12$ and sprayer is available for at the most 20 hours.
 We have: $3x + 2y \leq 20$.
 Profit from the sale of x lamps and y shades.
 $Z = 5x + 3y$
 So, our problem is to maximize $Z = 5x + 3y$ subject to constraints $3x + 2y \leq 20$, $2x + y \leq 12$, $x, y \geq 0$
 The feasible region determined by the system of constants is as follows.



The corner points are $O(0, 0)$, $P(6, 0)$, $C(4, 4)$ and $A(0, 10)$.
 The values of z at these corner points are as follows.
 Now $Z = 5x + 3y$
 At $O(0, 0)$, $Z = 0$
 At $P(6, 0)$, $Z = 30$
 At $C(4, 4)$, $Z = 5 \times 4 + 3 \times 4 = 32$
 At $A(0, 10)$, $Z = 5 \times 0 + 3 \times 10 = 30$
 Now maximum $Z = 32$ at $x = 4, y = 4$.
 Hence maximum $Z =$ Rs. 32 when he manufacture 4 pedestal lamps and 4 wooden shades.

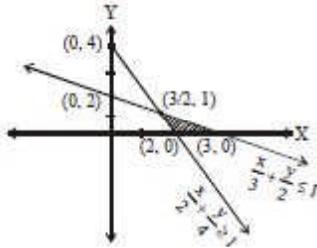
Q.17

Q.18

(3)
 (4)
 Here $(0, 2)$, $(0, 0)$ and $(3, 0)$ all are vertices of feasible region.

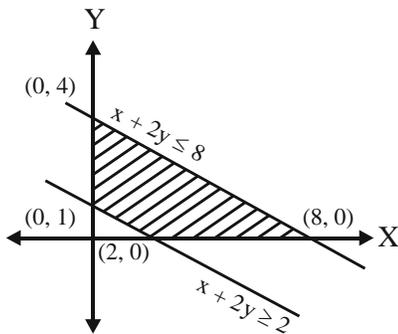
Q.19 (4)
Clearly point (2000, 0) is outside.

Q.20 (3)
Consider $\frac{x}{2} + \frac{y}{4} \geq 1, \frac{x}{3} + \frac{y}{2} \leq 1$ $x, y \geq 0$ convert them into equation and solve them and draw the graph of these equations we get $y=1$ and $x=3/2$



From graph region is finite but numbers of possible solutions are infinite because for different values of x and y we have different or different values solutions.

Q.21 (2)
Given : $x + 2y \geq 2$ (i)
 $x + 2y \leq 8$ (i)
and $x, y \geq 0$



From equation (i)

$$\frac{x}{2} + \frac{y}{1} = 1$$

and for equation (2)

$$\frac{x}{8} + \frac{y}{4} = 1$$

Given : $z = 3x + 2y$

At point (2, 0); $z = 3 \times 2 + 0 = 6$

At point (0, 1); $z = 3 \times 0 + 2 \times 1 = 2$

At point (8, 0); $z = 3 \times 8 + 2 \times 0 = 24$

At point (0, 4); $z = 3 \times 0 + 2 \times 4 = 8$

\therefore maximum value of z is 24 at point (8, 0).

Q.22 (3)

Q.23 (1)

Q.24 (2)

Q.25 (3)

Q.26 (4)

Q.27 (1)

Q.28 (2)

tickets sold be x and y .

Now, as the seating capacity of the aeroplane is 200, so $x + y \leq 200$.

As 20 tickets for executive class are to be reserved so we have $x \geq 20$.

And as the number of tickets of economy class should be at least 4 times that of executive class $y \geq 4x$. Profit on the sale of x tickets of Ist class and y tickets of economy class is

$$z = 1000x + 600y.$$

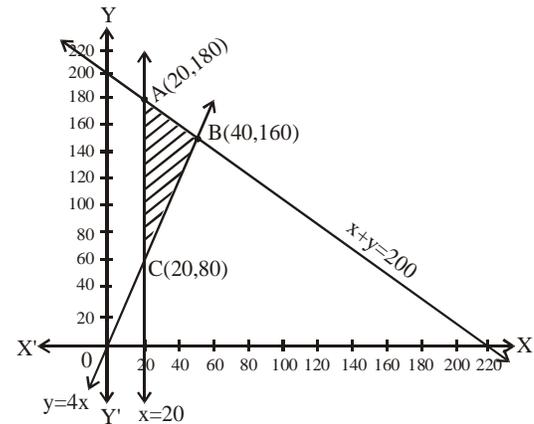
\therefore L.P.P is i.e. maximize, $(z) = 1000x + 600y$, subject to constraints are $x + y \leq 200, x \geq 20, y \geq 4x$ and $x, y \leq 0$. The region satisfying the inequality $x + y \leq 200, x \geq 20$ and $y \geq 4x$ is ABC.

$$z = 1000x + 600y \quad \text{At A}(20, 180), \quad z = 1000 \times 20 + 600 \times 180 = 20000 + 108000 = 128000$$

$$\text{At B}(40, 160), \quad z = 1000 \times 40 + 600 \times 160 = 40000 + 96000 = 136000 \text{ max}^m$$

$$\text{At C}(20, 80), \quad z = 1000 \times 20 + 600 \times 80 = 20000 + 48000 = 68000$$

The feasible region determined by the constraints is as follows.



$\Rightarrow z$ is maximum when $x = 40, y = 160$

\Rightarrow 40 tickets of executive class and 160 tickets of economy class should be sold to get the maximum profit of Rs. 136000.

Q.33 (1)

Let x kg of food P and y kg of food Q are mixed. we have

Food	Quantity	Vitamin A	Vitamin B	Cost
P	x kg	3units/kg	5units/kg	Rs. 60/kg
Q	y kg	4units/kg	2units/kg	Rs. 80/kg
least quantity of vitamin required		8unit	11unit	

Quantity of vitamin A in x kg food P and y kg food

$$Q = 3x + 4y$$

Quantity of vitamin required at least = 8 units

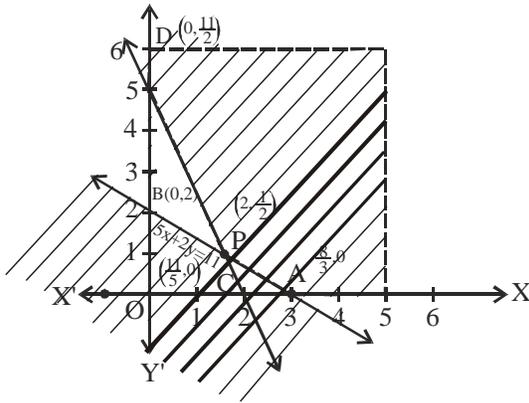
$$\Rightarrow Q = 3x + 4y \geq 8$$

Quantity of vitamin B in x kg food P and y kg of food

$$Q = 5x + 2y$$

Quantity of vitamin required at least = 11 units

constraints is as follows :



It can be seen that the feasible region is unbounded. The corner points of the region are

$$D\left(0, \frac{11}{2}\right), P\left(2, \frac{1}{2}\right), A\left(\frac{8}{3}, 0\right)$$

The value of z at these corner points are as follows. Now objective function $z = 60x + 80y$,

$$\text{At } D\left(0, \frac{11}{2}\right), z = 80 \times \frac{11}{2} = 440$$

$$\text{At } P\left(2, \frac{1}{2}\right), z = 120 \times 40 = 160 \text{ min}^m \text{ cost,}$$

$$\text{At } A\left(\frac{8}{3}, 0\right), z = 60 \times \frac{8}{3} + 0 = 160 \rightarrow \text{min}^m \text{ cost}$$

Minimum value of $z = 160$. But feasible region is unbounded

\therefore Consider the inequality $60x + 80y < 160$ or $3x + 4y < 8$

This shows that this region is below the line AD : $3x + 4y = 8$

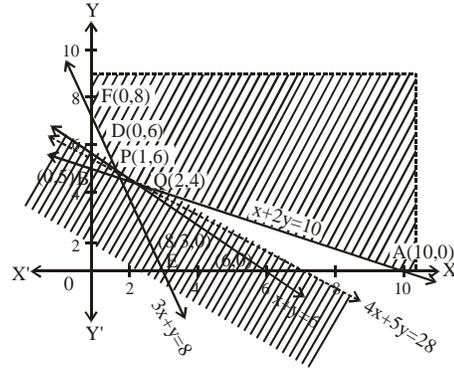
\Rightarrow There is no point in common between feasible region and $3x + 4y < 8$. Hence the minimum cost z is Rs. 160 at

all point lying on the segment joining $A\left(\frac{8}{3}, 0\right)$ and

$$P\left(2, \frac{1}{2}\right)$$

Q.34

(4) Let there be x kg food X and y kg food Y. Food X costs Rs. 16. and food Y costs Rs. 20 per kg. Objective function $Z = 16x + 20y$, subject to constraints are, $x + 2y \geq 10$, $2x + 2y \geq 12$ and $3x + y \geq 8$, $x, y \geq 0$
The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A(10, 0), Q(2, 4), P(1, 5) and F(0, 18).

The value of z at these corner points are as follows.

The objective function is $z = 16x + 20y$

$$\text{At } F(0, 8), Z = 16x + 20y = 0 + 160 = 160$$

$$\text{At } P(1, 5), Z = 16 + 100 = 116$$

$$\text{At } Q(2, 4), Z = 32 + 80 = 112 \quad \text{min}^m$$

$$\text{At } A(10, 0) z = 160 + 0 = 160$$

This shows that minimum value of z is Rs. 112 feasible region is unbounded.

Consider the inequality.

$$16x + 20y < 112 \text{ or } 4x + 5y = 28$$

The line $4x + 5y = 28$ passes through (7, 0) and

$$\left(0, \frac{28}{5}\right) \text{ Also putting } x=0, y=0 \text{ in } 4x + 5y < 28 \text{ we get}$$

$$0 < 28 \text{ which is true.}$$

$$\therefore 8x + 10y = 33 \text{ lies below the line } 4x + 5y = 28$$

Hence there is no common point between feasible region and $4x + 5y < 28$

\Rightarrow Minimum value of $z = 112$ at $x = 2, y = 4$ i.e. least value of mixture is Rs. 112 when 2 kg of food x and 4 kg of food y are mixed.

Q.35

(1)

The function is given by profit function

$$= x \cdot \frac{8}{100} + y \times \frac{10}{100} = 0.08x + 0.10y.$$

Q.36

(1)

$$x, y \geq 0 \text{ and } 4x + 3y \leq 24$$

Q.37

(3)

Constraints will be

$$x_{11} + x_{21} + \dots + x_{m1} = b_1$$

$$x_{12} + x_{22} + \dots + x_{m2} = b_2$$

$$x_{1n} + x_{2n} + \dots + x_{mn} = b_n$$

$$x_{11} + x_{12} + \dots + x_{1n} = b_1$$

$$x_{21} + x_{22} + \dots + x_{2n} = b_2$$

$$x_{m1} + x_{m2} + \dots + x_{mn} = b_n$$

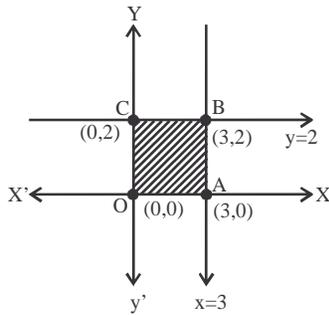
So, total number of constraints = $m + n$

PREVIOUS YEAR'S

MHT CET

- Q.1** (3) **Q.2** (2) **Q.3** (2) **Q.4** (1) **Q.5** (3)
Q.6 (3) **Q.7** (2) **Q.8** (1) **Q.9** (2) **Q.10** (2)
Q.11 (3) **Q.12** (1) **Q.13** (4) **Q.14** (3) **Q.15** (1)
Q.16 (3) **Q.17** (4) **Q.18** (1) **Q.19** (3) **Q.20** (4)
Q.21 (4) **Q.22** (3)
Q.23 (4)

Maximise $z = 11x + 7y$, subject to the constraints $x \leq 3$, $y \geq 2$, $y \geq 0$



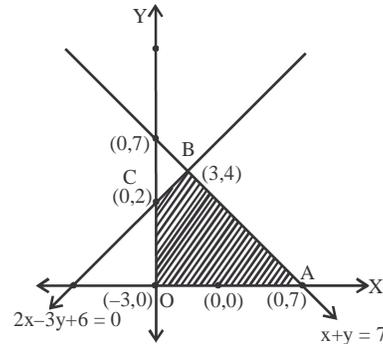
The shaded region as shown in the figure as OABC is bounded and the coordinates of corner points are (0,0), (3,0), (3,2) and (0,2) respectively.

