

SOLUTION RELATIONS AND FUNCTIONS

EXERCISE-I (MHT CET LEVEL)

Q.1 (2)
Obviously, the relation is not reflexive and transitive but it is symmetric, because $x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$.

Q.2 (2) Since R is an equivalence relation on set A , therefore $(a, a) \in R$ for all $a \in A$. Hence, R has at least n ordered pairs.

Q.3 (2)
Clearly, the relation is symmetric but it is neither reflexive nor transitive.

Q.4 (1)
Since R is reflexive relation on A , therefore $(a, a) \in R$ for all $a \in A$.
The minimum number of ordered pairs in R is n .
Hence, $m \geq n$.

Q.5 (1,2)
 $(1, 1)(2, 2)(3, 3)(4, 4) \in R$; $\therefore R$ is reflexive.
 $\therefore (1, 2)(3, 1)$ and also $(2, 1)(1, 3)$.
Hence, R is symmetric. But clearly R is not transitive.

Q.6 (2)
For any integer n , we have $n | n \Rightarrow n R n$
So, $n R n$ for all $n \in \mathbb{Z} \Rightarrow R$ is reflexive
Now $2 | 6$ but $6 \nmid 2, \Rightarrow (2, 6) \in R$ but $(6, 2) \notin R$
So, R is not symmetric.
Let $(m, n) \in R$ and $(n, p) \in R$
Then $\left. \begin{matrix} (m, n) \in R \Rightarrow m | n \\ (n, p) \in R \Rightarrow n | p \end{matrix} \right\} \Rightarrow m | p \Rightarrow (m, p) \in R$
So, R is transitive.
Hence, R is reflexive and transitive but it is not symmetric.

Q.7 (1)
For any $a \in \mathbb{N}$, we find that $a | a$, therefore R is reflexive but R is not transitive, because $a R b$ does not imply that $b R a$.

Q.8 (3)
It is obvious.

Q.9 (3)
It is obvious.

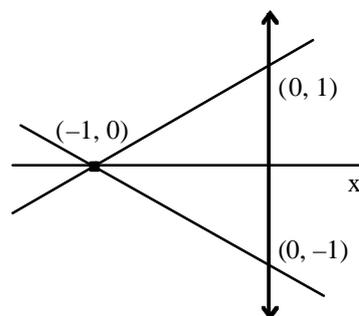
Q.10 (1) $A = \{2, 4, 6\}$; $B = \{2, 3, 5\}$
 $\therefore A \times B$ contains $3 \times 3 = 9$ elements.
Hence, number of relations from A to $B = 2^9$.

Q.11 (4)
Total number of reflexive relations in a set with n elements $= 2^n$.
Therefore, total number of reflexive relation set with 4 elements $= 2^4$.

Q.12 (4)
$$f(x) = \frac{x^2 + 2x + 5}{x^2 + x + 1} = \frac{1 + \frac{2}{x} + \frac{5}{x^2}}{1 + \frac{1}{x} + \frac{1}{x^2}}$$

 $f(\infty) = 1$ & $f(-\infty) = 1$
 \therefore many one function
if $f(x) \rightarrow \infty \Rightarrow x^2 + x + 1 \rightarrow 0$ which is false as $x^2 + x + 1 \neq 0 \forall x \in \mathbb{R}$
 \therefore into function.

Q.13 (4)
We have to choose two functions which are Bijective and mirror image about $y = 0$ i.e. about x -axis.



Let $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $-x + y = 1$
i.e. $y = x + 1$
 $f(x) = y = x + 1$ is one-one as well as onto.
Also $g : \mathbb{R} \rightarrow \mathbb{R}$ such that $-x - y = 1$

i.e. $y = -x - 1$

$g(x) = y = -x - 1$ is one-one as well as onto.

$f(x) = x + 1$ and $g(x) = -x - 1$ are image about line $y = 0$

From given condition, $h(x) = f(x) + g(x)$

$$h(x) = x + 1 - x - 1 = 0$$

$$h(x) = 0$$

which shows that $h(x)$ is neither one-one nor onto

\therefore Option (4) is correct Answer.

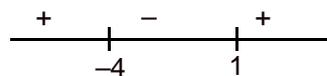
- Q.14 (3)
- Q.15 (1)
- Q.16 (1)
- Q.17 (1)
- Q.18 (1)
- Q.19 (3)
- Q.20 (1)
- Q.21 (4)
- Q.22 (3)

$$f: \mathbb{R} \rightarrow \mathbb{R}; f(x) = \frac{x^2 - 4}{x^2 + 1} \Rightarrow f(-x) = \frac{x^2 - 4}{x^2 + 1} = f(x)$$

$f(x)$ is even that's why many-one.

$$y = \frac{x^2 - 4}{x^2 + 1} \Rightarrow yx^2 + y = x^2 - 4$$

$$\Rightarrow x^2 = \frac{y + 4}{1 - y} \geq 0$$



$$\Rightarrow \frac{y + 4}{y - 1} \leq 0$$

$$\therefore y \in [-4, 1)$$

Range \neq Co-Domain \Rightarrow into

Q.23 (2) Width of both interval is same, which can mapped by these function $y = 1 - x$ and $y = 1 + x$.

Q.24 (1)

Let $x_1, x_2 \in \mathbb{R}$, then $f(x_1) = \cos x_1$, $f(x_2) = \cos x_2$,

so $f(x_1) = f(x_2)$

$$\Rightarrow \cos x_1 = \cos x_2 \Rightarrow x_1 = 2n\pi \pm x_2$$

$\Rightarrow x_1 \neq x_2$, so it is not one-one.

Again the value of f -image of x lies in between -1 to 1

$$\Rightarrow f[\mathbb{R}] = \{f(x) : -1 \leq f(x) \leq 1\}$$

So other numbers of co-domain (besides -1 and 1) is not f -image. $f[\mathbb{R}] \in \mathbb{R}$, so it is also not onto. So this mapping is neither one-one nor onto.

Q.25 (2)

We have $f(x) = (x - 1)(x - 2)(x - 3)$

$$\text{and } f(1) = f(2) = f(3) = 0$$

$\Rightarrow f(x)$ is not one-one.

For each $y \in \mathbb{R}$, there exists $x \in \mathbb{R}$ such that

$f(x) = y$. Therefore f is onto.

Hence $f: \mathbb{R} \rightarrow \mathbb{R}$ is onto but not one-one.

Q.26 (4)

$$f(-1) = f(1) = 1;$$

\therefore function is many-one function. Obviously, f is not onto so f is neither one-one nor onto.

Q.27 (1)

Let $x, y \in \mathbb{N}$ such that $f(x) = f(y)$

$$\text{Then } f(x) = f(y) \Rightarrow x^2 + x + 1 = y^2 + y + 1$$

$$\Rightarrow (x - y)(x + y + 1) = 0 \Rightarrow x = y \quad \text{or}$$

$$x = (-y - 1) \notin \mathbb{N}$$

$\therefore f$ is one-one.

Again, since for each $y \in \mathbb{N}$, there exist $x \in \mathbb{N}$

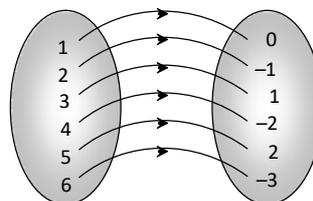
$\therefore f$ is onto.

Q.28 (3)

$$f: \mathbb{N} \rightarrow \mathbb{I}$$

$$f(1) = 0, f(2) = -1, f(3) = 1, f(4) = -2, f(5) = 2$$

and $f(6) = -3$ so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B . Hence f is one-one and onto function.

Q.29 (1)

$$-\sqrt{1 + (-\sqrt{3})^2} \leq (\sin x - \sqrt{3} \cos x) \leq \sqrt{1 + (-\sqrt{3})^2}$$

$$-2 \leq (\sin x - \sqrt{3} \cos x) \leq 2$$

$$-2 + 1 \leq (\sin x - \sqrt{3} \cos x + 1) \leq 2 + 1$$

$$-1 \leq (\sin x - \sqrt{3} \cos x + 1) \leq 3 \text{ i.e., range} = [-1, 3]$$

\therefore For f to be onto $S = [-1, 3]$

Q.30 (4)

$$\text{Let } f(x_1) = f(x_2) \Rightarrow [x_1] = [x_2] \Rightarrow x_1 = x_2$$

{For example, if $x_1 = 1.4, x_2 = 1.5$, then $[1.4] = [1.5] = 1$ }

$\therefore f$ is not one-one.