Corner points	Corresponding value of Z
(0,0)	0
(3,0)	33
(3,2)	47 ← Maximum
(0,2)	14

Hence, Z is maximum at (3,2) and its maximum value is 47.

Q.24 (3)

Minimize $Z = 13x - 15y$ subject to the constraints $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$, $y \geq 0$



Shaded region shown as OABC is bounded and coordinates of its corner points are (0,0), (7,0), (3,4) and (0,2) respectively,

Corner points	Corresponding value of Z
(0,0)	0
(7,0)	91
(3,4)	-21
(0,2)	-30 ← Minimum

Hence, the minimum value of Z is (-30) at (0,2).

Q.25 (2)

From the line $x + 2y = 6$, shaded region is away from the origin $\Rightarrow x + 2y \geq 6$

From the line $5x + 3y = 15$, shaded region is away from the origin $\Rightarrow 5x + 3y \geq 15$

From the line $x = 7$, shaded region is towards the origin $\Rightarrow y \leq 6$

\therefore Shaded region is in the first quadrant

$\Rightarrow x, y \geq 0$

PROBABILITY

EXERCISE-I (MHT CET LEVEL)

Q.1 (3)

It is obvious.

Q.2 (1)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{5}{6} = \frac{2}{3} + \frac{1}{2} - P(A \cap B) \Rightarrow P(A \cap B) = 0$$

∴ Events A and B are mutually exclusive.

Q.3 (2)

$$\frac{n!}{n^n} = \frac{3}{32} \Rightarrow \frac{n!}{n^n} = \frac{8 \times 3}{8 \times 32} = \frac{4!}{4^4}$$

∴ n = 4

Q.4 (4)

Q.5 (1)

Q.6 (3)

Q.7 (3)

Q.8 (3)

Q.9 (3)

Required probability is

$$P(\text{getting } 8) + P(9) + P(10) + P(11) + P(12)$$

$$= \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{15}{36} = \frac{5}{12}$$

Q.10 (2)

$$\text{Required probability} = \frac{4}{6} = \frac{2}{3}$$

Q.11 (3)

Since both heads and tails appears, so

$$n(S) = \{\text{HHT, HTH, THH, HTT, THT, TTH}\}$$

$$n(E) = \{\text{HTT, THT, TTH}\}$$

$$\text{Hence required probability} = \frac{3}{6} = \frac{1}{2}$$

Q.12 (3)

$$\text{Required probability} = \frac{16}{52} = \frac{4}{13}$$

(Since diamond has 13 cards including a king and there are another 3 kings).

Q.13 (4)

Let R stand for drawing red ball B for drawing black ball and W for drawing white ball.

Then required probability

$$= P(WWR) + P(BBR) + P(WBR) + P(BWR) + P(WRR) +$$

$$P(BRR) + P(RWR) + P(RBR).$$

$$= \frac{3.2.2}{8.7.6} + \frac{3.2.2}{8.7.6} + \frac{3.3.2}{8.7.6} + \frac{3.3.2}{8.7.6} + \frac{3.2.1}{8.7.6}$$

$$+ \frac{3.2.1}{8.7.6} + \frac{2.3.1}{8.7.6} + \frac{2.3.1}{8.7.6}$$

$$= \frac{2}{56} + \frac{2}{56} + \frac{3}{56} + \frac{3}{56} + \frac{1}{56} + \frac{1}{56} + \frac{1}{56} + \frac{1}{56} = \frac{1}{4}$$

Q.14 (3)

Total cases = 36. Favourable cases = 2 + 4 + 2 = 8

$$\therefore \text{The required probability} = \frac{8}{36} = \frac{2}{9}$$

Q.15 (4)

$$P(\text{at least 1H}) = 1 - P(\text{No head}) \\ = 1 - P(\text{four tail})$$

$$= 1 - \frac{1}{16} = \frac{15}{16}$$

Q.16 (4)

P(at least one head) = 1 - P(0 head)

$$= 1 - P(\text{All tail}) = 1 - \frac{1}{8} = \frac{7}{8}$$

Q.17 (2)

Let 100 students studying in which 60 % girls and 40 % boys.

Boys = 40, Girls = 60

$$25\% \text{ of boys offer Maths} = \frac{25}{100} \times 40 = 10 \text{ Boys}$$

$$10\% \text{ of girls offer Maths} = \frac{10}{100} \times 60 = 6 \text{ Girls}$$

It means, 16 students offer Maths.

$$\therefore \text{Required probability} = \frac{16}{100} = \frac{4}{25}$$

Q.18 (3)

Required probability

$$= \frac{{}^7C_3}{{}^9C_5} + \frac{{}^7C_5}{{}^9C_5} = \frac{56}{126} = \frac{4}{9}$$

Q.19 (2)

$$\text{Total ways of arrangements} = \frac{8!}{2! \cdot 4!}$$

• w • x • y • z •

Now 'S' can have places at dot's and in places of w, x, y, z we have to put 2A's, one I and one N.

Therefore favourable ways = $5 \binom{4!}{2!}$

Hence required probability

$$= \frac{5 \cdot 4! \cdot 2! \cdot 4!}{2! \cdot 8!} = \frac{1}{14}.$$

Q.20

(3)

n = Total number of ways = 6^5

A total of 12 in 5 throw can be obtained in following two ways –

(i) One blank and four 3's = ${}^5C_1 = 5$

or (ii) Three 2's and two 3's = ${}^5C_2 = 10$

Hence, the required probability = $\frac{15}{6^5} = \frac{5}{2592}$.

Q.21

(1)

Required probability = $\frac{{}^{12}C_1}{{}^{20}C_1} = \frac{3}{5}$.

Q.22

(1)

Required probability = $\frac{{}^5C_1 \times {}^8C_1 + {}^5C_2}{{}^{13}C_2} = \frac{25}{39}$.

Q.23

(4)

Let each of the friend have x daughters. Then the probability that all the tickets go to the daughters of A

is $\frac{{}^x C_3}{{}^{2x} C_3}$. Therefore $\frac{{}^x C_3}{{}^{2x} C_3} = \frac{1}{20} \Rightarrow x = 3$.

Q.24

(1)

Required probability = $\frac{{}^3C_1 \times {}^2C_1}{{}^7C_1} = \frac{1}{7}$

Q.25

(4)

3 cards are drawn out of 26 red cards (favourable)

$$= \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{26!}{3! \cdot 23!} \times \frac{3! \cdot 49!}{52!} = \frac{2}{17}.$$

Q.26

(3)

Required probability = $\frac{{}^{37}C_2}{{}^{38}C_3} = \frac{\binom{37}{2}}{\binom{38}{3}}$.

Q.27

(1)

Total number of ways = ${}^{15}C_{11}$

Favourable cases = ${}^8C_6 \times {}^7C_5$

Required probability = $\frac{{}^8C_6 \times {}^7C_5}{{}^{15}C_{11}}$.

Q.28

(2)

3 ball can be drawn in ${}^{18}C_3$ ways

Favourable cases = 6C_3

∴ Required probability

$$= \frac{{}^6C_3}{{}^{18}C_3} = \frac{6 \times 5 \times 4}{18 \times 17 \times 16} = \frac{5}{204}$$

Q.29

(4)

Required probability

$$= \frac{{}^5C_1 {}^4C_1}{{}^{12}C_1} + \frac{{}^7C_1 {}^8C_1}{{}^{12}C_1} = \frac{20 + 56}{144} = \frac{76}{144}$$

Q.30

(3)

Since we have $P(A + B) = P(A) + P(B) - P(AB)$

$$\Rightarrow 0.7 = 0.4 + P(B) - 0.2 \Rightarrow P(B) = 0.5.$$

Q.31

(4)

$$P(A + B) = P(A) + P(B) - P(AB) = \frac{1}{4} + \frac{1}{4} - 0 = \frac{1}{2}.$$

Q.32

(3)

The sample space is [LWW, WLW]

∴ $P(LWW) + P(WLW)$

= Probability that in 5 match series, it is India's second win

$$= P(L)P(W)P(W) + P(W)P(L)P(W)$$

$$= \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$$

Q.33

(3)

Q.34

(2)

Q.35

(1)

Let $A \equiv$ event of two socks being brown. $B \equiv$ event of two socks being white. Then

$$P(A) = \frac{{}^5C_2}{{}^9C_2} = \frac{5 \cdot 4}{9 \cdot 8} = \frac{5}{18}, P(B)$$

$$= \frac{{}^4C_2}{{}^9C_2} = \frac{4 \cdot 3}{9 \cdot 8} = \frac{3}{18}$$

Now, since A and B are mutually exclugin events, so required probability

$$= P(A) + P(B) = \frac{5}{18} + \frac{3}{18} = \frac{4}{9}$$

Q.36 (4)

Required probability is

$$\begin{aligned} & P(\text{Red} + \text{Queen}) - P(\text{Red} \cap \text{Queen}) \\ &= P(\text{Red}) + P(\text{Queen}) - P(\text{Red} \cap \text{Queen}) \\ &= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}. \end{aligned}$$

Q.37 (3) $P(A) + P(B)$ (Fundamental concept).**Q.38** (1)

$$P(A \cap B) = \frac{2}{8} + \frac{5}{8} - \frac{6}{8} = \frac{1}{8}.$$

Q.39 (2)

Let A be the event to be multiple of 4 and B be the event to be multiple of 6

$$\text{So, } P(A) = \frac{25}{100}, P(B) = \frac{16}{100} \text{ and}$$

$$P(A \cap B) = \frac{8}{100}$$

Thus required probability is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = \frac{25}{100} + \frac{16}{100} - \frac{8}{100} = \frac{33}{100}$$

Q.40 (1)

Since we have

$$P(A \cup B) + P(A \cap B) = P(A) + P(B) = P(A) + \frac{P(A)}{2}$$

$$\Rightarrow \frac{7}{8} = \frac{3P(A)}{2} \Rightarrow P(A) = \frac{7}{12}.$$

Q.41 (3) $1 - P(A' \cap B') = 0.6$, $P(A \cap B) = 0.3$, then

$$P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$$

$$\Rightarrow 1 - P(A \cap B) = P(A') + P(B') - 0.4$$

$$\Rightarrow P(A') + P(B') = 0.7 + 0.4 = 1.1.$$

Q.42 (3)

$$P(\overline{A} \cap B) = P(B) - P(A \cap B) = y - z.$$

Q.43 (2)

$$P(A') = 0.3, \therefore P(A) = 0.7$$

$$P(B') = 0.6, P(B) = 0.4 \text{ and } P(A \cap B') = 0.5$$

$$\begin{aligned} P(A \cup B') &= P(A) + P(B') - P(A \cap B') \\ &= 0.7 + 0.6 - 0.5 = 0.8. \end{aligned}$$

Q.44 (2)

Since we are given that 5 appears on first die so to get sum 11, six must be on the second and hence, the required probability = $\frac{1}{6}$.