Also, f is not onto as its range \mathbb{I} (set of integers) is a proper subset of its co-domain \mathbb{R} .

Q.31 (1)

It is obvious.

Q.32 (1)

$$(f-g)(x) = \begin{cases} x, & x \in \mathbb{Q} \\ -x, & x \notin \mathbb{Q} \end{cases}$$

Q.33 (3)

$$\begin{aligned} f(\theta) &= \sin\theta(\sin\theta + \sin 3\theta) \\ f(\theta) &= \sin^2\theta + \sin\theta(3\sin\theta - 4\sin^3\theta) \\ f(\theta) &= \sin^2\theta + 3\sin^2\theta - 4\sin^4\theta \\ f(\theta) &= 4\sin^2\theta - 4\sin^4\theta = 4\sin^2\theta(1 - \sin^2\theta) \\ &= 4\sin^2\theta \cos^2\theta \\ f(\theta) &= (\sin 2\theta)^2 \\ -1 &\leq \sin 2\theta \leq 1 \Rightarrow \sin^2 2\theta \leq 1 \\ f(\theta) &= \sin^2 2\theta \leq 1 \text{ for all real } \theta \\ \Rightarrow f(\theta) &\geq 0 \text{ for all real } \theta \\ \therefore \text{Option (3) is correct Answer.} \end{aligned}$$

Q.34 (2)

$$3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$$

$$\text{For } x = 7, 3f(7) + 2f(11) = 70 + 30 = 100$$

$$\text{For } x = 11, 3f(11) + 2f(7) = 140$$

$$\frac{f(7)}{-20} = \frac{f(11)}{-220} = \frac{-1}{9-4} \Rightarrow f(7) = 4.$$

Q.35 (3)

$$\frac{f(a)}{f(a+1)} = \frac{a/(a-1)}{(a+1)/a} = \frac{a^2}{a^2-1} = f(a^2).$$

Q.36 (4)

The given expression is

$$\begin{aligned} &\cos(\log x^2) \cos(\log y^2) - \frac{1}{2} \left[\cos \log \frac{x^2}{2} + \cos \log \frac{x^2}{y^2} \right] \\ &= \frac{1}{2} \left[\cos(\log x^2 + \log y^2) + \cos(\log x^2 - \log y^2) \right] \\ &\quad - \frac{1}{2} \left[\cos \log \frac{x^2}{2} + \cos(\log x^2 - \log y^2) \right] \\ &= \frac{1}{2} \left[\cos \log x^2 y^2 - \cos \log \frac{x^2}{2} \right]. \end{aligned}$$

Q.37 (4)

$$\begin{aligned} x^3 f\left(\frac{1}{x}\right) &= x^3 \left[\frac{4}{x^3} + \frac{3}{x^2} + \frac{3}{x} + 4 \right] \\ &= 4 + 3x + 3x^2 + 4x^3 = f(x). \end{aligned}$$

Q.38 (2)

$$f(2x) = 2(2x) + |2x| = 4x + 2|x|,$$

$$f(-x) = -2x + |-x| = -2x + |x|,$$

$$f(x) = 2x + |x| \Rightarrow f(2x) + f(-x) - f(x)$$

$$= 4x + 2|x| + |x| - 2x - 2x - |x|$$

Q.39 (2)

$$g(x) = 1 + \sqrt{x} \text{ and } f(g(x)) = 3 + 2\sqrt{x} + x \dots (i)$$

$$\Rightarrow f(1 + \sqrt{x}) = 3 + 2\sqrt{x} + x$$

$$\text{Put } 1 + \sqrt{x} = y \Rightarrow x = (y-1)^2$$

$$\text{then, } f(y) = 3 + 2(y-1) + (y-1)^2 = 2 + y^2$$

$$\text{therefore, } f(x) = 2 + x^2.$$

Q.40 (2)

$$f(x) = \frac{x}{\sqrt{1+x^2}}$$

$$(f \circ f)(x) = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1 + \frac{x^2}{1+x^2}}} = \frac{x}{\sqrt{2x^2+1}}$$

$$(f \circ f \circ f)(x) = \frac{\frac{x}{\sqrt{2x^2+1}}}{\sqrt{1 + \frac{x^2}{2x^2+1}}} = \frac{x}{\sqrt{1+3x^2}}$$