Q.45 (3)

$$P\left(\frac{\overline{A}}{\overline{B}}\right) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{P(\overline{A \cup B})}{P(\overline{B})} = \frac{1 - P(A \cup B)}{P(\overline{B})}.$$

Q.46 (2)

$$P(A) = 1/4, P(A/B) = \frac{1}{2}, P(B/A) = 2/3$$

By conditional probability,

$$P(A/B) = P(A)P(B/A) = P(A)P(B/A)$$

$$\Rightarrow \frac{1}{4} \times \frac{2}{3} = P(B) \times \frac{1}{2} \Rightarrow P(B) = \frac{1}{3}$$

Q.47 (4)**Q.48** (3)**Q.49** (1)**Q.50** (1)**Q.51** (1)

$$P\left(\frac{\overline{B}}{A}\right) = \frac{1 - P(A \cup B)}{P(A)} = \frac{1 - \frac{23}{60}}{1 - \frac{1}{3}} = \frac{37}{60} \times \frac{3}{2} = \frac{37}{40}.$$

Q.52 (1)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

Since A and B are mutually exclusive.

$$\text{So, } P(A \cap B) = 0. \text{ Hence } P\left(\frac{A}{B}\right) = \frac{0}{P(B)} = 0.$$

Q.53 (1)

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{(1/10)}{(1/4)} = \frac{2}{5}.$$

Q.54 (3)

Let A be the event that face 4 turns up and B be the event that face 5 turns up then $P(A) = 0.25$, $P(B) = 0.05$. Since A and B are mutually exclusive, so $P(A \cup B) = P(A) + P(B) = 0.25 + 0.05 = 0.30$

We have to find $P\left(\frac{A}{A \cup B}\right)$, which is equal to

$$P\left(\frac{A \cap (A \cup B)}{A \cup B}\right) = \frac{P(A)}{P(A \cup B)} = \frac{0.25}{0.30} = \frac{5}{6}$$

Q.55 (3)

Let $P(A)$ = probability of a boy in two children = $\frac{3}{4}$

Because cases are BB, BG, GB, GG = 4

Favourable cases are BB, BG, GB = 3

The probability that the second child is also boy is

$$P(A \cap B) = \frac{1}{4}$$

We have to find $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{3/4} = \frac{1}{3}$.

Q.56 (1)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.5}{0.6} = \frac{5}{6}.$$

Q.57 (1)

$$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{6}$$

$$P(\bar{A} \cap \bar{B}) = \frac{1}{3} = 1 - P(A \cup B)$$

$$\Rightarrow \frac{1}{3} = 1 - [P(A) + P(B)] + \frac{1}{6} \Rightarrow P(A) + P(B) = \frac{5}{6}.$$

Hence $P(A)$ and $P(B)$ are $\frac{1}{2}$ and $\frac{1}{3}$.

Q.58 (1)

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

$$\Rightarrow \frac{2}{3} = \frac{1}{3} + P(B) - \frac{1}{6} \Rightarrow P(B) = \frac{1}{2}$$

Now, $P(AB) = P(a)P(b)$, A and B are independent events.

Q.59 (1)**Q.60** (1)

(i) This question can also be solved by one student

(ii) This question can be solved by two students simultaneously

(iii) This question can be solved by three students all together.

$$P(A) = \frac{1}{2}, P(B) = \frac{1}{4}, P(C) = \frac{1}{6}$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$- [P(A) \cdot P(B) + P(B) \cdot P(C) + P(C) \cdot P(A)] + [P(A) \cdot P(B) \cdot P(C)]$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{6} - \left[\frac{1}{2} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{2} \right] + \left[\frac{1}{2} \times \frac{1}{4} \times \frac{1}{6} \right] = \frac{33}{48}$$

Q.61 (1)

Let E be the event in which all three coins shows tail and F be the event in which a coin shows tail.

$$\therefore F = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

and $E = \{TTT\}$ Required probability

$$= P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{1}{7}.$$

Q.62 (4)

$$P\left(\frac{B}{A}\right) = \frac{1}{2} \Rightarrow \frac{P(B \cap A)}{P(A)} = \frac{1}{2} \Rightarrow P(B \cap A) = \frac{1}{8}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{4} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \Rightarrow P(A \cap B) = \frac{1}{8}$$

$$P(A \cap B) = \frac{1}{8} = P(A) \cdot P(B)$$

\therefore Events A and B are independent.

$$\text{Now, } P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)} = \frac{P(A')P(B)}{P(B)} = \frac{3}{4}$$

$$\text{and } P\left(\frac{B'}{A'}\right) = \frac{P(B' \cap A')}{P(A')} = \frac{P(B')P(A')}{P(A')} = \frac{1}{2}.$$

Q.63 (4)

$$\text{Let } P(\text{fresh egg}) = \frac{90}{100} = \frac{9}{10} = p$$

$$P(\text{rotten egg}) = \frac{10}{100} = \frac{1}{10} = q; \quad n = 5, \quad r = 5$$

So the probability that none egg is rotten

$$= {}^5C_5 \left(\frac{9}{10}\right)^5 \cdot \left(\frac{1}{10}\right)^0 = \left(\frac{9}{10}\right)^5.$$

Q.64 (2)

$$\text{Required probability} = {}^5C_1 \left(\frac{4}{5}\right) \left(\frac{1}{5}\right)^4$$

{Here strictly one is swimmer}

Q.65 (1)

$$\text{Required probability} = {}^7C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^3.$$

Q.66 (3)

$$9 \cdot {}^6C_4 p^4 q^2 = {}^6C_2 p^2 q^4$$

Putting $q = 1 - p$, we get required result.

Q.67 (2)

$$\text{Since, } \sum P_i(X = x) = 1$$

$$\therefore K + 3K + 5K + 2K + K = 1$$

$$\therefore 12K = 1 \therefore K = \frac{1}{12}$$

Now,

$$P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4)$$

$$= 5K + 2K + K = 8K = 8 \left(\frac{1}{12} \right) = \frac{2}{3}$$

Q.68 (2)

The probability of hitting the target 5th time at the 10th throw = P(the probability of hitting the target 4 times in the first 9 throws) \times P (the probability of hitting the target at the 10th throw) =

$$\left[{}^9C_4 \left(\frac{1}{2} \right)^4 \left(\frac{1}{2} \right)^5 \right] \left(\frac{1}{2} \right) = \frac{9!}{4!5!} \times \left(\frac{1}{2} \right)^{10} = \frac{63}{2^9}$$

Q.69 (3)

The probability of showing same number by both dice

$$p = \frac{6}{36} = \frac{1}{6}$$

In binomial distribution here $n = 4, r = 2, p = \frac{1}{6}, q = \frac{5}{6}$

$$\therefore \text{req. probability} = {}^nC_r q^{n-r} p^r$$

$$= {}^4C_2 \left(\frac{5}{6} \right)^5 \left(\frac{1}{6} \right)^2$$

$$= 6 \left(\frac{25}{36} \right) \left(\frac{1}{36} \right) = \frac{25}{216}$$

Q.70 (2)

The man has to win at least 4 times.

\therefore Reqd. probability =

$${}^7C_4 \left(\frac{1}{2} \right)^4 \cdot \left(\frac{1}{2} \right)^3 + {}^7C_5 \left(\frac{1}{2} \right)^5 \cdot \left(\frac{1}{2} \right)^2$$

$$+ {}^7C_6 \left(\frac{1}{2} \right)^6 \cdot \frac{1}{2} + {}^7C_7 \left(\frac{1}{2} \right)^7$$

$$= \left({}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7 \right) \cdot \frac{1}{2^7} = \frac{64}{2^7} = \frac{1}{2}$$

Q.71 (3)

Q.72 (4)

Q.73 (2)

Q.74 (1)

Q.75 (2)

Q.76 (3)

Required probability

$$= {}^4C_3 \cdot \left(\frac{1}{2} \right)^3 \left(\frac{1}{2} \right) = \frac{1}{4}$$

Q.77 (1)

Required probability

$$= {}^8C_1 \left(\frac{1}{20} \right)^1 \left(\frac{19}{20} \right)^7 + {}^8C_0 \left(\frac{1}{20} \right)^0 \left(\frac{19}{20} \right)^8 = \frac{27}{20} \left(\frac{19}{20} \right)^7$$

Q.78 (1)

The probability that student is not swimmer

$$p = \frac{1}{5} \text{ and probability that student is swimmer } q = \frac{4}{5}$$

\therefore Probability that out of 5 students 4 are swimmer

$$= {}^5C_4 \left(\frac{4}{5} \right)^4 \left(\frac{1}{5} \right)^{5-4} = {}^5C_4 \left(\frac{4}{5} \right)^4 \left(\frac{1}{5} \right)$$

Q.79 (3)

$$\text{Probability of failure} = \frac{1}{3}$$

$$\text{Probability for getting success} = \frac{2}{3}$$

\therefore Required probability

$$= {}^4C_4 \left(\frac{2}{3} \right)^4 \left(\frac{1}{3} \right)^0 + {}^4C_3 \left(\frac{2}{3} \right)^3 \left(\frac{1}{3} \right)$$

$$= \left(\frac{2}{3} \right)^4 + 4 \left(\frac{2}{3} \right)^3 \left(\frac{1}{3} \right) = \frac{16}{27}$$

Q.80 (2)

$$p = P(\text{getting a head}) = \frac{1}{2}, q = \frac{1}{2}$$

\therefore Required probability = P(six successes)

$$= {}^{10}C_6 \left(\frac{1}{2} \right)^6 \left(\frac{1}{2} \right)^4 = \frac{10!}{6!4!} \cdot \frac{1}{2^{10}} = \frac{105}{512}$$

Q.81 (2)

Let X be the number of heads getting. X follows binomial distribution with parameters $n, p = 1/2$ Given

that $P(X \geq 1) \geq 0.8$

$$\Rightarrow 1 - P(X = 0) \geq 0.8 \Rightarrow P(X = 0) \leq 0.2$$

$$\Rightarrow {}^nC_0 (1/2)^n \leq 0.2 \Rightarrow \frac{1}{2^n} \leq \frac{1}{5} \Rightarrow 2^n = 5.$$

\therefore The least value of n is 3.

Q.82 (3)

Let X denote a random variable which is the number of aces. Clearly, X takes values, 1, 2.

$$\therefore p = \frac{4}{52} = \frac{1}{13}, q = 1 - \frac{1}{13} = \frac{12}{13}$$

$$P(X=1) = 2 \times \left(\frac{1}{13}\right) \times \left(\frac{12}{13}\right) = \frac{24}{169}$$

$$P(X=2) = 2 \cdot \left(\frac{1}{13}\right)^2 \left(\frac{12}{13}\right)^0 = \frac{1}{169}$$

$$\text{Mean} = \sum P_i X_i = \frac{24}{169} + \frac{2}{169} = \frac{26}{169} = \frac{2}{13}.$$

Q.83 (4)

$$np = 6$$

$$npq = 2 \Rightarrow q = \frac{1}{3}, p = \frac{2}{3} \text{ and } n = 9.$$

Hence the binomial distribution is $\left(\frac{1}{3} + \frac{2}{3}\right)^9$.

Q.84 (4)

We have mean $(X) = np = 2$ and variance

$$(X) = npq = 1 \Rightarrow q = \frac{1}{2} \text{ or } p = \frac{1}{2} \text{ and } n = 4$$

$$\text{Thus } p(X \geq 1) = 1 - p(X=0) = 1 - {}^4C_0 \left(\frac{1}{2}\right)^4 = \frac{15}{16}.$$

Q.85 (1)

Given $np = 6, npq = 4$

$$\therefore \frac{npq}{np} = \frac{4}{6} \Rightarrow q = \frac{2}{3} \text{ and } p = \frac{1}{3}$$

$$\therefore np = 6 \Rightarrow n \times \frac{1}{3} = 6 \Rightarrow n = 18.$$

Q.86 (1)

We have to find the bounded probability to get sum 15 when 4 appears first. Let the event of getting sum 15 of three thrown number is A and the event of appearing

4 is B . So we have to find $P\left(\frac{A}{B}\right)$.

$$\text{But } P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

When $n(A \cap B)$ and $n(B)$ respectively denote the number of digits in $A \cap B$ and B .

Now $n(B) = 36$, because first throw is of 4. So another two throws stop by $6 \times 6 = 36$ types. Three dices have only two throws, which starts from 4 and give sum 15 i.e., (4, 5, 6) and (4, 6, 5).