Q.41 (2)

$$f \circ f(x) = \begin{cases} f(x); & \text{when } f(x) \text{ is rational} \\ 1 - f(x); & \text{when } f(x) \text{ is irrational} \end{cases}$$

$$= \begin{cases} x; & \text{when } x \text{ is rational} \\ 1 - (1 - x); & \text{when } x \text{ is irrational} \end{cases} = x$$

Q.42 (2)

$$\text{We have } g(-3) = 0$$

$$\Rightarrow f(g(-3)) = f(0) = 7(0)^2 + 0 - 8 = -8$$

$$\therefore f \circ g(-3) = -8$$

$$g(9) = 9^2 + 4 = 85 \Rightarrow f(g(9)) = f(85)$$

$$= 8(85) + 3 = 683$$

$$\therefore f \circ g(9) = 683$$

$$f(0) = 7 \cdot 0^2 + 0 - 8 = -8$$

$$\Rightarrow g(f(0)) = g(-8) = |-8| = 8$$

$$\therefore g \circ f(0) = 8$$

$$f(6) = 4(6) + 5 = 29$$

$$\Rightarrow g(f(6)) = g(29) = (29)^2 + 4 = 845$$

$$\Rightarrow \text{gof}(6) = 845$$

Q.43 (2)

$$\begin{aligned} \text{Given that } f(x) &= (a - x^n)^{1/n} \\ \therefore \text{fof}(x) &= [a - \{(a - x^n)^{1/n}\}^n]^{1/n} \\ &= [a - (a - x^n)]^{1/n} \\ &= [x^n]^{1/n} = x \end{aligned}$$

Q.44 (4)

Q.45 (1)

Q.46 (2)

$$(\text{fog})(x) = f(g(x)) = a(cx + d) + b$$

$$\text{and } (\text{gof})(x) = g(f(x)) = c(ax + b) + d$$

Given that, $(\text{fog})(x) = (\text{gof})(x)$ and at $a = 1, b = 2$
 $\Rightarrow cx + d + 2 = cx + 2c + d \Rightarrow c = 1$ and d is arbitrary.

Q.47 (4)

$$f(f(x)) = \frac{\alpha f(x)}{f(x) + 1} = \frac{\alpha \left(\frac{\alpha x}{x + 1} \right)}{\left(\frac{\alpha x}{x + 1} + 1 \right)} = \frac{\alpha^2 \cdot x}{\alpha x + x + 1}$$

$$\therefore x = \frac{\alpha^2 \cdot x}{(\alpha + 1)x + 1} \text{ or } x((\alpha + 1)x + 1 - \alpha^2) = 0$$

$$\text{or } (\alpha + 1)x^2 + (1 - \alpha^2)x = 0.$$

This should hold for all x .

$$\Rightarrow \alpha + 1 = 0, 1 - \alpha^2 = 0, \therefore \alpha = -1.$$

Q.48 (4)

$$\text{Here } f(2) = \frac{5}{4}$$

$$\text{Hence } (\text{fof})(2) = f(f(2)) = f\left(\frac{5}{4}\right) = \frac{2 \times \frac{5}{4} + 1}{3 \times \frac{5}{4} - 2} = 2.$$

Q.49 (2)

$$(\text{gof})(x) = |\sin x| \text{ and } f(x) = \sin^2 x$$

$$\Rightarrow g(\sin^2 x) = |\sin x|; \therefore g(x) = \sqrt{x}.$$

Q.50 (3) $f[f(x)] = [a - \{f(x)\}^n]^{1/n} = [a - (a - x^n)]^{1/n} = x.$

Q.51 (4)

$$f(x) = \frac{\alpha x}{x + 1} \quad f(f(x)) = f\left(\frac{\alpha x}{x + 1}\right) = \frac{\alpha \left(\frac{\alpha x}{x + 1}\right)}{\frac{\alpha x}{x + 1} + 1}$$

$$\text{But } f(f(x)) = x \therefore \frac{\alpha^2 x}{\alpha x + x + 1} = x$$

$$\text{Put } \alpha = -1 \quad \frac{(-1)^2 x}{(-1)x + x + 1} = \frac{x}{-x + x + 1} = x;$$

$$\alpha = -1$$

Q.52 (2)

$$g(x) = 1 + \{x\}; f\{g(x)\} = f\{1 + \{x\}\} = f(k) = 1$$

where, $k = 1 + \{x\}, 1 \leq k < 2$

Q.53 (1)

$$(1) f(-x) = (-x) \frac{a^{-x} - 1}{a^{+x} + 1} + (-x) \frac{1 - a^x}{1 + a^x} = f(x)$$

$$(2) f(-x) = -f(x)$$

$$(3) f(-x) = -f(x)$$

$$(4) f(-x) = \frac{1 + a^x}{1 - a^x} = -f(x)$$

Hence (1) is an even function

Q.54 (1)

$$(1) f(-x) = \sqrt{1 - x + x^2} - \sqrt{1 + x + x^2} = -f(x)$$

$$(2) f(-x) = (-x) \left(\frac{1 + a^x}{1 - a^x} \right) = f(x)$$

$$(3) f(-x) = \log \left(\frac{1 + x}{1 + x^2} \right)$$

$$(4) f(x) = k \text{ (even)}$$

Q.55 (3)

$$f(x) = 0 \forall x \in R \Rightarrow f(3) - f(2) = 0$$

Q.56 (1)

We have, $\text{fog}(-x) = f[g(-x)] = f[-g(x)]$

(\because g is odd)

$$= f[g(x)] \quad (\because f \text{ is even})$$

$$= \text{fog}(x) = \forall x \in R.$$

\therefore fog is an even function.

Q.57 (1)

$$f(x) = 3x^2 - 4x + 8 \log(1 + |x|); x \in [0, 1]$$

$$f(-x) = 3x^2 + 4x + 8 \log(1 + |x|); x \in [-1, 0]$$

\therefore Option (1) is correct Answer.

Q.58 (2)

$$f(x) = 2x^6 + 3x^4 + 4x^2$$

$$f(-x) = 2(-x)^6 + 3(-x)^4 + 4(-x)^2 = f(x)$$

$\Rightarrow f(x)$ is an even function and derivative of an even function is always odd.

Q.59 (2)

$$f(x) = \log(x + \sqrt{x^2 + 1})$$

and $f(-x) = -\log(x + \sqrt{x^2 + 1}) = -f(x)$

$f(x)$ is odd function.

Q.60 (2)

$x \longrightarrow x + 2$
 (ii) $f(x+3) + f(x+5) = 2$
 (i) $f(x+1) + f(x+3) = 2$
 (ii) $-(i) \Rightarrow f(x+1) = f(x+5)$
 $f(x+1) = f((x+1)+4)$
 Hence $T = 4$

Q.61 (1)

$f(x) = e^{\cos\{x\}} + \sin\pi[x]$
 period of $\{x\} = x - [x]$ is 1

Also Period of $\sin\pi[x]$ is $\frac{\pi}{|\pi|} = 1$

L.C.M of (1, 1) is 1

\therefore Option (A) is correct Answer.

Q.62 (3)

$$\frac{\sin\theta + \sin 2\theta}{\cos\theta + \cos 2\theta}$$

$$= \frac{2\sin\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\cos\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)} = \tan\left(\frac{3\theta}{2}\right)$$

Hence period = $\frac{2\pi}{3}$

Q.63 (4)

$\tan\theta$ is of period π so that $\tan 3\theta$ is of period $\pi/3$.

Q.64 (4)

$f(x) = \cos(ax) + \sin bx$
 $\cos ax$ is periodic with period = $2\pi/a$
 $\sin bx$ is periodic with period = $2\pi/b$

$\frac{1}{2}$ L.C.M of $\left[\frac{2\pi}{a}, \frac{2\pi}{b}\right]$

Option (1), (2), (3) are correct
 \therefore Option (4) is correct Answer.