So, $n(A \cap B) = 2, n(B) = 36$

$$\therefore P\left(\frac{A}{B}\right) = \frac{2}{36} = \frac{1}{18}$$

Q.87 (2)

In a box, $B_1 = 1R, 2W$;

$B_2 = 2R, 3W$ and $B_3 = 3R, 4W$

Also, given that, $P(B_1) = \frac{1}{2}$,

$$P(B_2) = \frac{1}{3} \text{ and } P(B_3) = \frac{1}{6}$$

$$\begin{aligned} \therefore P\left(\frac{B_2}{R}\right) &= \frac{P(B_2)P\left(\frac{R}{B_2}\right)}{P(B_1)P\left(\frac{R}{B_1}\right) + P(B_2)P\left(\frac{R}{B_2}\right) + P(B_3)P\left(\frac{R}{B_3}\right)} \\ &= \frac{\frac{1}{3} \times \frac{2}{5}}{\frac{1}{2} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{5} + \frac{1}{6} \times \frac{3}{7}} = \frac{\frac{2}{15}}{\frac{1}{6} + \frac{2}{15} + \frac{1}{14}} = \frac{14}{39} \end{aligned}$$

Q.88 (1)

Total number of coins = $2n+1$

Consider the following events:

E_1 = Getting a coin having head on both sides from the bag.

E_2 = Getting a fair coin from the bag

A = Toss results in a head

$$\text{Given: } P(A) = \frac{31}{42}, P(E_1) = \frac{n}{2n+1}$$

$$\text{and } P(E_2) = \frac{n+1}{2n+1}$$

Then,

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$$

Q.89 (3)

Let $A \equiv$ event that drawn ball is red $B \equiv$ event that drawn ball is white Then AB and BA are two disjoint cases of the given event.

$$\therefore P(AB + BA) = P(AB) + P(BA)$$

$$= P(A) P\left(\frac{B}{A}\right) + P(B) P\left(\frac{A}{B}\right)$$

$$= \frac{3}{6} \cdot \frac{3}{5} + \frac{3}{6} \cdot \frac{3}{5} = \frac{3}{5}$$

Q.90

(3)
Let A be the event of selecting bag X, B be the event of selecting bag Y and E be the event of drawing a white ball, then $P(A) = 1/2, P(B) = 1/2,$
 $P(E/A) = 2/5 \quad P(E/B) = 4/6 = 2/3.$

$$P(E) = P(A)P(E/A) + P(B)P(E/B) = \frac{1}{2} \cdot \frac{2}{5} + \frac{1}{2} \cdot \frac{2}{3} = \frac{8}{15}$$

Q.91

(3)
It is based on Baye's theorem.

$$\text{Probability of picked bag A } P(A) = \frac{1}{2}$$

$$\text{Probability of picked bag B } P(B) = \frac{1}{2}$$

Probability of green ball picked from bag A

$$= P(A) \cdot P\left(\frac{G}{A}\right) = \frac{1}{2} \times \frac{4}{7} = \frac{2}{7}$$

Probability of green ball picked from bag B

$$= P(B) \cdot P\left(\frac{G}{B}\right) = \frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$$

$$\text{Total probability of green ball} = \frac{2}{7} + \frac{3}{14} = \frac{1}{2}$$

Probability of fact that green ball is drawn from bag B

$$P\left(\frac{G}{B}\right) = \frac{P(B)P\left(\frac{G}{B}\right)}{P(A)P\left(\frac{G}{A}\right) + P(B)P\left(\frac{G}{B}\right)} = \frac{\frac{1}{2} \times \frac{3}{7}}{\frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{3}{7}} = \frac{3}{7}$$

EXERCISE-II (JEE MAIN LEVEL)

Q.1

(3)
Since sum of $1+2+3+\dots+9 = \frac{9 \times 10}{2} = 45$ is divisible by 9, hence all no. will be divisible by 9.

Q.2

(1)
Max sum = 12

$$\left. \begin{array}{l} 6+6=12 \\ 6+5=11 \\ 6+4=10 \\ 5+5=10 \\ 5+6=11 \\ 4+6=10 \end{array} \right\} 6 \text{ cases}$$

$$P = \frac{6}{36} = \frac{1}{6} = \frac{1}{6}$$

Q.3

(2)
Exhaustive no. of cases = 6^3
10 can appear on three dice either as distinct number as following (1, 3, 6); (1, 4, 5); (2, 3, 5) and each can occur in $3!$ ways. Or 10 can appear on three dice as repeated digits as following (2, 2, 6), (2, 4, 4), (3, 3, 4) and each can occur in $\frac{3!}{2!}$ ways.

$$\therefore \text{No. of favourable cases} = 3 \times 3! + 3 \times \frac{3!}{2!} = 27$$

Q.4

(1)

Q.5

(1)

Q.6

(3)

Q.7

(3)

Q.8

(1)

$$\frac{\frac{2n-2!}{n-1!n-1!2!} \times 2!}{\frac{2n!}{n!n!2!}} = P$$

Q.9

(1)
Given that,
 $\alpha^2 + \beta^2 = \alpha + \beta$ & $\alpha^2 \beta^2 = \alpha \beta$
4 possibilities
(1, 1), (1, 0), (0, 0), (ω, ω^2)

$$\text{Required probability} = \frac{2}{4} = \frac{1}{2}$$

Q.10

(4)
Total ways in which 5 persons can exit at 8 floor = 8^5
(each has 8 options)
No. of ways of selecting 5 floor out of 8 = 8C_5
No. of ways of exiting at 5 selected floor = $5!$

$$\therefore \text{Probability} = \frac{{}^8C_5 \times 5!}{8^5}$$

Q.11

(1)
Required probability

$$= \frac{{}^5C_4 \times {}^3C_2 \times {}^2C_1}{{}^{10}C_7} = \frac{1}{4}$$

Q.12

(1)
 ${}^4C_1 \times {}^{13}C_9 \times {}^{39}C_4 = \text{Formula card}$
site any 9 cards any 4 cards from 39 cards
 ${}^{52}C_{13} = \text{total case}$

$$P = \frac{{}^4C_1 \times {}^{13}C_9 \times {}^{39}C_4}{{}^{52}C_{13}}$$

Q.13 (4)

$P(A) = \frac{13}{52} = \frac{1}{4}$ Let P(A) is prob of card drawn is spade & P(B) is card drawn is an ace then

$$P(B) = \frac{4}{52} = \frac{1}{13}$$

Q.14 (3)

Throughing both cubes is an independent event and occurring of red 8 blue color on top face is mutually exclusive let there be 'x' blue faces on the second cube. Given,

$$\underbrace{\frac{1}{6} \times \frac{x}{6}}_{\text{Blue at the top}} + \underbrace{\frac{5}{6} \times \frac{(6-x)}{6}}_{\text{Red at the top}} = \frac{1}{2}$$

$$\Rightarrow x + 30 - 5x = 18$$

$$\Rightarrow 4x = 12 \Rightarrow x = 3$$

$$\therefore \text{No. of red faces} = (6 - x) = (6 - 3) = 3$$

Q.15 (4)

$p_1 + p_2 + p_3 + p_4 = 1$ in D obvious solution follows

Q.16 (1)

Required Probability

$$= \frac{3}{9} \times \frac{6}{8} + \frac{6}{9} \times \frac{5}{8} = \frac{6}{9 \times 8} (5 + 3) = \frac{2}{3}$$

Q.17 (1)

Total = m + n

$$(w, w) \Rightarrow \frac{m}{m+n} \times \frac{m-1}{m+n-1} \dots\dots(1)$$

$$(B, w) \Rightarrow \frac{n}{m+n} \times \frac{m}{m+n-1} \dots\dots(2)$$

Total proof = (w, w) + (B, w)

$$= \frac{m}{m+n(m+n-1)} (m+n-1) = \frac{m}{m+n}$$

Q.18 (4)

Required Probability

$$= \frac{{}^7C_3 - 2}{{}^{10}C_3} = \frac{11}{40}$$

Q.19 (1)

Required probability

$$= \frac{{}^3C_2 \times 1 \times 1}{2^3 - 1} = \frac{3}{7}$$

Q.20 (1)

squared of a no. can have 1, 4, 6, 9, 5
So P = (9/25)

Q.21 (2)

Required probability

$$= \frac{{}^3C_1 {}^3C_1 {}^3C_1 + {}^2C_1 {}^2C_1 {}^2C_1 + {}^3C_1 {}^2C_1 {}^2C_1 \times 2}{{}^{5 \times 5 \times 5}} = \frac{59}{125}$$

Q.22 (2)

Probability of detection = P = P(A)

Probability of non-detection = 1 - P = P(B)

1st cycle = P

2nd cycle = P(B) . P(A)

$$= (1 - p) P$$

3rd cycle = P(B) . P(B) . P(A)

$$= (1 - P) (1 - P) . P$$

4th cycle = P(B) . P(B) . P(B) . P(A)

$$= (1 - P)^3 . P$$

4th cycle = [P(B)]ⁿ⁻¹ P(A)

$$= (1 - P) . P$$

$$= P^n$$

Q.23 (1)

Foce cards = 12

Tens = 4

Total removed cards = 16

Remaining cards = 36

$$\therefore P(A) = \frac{4}{36}; P(H) = \frac{9}{36}; P(S) = \frac{9}{36}$$

$$\Rightarrow 9P(A) = 4P(H)$$

Q.24 (1)

Since, P(exactly one of A, B occurs) = q.

$$\therefore P(A \cup B) - P(A \cap B) = q$$

$$\Rightarrow p - P(A \cap B) = q \Rightarrow P(A \cap B) = p - q$$

$$\Rightarrow 1 - P(A' \cup B') = p - q \Rightarrow P(A' \cup B')$$

$$= 1 - p + q$$

$$\Rightarrow P(A') + P(B') - P(A' \cap B') = 1 - p + q$$

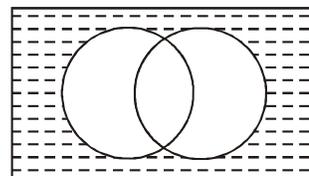
$$\Rightarrow P(A') + P(B') = (1 - p + q) + [1 - P(A \cup B)]$$

$$= (1 - p + q) + (1 - p) = 2 - 2p + q$$

Q.25 (3)

Q.26 (3)

$A^c - B$



$$= (A \cup B)^c$$

Q.27 (1)

$$p(A) = \frac{1}{6}, p(B) = \frac{2}{6}$$

$$A \equiv \{1, 3, 5\} B \equiv \{3, 6\}$$

$B \not\subset A$.

$B - A = \{6\}$ as follows

Q.28 (4)

Let the probabilities of $A \cap B$, A , B & $A \cup B$ be $a - 3d$,

$a - d$, $a + d$, $a + 3d$ in AP

Given, $(a - d) = 2d$

$a = 3d$

$$2P(A) = 2(a - d) = 2(2d) = 4d$$

$$\& P(B) = a + d = 3d + d = 4d$$

$$\Rightarrow 2P(A) = P(B)$$

Q.29 (1)

$$P(\text{atleast} / W) = P(1W, 1M) + P(2W_1, 0M)$$

$$= \frac{5 \times 8}{{}^{13}C_2} + \frac{{}^5C_2}{{}^{13}C_2} = \frac{25}{39}$$

Q.30 (1)

Q.31 (1)

$$A = (1, 3, 5), A \cap B = (3, 5)$$

$$P(B/A) = P(B \cap A) / p(A) = 2/3$$

Q.32 (3)

$$P(A \text{ Late}) = \frac{1}{5} \quad P(B \text{ Late}) = \frac{7}{25}$$

$$P(B \text{ is late given that } A \text{ is late}) = \frac{9}{10}$$

(i) neither bus is late

$$P(\bar{A} \cap \bar{B}) = P(\bar{A} \cup \bar{B}) = 1 - P(A \cup B)$$

$$\frac{P(B \cap A)}{P(A)} = \frac{9}{10}$$

$$P(B \cap A) = \frac{9}{10} \times P(A)$$

$$= \frac{9}{10} \times \frac{1}{5} = \frac{9}{50}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{5} + \frac{7}{25} - \frac{9}{50} = \frac{3}{10}$$

$$1 - P(A \cup B) = 7/10$$

$$(ii) P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{9}{50}}{7/15} = \frac{9}{14}$$

Q.33 (2)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore P(A \cap B) = P(A) \cdot P(B) \text{ because}$$

A & B are independent events

$$\Rightarrow P(A \cup B) = 1 - P(\bar{A}) \cdot P(\bar{B})$$

$$\Rightarrow 0.8 = 1 - (0.7) \cdot a$$

$$\Rightarrow (0.7)a = 1 - 0.8 = 0.2$$

$$\Rightarrow a = \frac{0.2}{0.7} \Rightarrow a = \frac{2}{7}$$

Q.34 (3)

$$1 - P(BB)$$

$$1 - 1/2 \times 1/2 = 1 - 1/4 = 3/4$$

Q.35 (2)

Q.36 (3)

$$p(A) = \frac{1 \times 6}{36} = \frac{1}{36}, \quad p(B) = \frac{6}{36} = \frac{1}{6}$$

$$\left. \begin{array}{l} 6+1 \\ 5+2 \\ 4+3 \end{array} \right\} A \cap B = \frac{1}{36}; p(A \cap B) = p(A) \times p(B)$$

Q.37 (2)

A and B are independent events

$$\therefore P(A) = P(B) = \frac{1}{13}$$

Q.38 (1)

$${}^3C_2 P^2 (1-P) = 12 {}^3C_3 P^3$$

$$1 - P = 4P \Rightarrow \frac{1}{5} = P$$

Q.39 (2)

As $0.4 + 0.6 = 1$, the man either takes a step forward or a step backward. Let a step forward be a success and a step backward be a failure.

Then, the probability of success in one step

$$= P = 0.4 = \frac{2}{5}$$

The probability of failure in one step

$$= P = 0.6 = \frac{3}{5}$$

In 11 steps he will be one step away from the starting point if the numbers of successes and failures differ by 1. So, the number of successes = 6

The number of failures = 5 or the number of successes = 5, The number of failures = 6

\therefore the required probability

$$= {}^{11}C_6 p^6 q^5 + {}^{11}C_5 p^5 q^6$$

$$= {}^{11}C_6 \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^5 + {}^{11}C_5 \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^6$$

$$= \frac{11!}{6!5!} \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^5 \left\{ \frac{2}{5} + \frac{3}{5} \right\}$$

$$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{120} \cdot \frac{2^5 \cdot 3^5}{5^{10}} = 462 - \left(\frac{6}{25}\right)^5$$

Q.40 (4)

Since, $P = \frac{e^{-\lambda} \lambda^r}{r!}$ (where $\lambda = \text{mean}$)

$$\therefore P(X = r > 1.5) = P(2) + P(3) + \dots \infty$$

$$= 1 - P(X = r \leq 1) = 1 - P(0) - P(1)$$

$$= 1 - \left(e^{-2} + \frac{e^{-2} \times 2}{1!} \right) = 1 - \frac{3}{e^2}$$

Q.41 (3)

In Binomial distribution, Mean = np,
Variance = npq and the mode is r if x = r, the
probability function p(x) is maximum.

Given np = 4 and npq = 3

$$\therefore q = \frac{3}{4} \text{ and } p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\text{Also, } n = \frac{4}{p} = \frac{4}{1/4} = 16$$

$$\text{Now, } (n+1)p = (16+1) \frac{1}{4} = \frac{17}{4} = 4 + \frac{1}{4}$$

\Rightarrow The distribution will have unique mode
(unimodal) & the mode = 4

Q.42 (2)

$$P(E) = P(2 \text{ or } 3 \text{ or } 5 \text{ or } 7) \\ = 0.23 + 0.12 + 0.20 + 0.07 = 0.62$$

$$P(F) = P(1 \text{ or } 2 \text{ or } 3) \\ = 0.15 + 0.23 + 0.12 = 0.50$$

$$P(E \cap F) = P(2 \text{ or } 3) = 0.23 + 0.12 = 0.35$$

$$\therefore P(E \cup F) = P(E) + P(F) - P(E \cap F) \\ = 0.62 + 0.50 - 0.35 = 0.77$$

Q.43 (2)

Q.44 (3)

2W & 4B

$$P = {}^5C_4 \times \left(\frac{2}{6}\right)^4 \left(\frac{4}{6}\right)^1 + {}^5C_5 \left(\frac{2}{6}\right)^5 = \frac{11}{243}$$

Q.45 (3)

$$\frac{3}{5} \times \frac{2}{4} \times 30 + \frac{3}{5} \times \frac{2}{4} \times 40 + \frac{2}{5} \times \frac{1}{4} \times 20$$

Q.46 (2)

$$E_A \left[\frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \dots \right] \times 99$$

$$= \frac{99}{6} \frac{B6}{1 - \frac{25}{36}} = 54$$

Q.47 (3)

A = event it is chosen from A

$$P(A) = 3/5$$

$$P(B) = 2/5$$

$$P(D) = P(A) \cdot P\left(\frac{D}{A}\right) + P(B) \cdot P\left(\frac{D}{B}\right)$$

$$= \frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{1}{5}$$

$$= \frac{6}{25} + \frac{2}{25} = \frac{8}{25}$$

Q.48 (4)

Let A_i ($i = 2, 3, 4, 5$) be the event that urn contains 2, 3, 4, 5 white balls and let B be the event that two white balls have been drawn then we have to find $P(A_i/B)$. Since the four events A_2, A_3, A_4 and A_5 are equally likely we have P

$$P(A_4) = P(A_5) = \frac{1}{4}$$

$P(B/A_2)$ is probability of event that the urn contains 2 white balls and both have been drawn.

$$\therefore P(B/A_2) = \frac{{}^2C_2}{{}^5C_2} = \frac{1}{10}$$

$$\text{Similarly } P(B/A_3) = \frac{{}^3C_2}{{}^5C_2} = \frac{3}{10}$$

$$P(B/A_4) = \frac{{}^4C_2}{{}^5C_2} = \frac{3}{5}$$

$$P(B/A_5) = \frac{{}^4C_2}{{}^5C_2} = 1$$

By Baye's theorem,

$$P(A_5/B) = \frac{P(A_5)P(B/A_5)}{P(A_2)P(B/A_2) + P(A_3)P(B/A_3) + P(A_4)P(B/A_4) + P(A_5)P(B/A_5)}$$

$$= \frac{\frac{1}{4} \cdot 1}{\frac{1}{10} + \frac{3}{10} + \frac{3}{5} + 1} = \frac{10}{25} = \frac{1}{2}$$

Q.49 (3)

Let E_1 denote the event "a coin with head on both sides is selected" and E_2 denotes the event "a fair coin is selected". Let A be the event "he toss, results in heads".

$$\therefore P(E_1) = \frac{1}{n+1}, P(E_2) = \frac{n}{n+1} \text{ and}$$

$$P\left(\frac{A}{E_1}\right) = 1, P\left(\frac{A}{E_2}\right) = \frac{1}{2}$$

$$\therefore P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$\Rightarrow \frac{7}{12} = \frac{1}{n+1} \times 1 + \frac{n}{n+1} \times \frac{1}{2}$$

$$\Rightarrow 14n + 14 = 24 + 12n \Rightarrow n = 5$$

Q.50 (2)

Let E_1, E_2 and A be the events defined as follows:

E_1 = red ball is transferred from bag P to bag Q

E_2 = blue ball is transferred from bag P to bag Q

A = the ball drawn from bag Q is blue

As the bag P contains 6 red and 4 blue balls,

$$P(E_1) = \frac{6}{10} = \frac{3}{5} \text{ and } P(E_2) = \frac{4}{10} = \frac{2}{5}$$

Note that E_1 and E_2 are mutually exclusive and exhaustive events.

When E_1 has occurred i.e., a red ball has already been transferred from bag P to Q, then bag Q will contain 6

red and 6 blue balls, So, $P(A | E_1) = \frac{6}{12} = \frac{1}{2}$

When E_2 has occurred i.e., a blue ball has already been transferred from bag P to Q then bag Q will contain 5

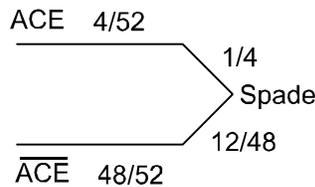
red and 7 blue balls, So, $P(A | E_2) = \frac{7}{12}$

By using law of total probability, we get

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2)$$

$$= \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times \frac{7}{12} = \frac{8}{15}$$

Q.51 (1)



$$\text{Required probability} = \frac{\frac{4}{52} \times \frac{1}{4}}{\frac{4}{52} \times \frac{1}{4} + \frac{48}{52} \times \frac{12}{48}} = \frac{1}{13}$$

Q.52 (2)

$$\frac{np}{npq} = \frac{3}{2} \Rightarrow q = \frac{2}{3} \Rightarrow p = \frac{1}{3}$$

$$r \leq \frac{11}{1 + \frac{2}{3}} \Rightarrow r \leq \frac{10}{3} \Rightarrow r \leq 3.33$$

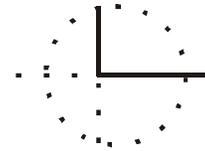
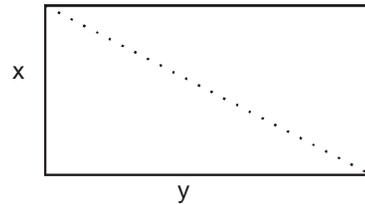
thus 3 succes is most parallal.

Q.53 (3)

$$0 < x < 10, x^2 + y^2 < 100$$

$$0 < y < 10$$

$$P = \frac{\frac{1}{4} \pi \times 10^2}{10 \times 10}$$



$$P = \frac{\pi}{4}$$

Q.54 (2)

If set A has m elements and set B has n elements then number of onto functions from A to B is

$$\sum_{r=1}^n (-1)^{n-r} {}^n C_r r^m \text{ where } 1 \leq n \leq m$$

Here $E = \{1, 2, 3, 4\}, F = \{1, 2\}$

$m = 4, n = 2$

\therefore No of onto functions from E to F

$$= \sum_{r=1}^2 (-1)^{2-r} {}^2 C_r (r)^4$$

$$= (-1)^2 {}^2 C_1 + {}^2 C_2 (2)^4 = -2 + 16 = 14$$

Q.55 (4)

$$P(i) = \frac{k}{1} \Rightarrow 1 = \sum_{i=1}^6 P(i) = k \sum_{i=1}^6 \frac{1}{i} = k \frac{49}{20}$$

$$\Rightarrow k = \frac{20}{49} \cdot P(3) = \frac{20}{147}$$

EXERCISE-III

Q.1 0.39

$$\begin{aligned}
 P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
 &= 0.25 + 0.50 - 0.14 = 0.61 \\
 \therefore P(A' \cap B') &= P((A \cup B)') = 1 - P(A \cup B) \\
 &= 1 - 0.61 = 0.39
 \end{aligned}$$

Q.2 0.6

Let A \equiv event that drawn ball is red
 B \equiv event that drawn ball is white
 Then AB and BA are two disjoint cases of the given event.

$$\begin{aligned}
 \therefore P(AB + BA) &= P(AB) + P(BA) \\
 &= P(A)P\left(\frac{B}{A}\right) + P(B)P\left(\frac{A}{B}\right) \\
 &= \frac{3}{6} \cdot \frac{3}{5} + \frac{3}{6} \cdot \frac{3}{5} = \frac{3}{5}
 \end{aligned}$$

Q.3 0.892

Let the events of hitting the enemy plane at the first, second and third shot are respectively A, B and C. Then as given

$$P(A) = 0.6, P(B) = 0.7, P(C) = 0.1$$

Since events A, B, C are independent, so

$$\begin{aligned}
 \text{reqd. prob.} &= P(A + B + C) \\
 &= 1 - P(\bar{A})P(\bar{B})P(\bar{C}) \\
 &= 1 - (1 - 0.6)(1 - 0.7)(1 - 0.1) \\
 &= 1 - (0.4)(0.3)(0.9) \\
 &= 1 - 0.108 = 0.892
 \end{aligned}$$

Q.4 1.2

As given $P(A + B) = 0.6$ and $P(AB) = 0.2$

$$\therefore P(A + B) = P(A) + P(B) - P(AB)$$

$$\Rightarrow 0.6 = P(A) + P(B) - 0.2$$

$$\Rightarrow -P(A) - P(B) = -0.8$$

$$\Rightarrow [1 - P(A)] + [1 - P(B)] = 2 - 0.8 = 1.2$$

$$\Rightarrow P(\bar{A}) + P(\bar{B}) = 1.2$$

Q.5 0.5049

$${}^{100}C_{50} p^{50} (1-p)^{50} = {}^{100}C_{51} p^{51} (1-p)^{49}$$

$$\Rightarrow 51(1-p) = 50p$$

$$\Rightarrow 51 = 101p$$

$$p = \frac{51}{101}$$

Q.6 0.057

SUCCESS \rightarrow 3S, 2C, 1U, 1E

$$\text{Total arrangements} \rightarrow \frac{7!}{3! 2!} = \frac{2}{35}$$

Now, similar letters are together (consider 3S one letter and 2C's one letter) = 4! (favourable cases)

$$\therefore \text{Probability} = \frac{4! 3! 2!}{7!} = \frac{2}{35}$$

Q.7 0.2

Given:

x	P(x)	xP(x)	x ² P(x)
1	$\frac{1}{5}$	$-\frac{1}{5}$	$\frac{1}{5}$
0	$\frac{2}{5}$	0	0
1	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$
Total			$\frac{1}{5}$ $\frac{3}{5}$

$$E(X) = \sum xP(x) = \frac{1}{5} = 0.2$$

Q.8 0007

When two dice are rolled, the possible out comes are 36 in number viz. (1, 1) (1, 2), ... (6, 6).