Q.65 (2)

$f(x) = 2(x - [x]) + \sin^2\pi(x - [x])$
 $x - [x]$ is periodic with period 1

$\sin^2\pi(x - [x])$ is periodic with period 1
 $f(x)$ is periodic with period 1
 \therefore Option (2) is correct Answer.

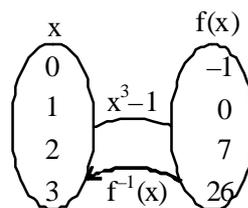
Q.66 (1)

$f(x) = \sin x + \tan x + \operatorname{sgn}(x^2 - 6x + 10)$
 $\downarrow \quad \downarrow \quad \downarrow$
 L.C.M.: $T = 2\pi \quad 2\pi, \quad 2\pi$

Q.67 (3)

Let $f(x)$ be periodic with period T .
 Then, $f(x+T) = f(x)$ for all $x \in \mathbb{R}$
 $\Rightarrow x+T - [x+T] = x - [x]$, for all $x \in \mathbb{R}$
 $\Rightarrow x+T - x = [x+T] - [x]$
 $\Rightarrow [x+T] - [x] = T$ for all $x \in \mathbb{R}$
 $\Rightarrow T = 1, 2, 3, 4$
 The smallest value of T satisfying
 $f(x+T) = f(x)$ for all $x \in \mathbb{R}$ is 1.
 Hence $f(x) = x - [x]$ has period 1.

Q.68 (3)



\therefore Domain of $f^{-1}(x)$
 = range of $f(x)$
 = $\{-1, 0, 7, 26\}$

Q.69 (3)

$y = \frac{e^{2x} - 1}{e^{2x} + 1}$; Let $e^{2x} = t$

$y = \frac{t-1}{t+1} \Rightarrow ty + y = t - 1$

$\Rightarrow \frac{y+1}{1-y} = t$

$\Rightarrow e^{2x} = \frac{y+1}{1-y} \Rightarrow 2x = \log\left(\frac{y+1}{1-y}\right)$

$\Rightarrow f^{-1}(x) = \frac{1}{2} \log\left(\frac{1+x}{1-x}\right)$

Q.70 (2)

Let $f(x) = y$. Then $\frac{1-x}{1+x} = y$

$$\Rightarrow x = \frac{1-y}{1+y}$$

$$\Rightarrow f^{-1}(y) = \frac{1-y}{1+y}$$

Thus, $f^{-1}(x) = \frac{1-x}{1+x}$

Clearly, $f^{-1}(x)$ is defined for $1+x \neq 0$

Hence, domain of $f^{-1}(x)$ is $R - \{-1\}$

- Q.71** (3)
Q.72 (1)
Q.73 (2)

From given statement, following combinations may be generated as –

- $f(x) = 1, f(y) = 1, f(z) = 2$ one-one not exist
 $f(y) = 2, f(z) = 2, f(x) = 3$ one-one not exist
 $f(y) = 3, f(z) = 2, f(x) = 3$ one-one not exist
 $f(z) = 1, f(x) = 2, f(y) = 1$ one-one not exist
 $f(z) = 3, f(y) = 1, f(x) = 2$ one-one mapping

Hence, $f^{-1}(1) = y$

\therefore Option (2) is correct Answer.

- Q.74** (4)

Given $f : (2,3) \rightarrow (0,1)$ and $f(x) = x - [x]$

$$.f(x) = y = x - 2 \Rightarrow x = y + 2 = f^{-1}(y) \Rightarrow f^{-1}(x) = x + 2$$

- Q.75** (3)

It is direct consequence of the definition.

- Q.76** (4)

$A = N \times N$ Binary operation $*$ is defined as

$$(a, b) * (c, d) = (a + c, b + d)$$

(a) Now $(c, d) * (a, b) = (c + a, d + b) = (a + c, b + d) = (a, b) * (c, d)$

$$\Rightarrow (a, b) * (c, d) = (c, d) * (a, b)$$

\therefore This operation $*$ is commutative

(b) Next $(a, b) * [(c, d) * (e, f)]$
 $= (a, b) * (c + e, d + f) = ((a + c + e), (b + d + f))$ and
 $[(a, b) * (c, d)] * (e, f)$
 $= (a + c, b + d) * (e, f) = ((a + c + e), (b + d + f))$
 $\Rightarrow (a, b) * [(c, d) * (e, f)] = [(a, b) * (c, d)] * (e, f)$

\therefore The binary operation given is associative

(c) An element $e = (e_1, e_2) \in A$ will be an identify element for the operation it

$a * e = a = e * a \forall a = (a_1, a_2) \in A$
 i.e. $(a_1 + e_1, a_2 + e_2) = (a_1, a_2) = (e_1 + a_1, e_2 + a_2)$
 Which is not true for any element therefore e .
 Identity element does not exists.