Let X = sum of the two numbers.

Then X assumes the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

x	Favourable cases	P(x)	xP(x)
2	(1, 1)	$\frac{1}{36}$	$\frac{2}{36}$
3	(1, 2), (2, 1)	$\frac{2}{36}$	$\frac{6}{36}$
4	(1, 3), (2, 2), (3, 1)	$\frac{3}{36}$	$\frac{12}{36}$
5	(1, 4), (2, 3), (3, 2), (4, 1)	$\frac{4}{36}$	$\frac{20}{36}$
6	(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)	$\frac{5}{36}$	$\frac{30}{36}$
7	(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)	$\frac{6}{36}$	$\frac{42}{36}$
8	(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)	$\frac{5}{36}$	$\frac{40}{36}$
9	(3, 6), (4, 5), (5, 4), (6, 3)	$\frac{4}{36}$	$\frac{36}{36}$
10	(4, 6), (5, 5), (6, 4)	$\frac{3}{36}$	$\frac{30}{36}$
11	(5, 6), (6, 5)	$\frac{2}{36}$	$\frac{22}{36}$
12	(6, 6)	$\frac{1}{36}$	$\frac{12}{36}$
Total			$\frac{252}{36}$

$$E(X) = \sum x_i P(x_i) = \frac{252}{36} = 71$$

Q.9 0.6976
 Probability that at least one shot hits the plane
 = $1 - P(\text{none of the shot hits the plane})$
 = $1 - 0.6 \times 0.7 \times 0.8 \times 0.9$
 = $1 - 0.3024 = 0.6976$

Q.10 0.703
 Probability of defective transistor = $\frac{5}{15} = \frac{1}{3}$ and
 probability of non-defective transistor = $1 - \frac{1}{3} = \frac{2}{3}$
 Probability that the inspectors finds non-defective
 transistors = $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$
 Hence, probability that atleast one of the inspectors
 finds a defective transistor = $1 - \frac{8}{27} = \frac{19}{27}$

PREVIOUS YEAR'S

MHT CET

- | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Q.1 (2) | Q.2 (3) | Q.3 (2) | Q.4 (1) | Q.5 (3) |
| Q.6 (3) | Q.7 (1) | Q.8 (1) | Q.9 (3) | Q.10 (1) |
| Q.11 (2) | Q.12 (2) | Q.13 (3) | Q.14 (1) | Q.15 (2) |
| Q.16 (2) | Q.17 (2) | Q.18 (2) | Q.19 (1) | Q.20 (2) |
| Q.21 (1) | Q.22 (1) | Q.23 (2) | Q.24 (4) | Q.25 (3) |
| Q.26 (1) | Q.27 (1) | Q.28 (2) | Q.29 (1) | Q.30 (4) |
| Q.31 (4) | Q.32 (4) | Q.33 (4) | Q.34 (4) | Q.35 (4) |
| Q.36 (1) | Q.37 (2) | | | |

Q.38 (3)
 Total number of cases = ${}^{50}C_1 = 50$
 Let A be the event of selecting ticket with sum of digits '8'.
 Favourable cases to A are {08, 17, 26, 35, 44}.
 Let B be the event of selecting ticket with product of its digits '7'.
 Favourable cases to B is only {17}.

$$\text{Now, } P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/50}{5/50} = \frac{1}{5}$$

Q.39 (3)
 Given, probability of winning a test match, $P(W) = \frac{1}{2}$
 Probability of losing a match, $P(L) = 1 - \frac{1}{2} = \frac{1}{2}$
 Probability that India's second win occurs at the third day
 = $P(L) \cdot P(W) \cdot P(W) + P(W) \cdot P(L) \cdot P(W)$
 = $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

Q.40 (4)
 We have, $f(x) = \frac{k}{2^x}$, $x = 0, 1, 2, 3, 4$

Since, $f(x)$ is a probability distribution of a random variable X, therefore we have

$$\sum_{x=0}^4 f(x) = 1 \Rightarrow \sum_{x=0}^4 \left(\frac{k}{2^x}\right) = 1$$

$$\Rightarrow k \sum_{x=0}^4 \frac{1}{2^x} = 1$$

$$\Rightarrow k \left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}\right) = 1$$

$$\Rightarrow k \left(\frac{16 + 8 + 4 + 2 + 1}{2^4}\right) = 1$$

$$\Rightarrow k \times \left(\frac{31}{16}\right) = 1$$

$$\therefore k = \frac{16}{31}$$

Q.41 (3)
 Let probability of defective bulb,

$$p = \frac{10}{100} = \frac{1}{10} = 0.1$$

and probability of non - defective bulb,
 $q = 1 - 0.1 = 0.9$

Here, $n = 5$

$$\therefore P(\text{none id deffective}) = P(X = 0) = {}^5C_0 (0.1)^0 (0.9)^5$$

$$= 1 \times (0.9)^5 = \left(\frac{9}{10}\right)^5$$

Q.42 (2)
 Given distribution is

x	1	2	3	4	5
p(x = x)	K	2K	3K	2K	K

$$\therefore \text{Variance} = \sum x_i^2 p - (\sum x_i p)^2 = (1k + 8k + 27k + 32k + 25k) - (k + 4k + 9k + 8k + 5k)^2$$

$$= (93k) - (27k)^2 = \left(93 \times \frac{1}{9}\right) - \left(27 \times \frac{1}{9}\right)^2$$

$$\left[\because \sum p = 1, \text{so } k = \frac{1}{9}\right]$$

$$= \frac{93}{9} - 9 = \frac{93 - 81}{9} = \frac{12}{9} = \frac{4}{3}$$

Q.43 (3)
 Let E_1 = Student does not know the answer
 E_2 = Student knows the answer

and E = student answer correctly

$$\therefore P(E_1) = 1 - p \quad P(E_2) = p$$

$$\Rightarrow P\left(\frac{E}{E_1}\right) = 1 \quad \text{and} \quad P\left(\frac{E}{E_2}\right) = \frac{1}{5}$$

Note that, the probability that student did not know the answer randomly = The probability that student know the answer.

$$\therefore P\left(\frac{E_2}{E}\right) = \frac{P(E_2)P\left(\frac{E}{E_2}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right)}$$

$$= \frac{p(1)}{(-p)\frac{1}{5} + p(1)} = \frac{p}{1-p+5p} = \frac{5p}{1+4p}$$

Q.44

(2)

Let probability of choosing box, $B, P(B) = p$

According to the given condition,

$$P(A) = 2P(B) = 2p$$

$$\text{Now, } P\left(\frac{R}{A}\right) = \frac{{}^3C_1}{{}^5C_1} = \frac{3}{5}$$

$$\text{and } P\left(\frac{R}{B}\right) = \frac{{}^4C_1}{{}^7C_1} = \frac{4}{7}$$

$$\therefore P\left(\frac{B}{R}\right) = \frac{P(B) \cdot P\left(\frac{R}{B}\right)}{P(A) \cdot P\left(\frac{R}{A}\right) + P(B) \cdot P\left(\frac{R}{B}\right)}$$

$$\frac{p \cdot \frac{4}{7}}{2p \cdot \frac{3}{5} + p \cdot \frac{4}{7}} = \frac{10}{31}$$

Q.45

(1)

Probability of getting head in one trial, $p = \frac{1}{2}$

and probability of not getting head,

$$q = \frac{1}{2}$$

Probability of getting head odd times

$$= {}^{20}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{19} + {}^{20}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{17}$$

$$+ \dots + {}^{20}C_{19} \left(\frac{1}{2}\right)^{19} \left(\frac{1}{2}\right)^1$$

$$= \frac{1}{2^{20}} [{}^{20}C_1 + {}^{20}C_3 + \dots + {}^{20}C_{19}]$$

$$= \frac{1}{2^{20}} \times 2^{20-1} = \frac{2^{19}}{2^{20}} = \frac{1}{2}$$

Q.46

(2)

Since, the probability of person die, due to suffering from a disease is 10%.

$$\therefore p = \frac{10}{100} = \frac{1}{10} \quad \text{and} \quad q = \frac{9}{10}$$

Total number of patients, $n = 6$

$$\begin{aligned} \therefore \text{Required probability} &= {}^6C_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^3 \\ &= \frac{6.5.4}{3.2.1} \times \frac{1}{1000} \times \frac{9 \times 9 \times 9}{1000} \\ &= \frac{2}{10^5} \times 729 = 1458 \times 10^{-5} \end{aligned}$$

Q.47

(3)

Given, $P(A^c) = 0.3, P(B) = 0.4$

and $P(A \cap B^c) = 0.5$

$$\therefore P(A^c) = 0.3$$

$$\Rightarrow P(A) = 1 - P(A^c) = 0.7$$

and $P(B) = 0.4 \Rightarrow P(B^c) = 1 - P(B) = 0.6$

Consider, $P(A \cap B^c) = P(A) - P(A \cap B)$

$$0.5 = 0.7 - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = 0.2$$

$$\text{Now, } P\left[\frac{B}{(A \cup B^c)}\right] = \frac{P[B \cap (A \cup B^c)]}{P(A \cup B^c)}$$

$$= \frac{P[B \cap A] \cup (A \cap B^c)}{P(A) + P(B^c) - P(A \cap B^c)}$$

$$= \frac{P(B \cap A)}{0.7 + 0.6 - 0.5} = \frac{0.2}{0.8} = \frac{1}{4}$$

Q.48

(1)

Given, box contains 15 green and 10 yellow balls.

\therefore Total number of balls in box = $15 + 10 = 25$

$$P(\text{green balls}) = \frac{15}{25} = \frac{3}{5} = p$$

(probability of success)

$$P(\text{yellow balls}) = \frac{10}{25} = \frac{2}{5} = q$$

(probability of unsuccess)

and $n = 10$ (number of trials)

\therefore Variance of the number of green balls drawn

$$= npq = 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$$

Q.49

(3)

\therefore Sample space = ${}^{12}C_2$

Number of possibilities for z is even,

$$\begin{aligned} z=0 &\Rightarrow {}^{11}C_1 \\ z=2 &\Rightarrow {}^9C_1 \\ z=4 &\Rightarrow {}^7C_1 \\ z=6 &\Rightarrow {}^5C_1 \\ z=8 &\Rightarrow {}^3C_1 \\ z=10 &\Rightarrow {}^1C_1 \end{aligned}$$

Total possibilities = 36

\therefore Required probability = $36 / 66 = 6 / 11$

Q.50

(2)

Given, total coupons = 15

$1 \leq$ selected coupon ≤ 9

i.e., 1, 2, 3, 4, 5, 6, 7, 8, 9

\therefore Probability of one selected coupon

= $9 / 15 = 3 / 5$

Hence, the required probability

$$\begin{aligned} &= \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \\ &= \left(\frac{3}{5}\right)^7 \end{aligned}$$

Q.51

(4)

Given, A be the event that the number obtained is greater than 3.

$\therefore A = \{4, 5, 6\}$

So, $A \cap B = \{4\}$

By addition theorem of probability, we have

$$P(A \cup B) = P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = (3/6) + (4/6) - (1/6) = 1$$

Q.52

(1)

Given, $P(A) = 0.25$, $P(B) = 0.50$

and $P(A \cap B) = 0.14$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.25 + 0.50 - 0.14 = 0.61$$

$$\text{Now, } P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.61 = 0.39$$

Q.53

(3)

$$\text{Given that, } P(A) = \frac{1}{4}, P\left(\frac{A}{B}\right) = \frac{1}{2}$$

$$\text{and } P\left(\frac{B}{A}\right) = \frac{3}{2}$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \quad \dots(i)$$

$$\text{and } P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} \quad \dots(ii)$$

$$\therefore P(B) = \frac{P\left(\frac{B}{A}\right) \cdot P(A)}{P\left(\frac{A}{B}\right)} = \frac{\left(\frac{2}{3}\right) \left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{3}$$

Q.54 (2)

$$\text{Given that, } P(A \cap B) = \frac{1}{4}, P(\overline{A}) = \frac{1}{4}$$

$$\text{and } P(\overline{A \cup B}) = \frac{1}{6}$$

$$\Rightarrow 1 - P(A \cap B) = \frac{1}{6} [\because P(A) + P(\overline{A}) = 1]$$

$$\Rightarrow 1 - P(A) - P(B) + P(A \cap B) = \frac{1}{6}$$

$$\Rightarrow P(\overline{A}) - P(B) + \frac{1}{4} = \frac{1}{6} \Rightarrow P(B) = \frac{1}{4} + \frac{1}{4} - \frac{1}{6}$$

$$\Rightarrow P(B) = \frac{1}{3} \text{ and } P(A) = \frac{3}{4}$$

$$\text{Now, } P(A \cap B) = \frac{1}{4} = \frac{3}{4} \times \frac{1}{3} = P(A) \cdot P(B)$$

Hence, the events A and B are independent events.

Q.55

(1)

Here, 5 Physics, 3 Biology @ shelf 1

4 Physics, 2 Biology @ shelf 2

$P(\text{choosing shelf 1}) = P(\text{choosing shelf 2}) = 1/2$

$P(\text{physics from shelf 2}) = 4/6$

$$\therefore \text{ Required probability} = \left(\frac{1}{2} \times \frac{5}{8}\right) + \left(\frac{1}{2} \times \frac{4}{6}\right)$$

$$= \frac{5}{16} + \frac{1}{3} = \frac{15+16}{48} = \frac{31}{48}$$

Q.56

(3)

Given, $n = 4$

and $2P(X=3) = 3P(X=2)$

$$\Rightarrow 2 \cdot {}^4C_3 p^3 q = 3 \cdot {}^4C_2 p^2 q^2$$

$$\Rightarrow \frac{2 \cdot 4!}{3!1!} p = \frac{3 \cdot 4!}{2!2!} q$$

$$\Rightarrow p = \frac{9}{4} q$$

$$\therefore p + q = 1$$

$$\Rightarrow \frac{9}{4} q + q = 1$$

$$\Rightarrow \frac{13q}{4} = 1 \Rightarrow q = \frac{4}{13}$$

JEE MAIN

Q.1 (479)

$$(40)(40) + (30)(50) + (20)(60)$$

$$({}^4C_4) \left[\frac{3}{4}\right]^4 \cdot {}^6C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^2 + 4 \cdot {}^6C_3 \cdot \left(\frac{3}{4}\right)^3 \cdot \frac{1}{4} \cdot {}^6C_3 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)$$

$$\begin{aligned}
 &+ \left(4C_2 \cdot \left(\frac{3}{4}\right)^2 \cdot \left(\frac{1}{4}\right)^2 \right) \left(6C_6 \left(\frac{1}{4}\right)^6 \right) \\
 &= \frac{3^6 \times 15 + 4 \times 3^3 \times 6 \times 3 + 6 \times 3^2}{4^{10}} \\
 &= \frac{3^3(15 \times 27 + 4 \times 6 \times 3 + 2)}{4^{10}} \Rightarrow \frac{27}{4^{10}}(405 + 72 + 2) \\
 &= \frac{27 \cdot 479}{4^{10}} \Rightarrow k = 479
 \end{aligned}$$

Q.2

(4)

$$n(s) = 5C_4 \times 4! = 120$$

f(a)	+2f(b)	=	f(c)	+f(d)
5	2×1		3	4
4	2×2		3	5
1	2×3		2	5

$$n(A) = 2! \times 3 = 6$$

$$\therefore P(A) = \frac{n(A)}{n(s)} = \frac{6}{120} = \frac{1}{20}$$

Q.3

(3)

Let matrix A is singular then $|A| = 0$

Number of singular matrix = All entries are same + only two prime number are used in matrix
 $= 10 + 10 \cdot 9 \cdot 2$
 $= 190$

$$\text{Required probability} = \frac{190}{10^4} = \frac{19}{10^3}$$

Q.4

(1)

Let Set A = {x, y}

$$B = \{x, y\}$$

$$A \times B = \{(x, x), (x, y), (y, x), (y, y)\}$$

$$\text{Total number of relations} = 2^4 = 16$$

The relations which are symmetric as well as transitive are

ϕ , $\{(x, x)\}$, $\{(y, y)\}$, $\{(x, x), (y, y)\}$ and $\{(x, x), (x, y), (y, x), (y, y)\}$

$$\therefore \text{Probability} = \frac{5}{16}$$

Q.5

(3)

 E_1 = denotes selection for 1st bag E_2 = denotes selection for 2nd bag

$$P(E_1) = \frac{1}{2}; P(E_2) = \frac{1}{2}$$

A = selected balls are 1 red & 1 black

$$P\left(\frac{A}{E_1}\right) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{1}{5}$$

$$P\left(\frac{A}{E_2}\right) = \frac{{}^3C_1 \times {}^2C_1}{n + {}^5C_2} = \frac{12}{(n+5)(n+4)}$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \times P\left(\frac{A}{E_1}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)} = \frac{6}{11}$$

$$\Rightarrow \frac{\frac{1}{10}}{\frac{1}{10} + \frac{6}{(n+5)(n+4)}} = \frac{6}{11}$$

$$\Rightarrow n = 4$$

Q.6

(1)

Given n = 33

success is p & failure is q = 1 - p { $\because p + q = 1$ }

$$3P(x=0) = P(x=1)$$

$$3 \cdot {}^{33}C_0 (q)^{33} = {}^{33}C_1 p^1 q^{32}$$

$$3 \cdot (1)(q) = 33(p)$$

$$3q = 33p \Rightarrow q = 11p$$

$$p + q = 1 \Rightarrow 12p = 1 \quad \Rightarrow p = 1/12 \text{ \& } q = \frac{11}{12}$$

$$q/p = 11$$

$$\frac{P(x=15)}{P(x=18)} = \frac{P(x=16)}{P(x=17)}$$

$$\frac{{}^{33}C_{15} p^{15} q^{18}}{{}^{33}C_{18} p^{18} q^{15}} = \frac{{}^{33}C_{16} p^{16} q^{17}}{{}^{33}C_{17} p^{17} q^{16}}$$

$$= \left(\frac{q}{p}\right)^3 - \left(\frac{q}{p}\right) = (11)^3 - 11 = 1320$$

Q.7

(1)

$$L = K + 2K + 4K + 6K + 8K = 1$$

$$K = \frac{1}{21}$$

$$= \frac{[P(1 < x < 4 / x \leq 2)]}{p(x \leq 2)}$$

$$= \frac{P(2)}{P(0) + P(1) + P(2)}$$

$$= \frac{4K}{K + 2K + 4K}$$

$$= \frac{4}{7}$$

Q.8 (4)

$$\begin{aligned}
 P(\text{sum} = 48) &= P(16) \times P(16) \times P(16) \\
 &\quad + P(32) \times P(8) \times P(8) \\
 &= \frac{1}{16} \times \frac{1}{16} \times \frac{1}{16} + 3 \times \frac{2}{32} \times \frac{1}{8} \times \frac{1}{8} \\
 &= \frac{1}{16^3} + \frac{6+2}{16 \times 16 \times 16} \\
 &= \frac{13}{16^3} = \frac{13}{2^{12}}
 \end{aligned}$$

Q.9 (3)

No. of ways to select and arrange x_1, x_2, x_3, x_4, x_5 , from 1, 2, 3, ..., 18

$$n(s) = {}^{18}C_5$$

x_1	(x_2)	x_3	(x_4)	x_5
7			11	

$$n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$$

$$P(E) = \frac{6 \times 3 \times 7}{{}^{18}C_5}$$

$$\frac{1}{17 \times 4} = \frac{1}{68}$$

Q10 (3)

$B(7, p)$
 $n = 7$ $p = p$
 given
 $P(x = 3) = 5P(x = 4)$
 ${}^7C_3 \times p^3 (1-p)^4 = 5 \cdot {}^7C_4 (1-p)^3$

$$\frac{{}^7C_3}{5 \cdot {}^7C_4} = \frac{p}{1-p}$$

$$1 - p = 5p$$

$$6p = 1$$

$$p = \frac{1}{6} \Rightarrow q = \frac{5}{6}$$

$$n = 7$$

$$\text{Mean} = np = 7 \times \frac{1}{6} = \frac{7}{6}$$

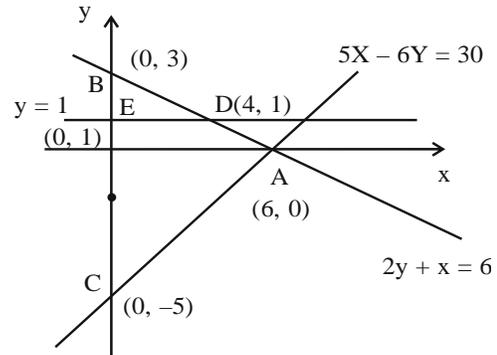
$$\text{Var} = npq = 7 \times \frac{1}{6} \times \frac{5}{6} = \frac{35}{36}$$

Sum

$$= \frac{7}{6} + \frac{35}{36}$$

$$= \frac{42 + 35}{36} = \frac{77}{36}$$

Q.11 (2)



$$\begin{aligned}
 \text{Required probability} &= \frac{\text{ar}(ADEC)}{\text{ar}(ABC)} \\
 &= 1 - \frac{\text{ar}(BDC)}{\text{ar}(ABC)} \\
 &= 1 - \frac{\frac{1}{2} \times 2 \times 4}{\frac{1}{2} \times 8 \times 6} = 1 - \frac{1}{6} = \frac{5}{6}
 \end{aligned}$$

Q.12 (19)

$$P(A') < \frac{1}{5} = \frac{36}{180}$$

5 times the sum of missing number should be less than 36.

If 1 digit is missing = 7
 If 2 digit is missing = 9
 If 3 digit is missing = 2
 If 0 digit is missing = 1

Alternate
 A is subset of S hence
 A can have elements :

type 1 : { }
 type 2 : { E_1 }, { E_2 }, ..., { E_8 }
 type 3 : { E_1, E_2 }, { E_1, E_3 } ... { E_1, E_8 }
 ⋮
 ⋮
 type 6 : { E_1, E_2, \dots, E_5 }, ... { E_4, E_5, E_6, E_7, E_8 }
 type 7 : { E_1, E_2, \dots, E_6 }, ... { E_3, E_4, \dots, E_8 }
 type 8 : { E_1, E_2, \dots, E_7 }, ... { E_2, E_3, \dots, E_8 }
 type 9 : { E_1, E_2, \dots, E_8 }

As $P(A) \geq \frac{4}{5}$;

Note : Type 1 to Type 4 elements can not be in set A as maximum probability of type 4 elements

$$\{E_5, E_6, E_7, E_8\} \text{ is } \frac{5}{36} + \frac{6}{36} + \frac{7}{36} + \frac{8}{36} = \frac{13}{18} < \frac{4}{5}$$

Now for Type 5 acceptable elements let's call

probability as P_5

$$P_5 = \frac{n_1 + n_2 + n_3 + n_4 + n_5}{36} \leq \frac{4}{5}$$

$$\Rightarrow n_1 + n_2 + n_3 + n_4 + n_5 \Rightarrow 28.8$$

Hence, 2 possible ways $\{E_5, E_6, E_7, E_8, E_3 \text{ or } E_4\}$

$$P_6 = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 \geq 28.8$$

\Rightarrow 9 possible ways

$$P_7 = n_1 + n_2 + \dots + n_7 \geq 28.8$$

\Rightarrow 7 possible ways

$$P_8 = n_1 + n_2 + \dots + n_8 \geq 28.8$$

\Rightarrow 1 possible ways

Total = 19

Q.13

(3)

$$P(E_1/E_2) = 1/2$$

$$\Rightarrow \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1}{2}$$

$$\Rightarrow P(E_2) = \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2}$$

$$P(E_2/E_1) = \frac{3}{4} \Rightarrow P(E_1) = \frac{\frac{3}{4}}{\frac{3}{6}} = \frac{1}{6}$$

$$(A) P(E_1) \cdot P(E_2) = \frac{1}{6} \cdot \frac{1}{4} = \frac{1}{24} \text{ Wrong}$$

$$(B) P(\bar{E}_1 \cap \bar{E}_2) = P(\bar{E}_1 \cup \bar{E}_2) = 1 - P(E_1 \cup E_2)$$

$$= 1 - \left\{ \frac{1}{6} + \frac{1}{4} - \frac{1}{8} \right\}$$

$$= 1 - \left\{ \frac{4+6-3}{24} \right\} = 1 - \frac{7}{24} = \frac{17}{24}$$

$$P(\bar{E}_1) \cdot P(E_2) = \frac{5}{6} \times \frac{1}{4} = \frac{5}{24}$$

$$(C) P(E_1 \cap \bar{E}_2) = P(E_1) - P(E_1 \cap E_2)$$

$$= \frac{1}{6} - \frac{1}{8} = \frac{1}{24} = P(E_1) \cdot P(E_2) \text{ Correct}$$

Q.14

(4)

$$P(H) = x, P(T) = 1-x$$

$$P(4H, 1T) = P(5H)$$

$${}^5C_1 (x)^4 (1-x)^1 = {}^5C_5 x^5$$

$$5(1-x) = x$$

$$6x = 5 \Rightarrow x = \frac{5}{6}$$

P(atmost 2H)

$$= P(0H, 5T) + P(1H, 4T) + P(2H, 3T)$$

$$= {}^5C_0 \left(\frac{1}{6}\right)^5 + {}^5C_1 \frac{5}{6} \cdot \left(\frac{1}{6}\right)^4 + {}^5C_2 \left(\frac{5}{6}\right)^3 \cdot \left(\frac{1}{6}\right)^2$$

$$= \frac{1}{6^5} (1 + 25 + 250) = \frac{276}{6^5} = \frac{46}{6^4}$$

Q.15 (1)

At least two digits are odd = exactly two digits are odd + exactly three digits are odd

For exactly three digits are odd : $5 \times 5 \times 5 = 125$

For exactly two digits odd :

If 0 is used then : $2 \times 5 \times 5 = 50$

If 0 is not used then : ${}^3C_1 \times 4 \times 5 \times 5 = 300$

$$\text{Required probability} = \frac{475}{900} = \frac{19}{36}$$

Q.16 (1)

$$P(A|B) = \frac{1}{7} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{7}$$

$$P(B) = \frac{7}{9}$$

$$P(B|A) = \frac{2}{5} \Rightarrow \frac{P(A \cap B)}{P(A)} = \frac{2}{5}$$

$$\Rightarrow P(A) = \frac{5}{18}$$

Now, $P(A' \cup B) = 1 - P(A \cap B) + P(B)$

$$= 1 - P(A) + P(A \cap B) = \frac{5}{6}$$

$$P(A' \cup B') = 1 - P(A \cap B)$$

$$= 1 - P(A) - P(B) + P(A \cap B) = \frac{1}{18}$$

\Rightarrow Both (S1) and (S2) are true

Q.17

(56)

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

$$\sigma^2 = \sum X^2 P(X) - (\sum X P(X))^2 = \frac{56}{100}$$

$$100\sigma^2 = 56$$

Q.18

(3)

$$\text{Mean} = np$$

$$np + npq = 24$$

$$\text{variance} = npq$$

$$\& \quad np \cdot npq = 128$$

$$\left. \begin{aligned} np(1+q) &= 24 \\ (np)^2 q &= 128 \end{aligned} \right\} \Rightarrow \frac{(1+q)^2}{q} = \frac{(24)^2}{128}$$

$$1+q^2+2q = q \left[\frac{24 \times 24}{128} \right]$$

$$1+q^2+2q = \frac{9q}{2}$$

$$2q^2 - 5q + 2 = 0$$

$$(2q-1)(q-2) = \begin{cases} q = \frac{1}{2} \\ q = 2 \end{cases}$$

Now $np = 1 + \frac{1}{2} = \boxed{np = 16}$

& $p + q = 1$

$$\boxed{n = 32} \quad \boxed{p = \frac{1}{2}}$$

Prob. $(1 + (su)vn) + 2(su)vn$
 $= {}^n C_1 \cdot (p)^1 (q)^{n-1} + {}^n C_2 \cdot (p)^2 (q)^{n-2}$
 $= npq^{n-1} + \frac{n(n-1)p^2}{2} q^{n-2}$

$$= 32 \cdot \frac{1}{2} \left(\frac{1}{2}\right)^{31} + \frac{32 \cdot 31}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{30}$$

$$= \frac{33}{2^{28}}$$

Q.19 (1)

$$x^2 + \alpha x + \beta > 0 \quad \forall \quad x \in \mathbb{R}$$

$$\Rightarrow D < 0$$

$$\Rightarrow \alpha^2 - 4\beta < 0$$

$$\Rightarrow \alpha^2 < 4\beta$$

$n(S) : 6 \times 6$

α	β	}	$n(A) = 17$
1	1, 2, 3, 4, 5, 6		
2	2, 3, 4, 5, 6		
3	3, 4, 5, 6		
4	5, 6		

$$P(A) = \frac{n(A)}{n(S)} = \frac{17}{36}$$

Q.20 (2)

3R	2R
4B	5B
3W	2W

A : Drown ball from boy II is black
 B : Red ball transferred

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{3}{9} \times \frac{5}{10}}{\frac{3}{9} \times \frac{5}{10} + \frac{4}{9} \times \frac{6}{10} + \frac{3}{9} \times \frac{5}{10}}$$

$$= \frac{15}{15 + 24 + 15} = \frac{15}{54} = \frac{5}{18}$$

Q.21

(96)
 Let, mean = $m = np$
 & variance = $v = npq$, $p + q = 1$

$$\text{Sum} = m + v = \frac{165}{2}$$

$$\text{Product} = mv = 1350$$

On solving,

$$m = np = 60 \quad \& \quad v = npq = \frac{45}{2}$$

$$\therefore q = \frac{3}{8} \quad \therefore P = \frac{5}{8}$$

Hence, $n = 96$

Q.22

(4)
 Total number of elements = 2022
 $2022 = 2 \times 3 \times 337$
 $\text{HCF}(n, 2022) = 1$
 is feasible when the value of 'n' and 2022 has no common factor.

A = Number which are divisible by 2 from {1, 2, 3, ..., 2022}
 $n(A) = 1011$

B = Number which are divisible by 3 from {1, 2, 3, ..., 2022}
 $n(B) = 674$

$A \cap B$ = Number which are divisible by 6 from {1, 2, 3, ..., 2022}
 6, 12, 18, ..., 2022

$$337 = n(A \cap B)$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 1011 + 674 - 337$$

$$= 1348$$

C = Number which divisible by 337 from {1, ..., 1022}

$$C = \{337, 674, 1011, 1348, 1685, 2022\}$$

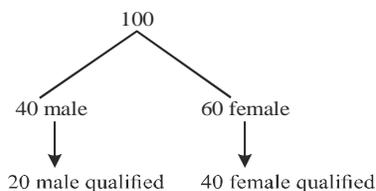
\swarrow	\downarrow	\searrow
Already counted in Set $(A \cup B)$	Already counted in Set $(A \cup B)$	Already counted in Set $(A \cup B)$

Total elements which are divisible by 2 or 3 or 337 = $1348 + 2 = 1350$

Favourable cases = Element which are neither divisible by 2, 3 or 337
 $= 2022 - 1350$
 $= 672$

$$\text{Required probability} = \frac{672}{2022} = \frac{112}{337}$$

Q.23 (1)



$$\text{Probability that chosen candidate is female} = \frac{40}{60} = \frac{2}{3}$$

Q.24 (3)

five digit number
 $\therefore 10000 + 99999$
 therefore $S = 90000$

$$\text{Number divisible by 7} = \frac{9000}{7}$$

$$\text{Number divisible by 7 and multiple by 5} = \frac{90000}{35}$$

$$\therefore \text{revised probability} = \frac{\frac{90000}{7} - \frac{90000}{35}}{90000}$$

$$P = \frac{35 - 7}{35 \times 7}$$

$$= \frac{4}{35}$$

$$\text{then } 9P = \frac{4 \times 9}{35} = 1.0285$$

Q.25 (2)

$np = 4$
 $npq = 4/3$
 $n = 6, p = 2/3, q = 1/3$
 $54 (P(X=2) + P(X=1) + P(X=0))$

$$54 \left({}^6C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^4 + {}^6C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^5 + {}^6C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^6 \right)$$

$$= \frac{146}{27}$$

Q.26 (4)

$$0 \leq P(E) \leq 1$$

$$0 \leq \frac{2+3p}{6} + \frac{2-p}{8} + \frac{1-p}{2} \leq 1$$

$$\frac{2}{3} \leq p \leq 1$$

$$p_1 = 1, p_2 = \frac{2}{3}$$

$$p_1 + p_2 = \frac{5}{3}$$

Q.27 [2]

Mean (m) + variance (v) = 24 (given)
 Mean (m) \times variance (v) = 128 (given)
 i.e. $m + v = 24$ (i)

$$mv = 128 \dots \text{(ii)}$$

$$\text{Mean (np)} = 16, \text{ variance (npq)} = \theta$$

$$q = \frac{1}{2}, p = \frac{1}{2}, n = 32$$

$$\begin{aligned} P(x > 32 - 3) &= P(x > 29) \\ &= P(x = 30) + p(x = 31) + p(x = 32) \\ &= {}^{32}C_{30} \left(\frac{1}{2}\right)^{32} + {}^{32}C_{31} \left(\frac{1}{2}\right)^{32} + {}^{32}C_{32} \left(\frac{1}{2}\right)^{32} \end{aligned}$$

$$\frac{\frac{32 \times 31}{2} + 32 + 1}{2^{32}} = \frac{16 \times 31 + 33}{2^{32}} = \frac{k}{2^{32}} \Rightarrow k = 16 \times 31 + 33$$

$$k = 529$$

Q.28

[4]

Let P (Prime number) = a
 P (composite number) = b
 $P(1) = C$
 $3a = 6b = 2c = k$

$$a = \frac{k}{3}, b = \frac{k}{6}, c = \frac{k}{2}$$

$$P(1) + P(2) + P(6) = 1$$

$$\frac{k}{2} + \frac{k}{3} \times 3 + \frac{k}{6} \times 2 = 1 \Rightarrow k = \frac{6}{11}$$

$$1^2, 2^2 = \Rightarrow (1), (4)$$

$$P(1) = C = \frac{K}{2} = \frac{3}{11}$$

$$P(1,4) = \frac{3}{11} + \frac{1}{11} = \frac{4}{11}$$

$$P(4) = \frac{k}{6} = \frac{1}{11}$$

$$\text{Mean} = nP = 2 \times \frac{4}{11} = \frac{8}{11}$$

Q.29 (2)

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{1}{3} + \frac{1}{5} - \frac{1}{2} = \frac{1}{30}$$

$$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{\frac{1}{3} - \frac{1}{30}}{\frac{4}{5}} = \frac{\frac{9}{30}}{\frac{4}{5}} = \frac{3}{8}$$

$$P(B|A') = \frac{P(B \cap A')}{P(A')} = \frac{\frac{1}{5} - \frac{1}{30}}{\frac{2}{3}} = \frac{\frac{5}{30}}{\frac{2}{3}} = \frac{5}{4} = \frac{1}{4}$$

$$\therefore P(A|B') + P(B|A') = \frac{3}{8} + \frac{1}{4} = \frac{5}{8}$$