- Q.77** (4)
Q.78 (3)

- Q.79** (3)
Q.80 (4)
Q.81 (2)
Q.82 (4)
Q.83 (2)
Q.84 (1)
Q.85 (3)
Q.86 (4)
Q.87 (4)

A binary operation $*$ on $\{a, b\}$ is a function form $\{a, b\} \times \{a, b\} \rightarrow \{a, b\}$

i.e. $*$ is a function form

$$\{(a,a), (a,b), (b,a), (b,b)\} \rightarrow \{a, b\}$$

Hence, the total number of binary operations on the set $\{a, b\}$ is 2^4 i.e 16

EXERCISE-II (JEE MAIN LEVEL)

- Q.1** (2)
 Reflexive relation : $a R a$
 but identity relation is $y = x : x \in A \& y \in A$
 so $I \subset R$
- Q.2** (2)
 $R = \{(1, 2), (2, 3)\}$
 for Reflexive : $a R a$
 for symmetric : $a R b \Rightarrow b R a$
 for transitive : $a R b, b R c \Rightarrow a R c$
 So elements to be added
 $\{(1, 1), (2, 2), (3, 3), (2, 1), (3, 2), (1, 3), (3, 1)\}$
- Q.3** (2)
 Since $x < y, y < z \Rightarrow x < z \forall x, y, z \in N$
 $\therefore x R y, y R z \Rightarrow x R z$, Relation is transitive,
 does not give ,
 Relation is not symmetric.
 Since does not hold, hence relation is not reflexive.
- Q.4** (2)
 Let $(a, b) \in R$
 Then, $(a, b) \in R \Rightarrow (b, a) \in R^{-1}$, [By def. of R^{-1}]
 $\Rightarrow (b, a) \in R$, [$\because R = R^{-1}$]
 So, R is symmetric.
- Q.5** (3)
 Since $(1, 1) \notin R$ so, is not reflexive.
 Now $(1, 2) \in R$ but $(2, 1)$, therefore R is not symmetric Clearly R is transitive.
- Q.6** (4)
 It is obvious.
- Q.7** (3)
 Given $A = \{1, 2, 3, 4\}$
 $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$
 $(2, 3) \in R$ but $(3, 2) \notin R$. Hence R is not symmetric.

R is not reflexive as $(1, 1) \notin R$.
 R is not a function as $(2, 4) \in R$ and $(2, 3) \in R$.
 R is not transitive as $(1, 3) \in R$ and $(3, 1) \in R$
 but $(1, 1) \notin R$.

Q.8 (1)
 Since $1 + a.a = 1 + a^2 > 0, \forall a \in S, \therefore (a, a) \in R$
 $\therefore R$ is reflexive.
 Also $(a, b) \in R \Rightarrow 1 + ab > 0$
 $\Rightarrow 1 + ba > 0 \Rightarrow (b, a) \in R,$
 $\therefore R$ is symmetric.
 $\therefore (a, b) \in R$ and $(b, c) \in R$ need not imply
 $(a, c) \in R$. Hence, R is not transitive.

Q.9 (1)
 $A = \{3, 6, 9, 12\}$
 (i) $\therefore 3R3, 6R6, 9R9, 12R12$ so R is reflexive.
 (ii) $\therefore 6R12$ but $12 \not R 6$ so R is not symmetric.
 (iii) Also there are not pairs for whom transitivity fails.
 $3R6$ & $6R12 \Rightarrow 3R12$
 $\therefore R$ is transitive.

Q.10 (1)
 xRx : As word x has all letters common with itself, so reflexive.
 (i) $xRy \Rightarrow yRx$ so symmetric
 (ii) $xRy, yRz \not\Rightarrow xRz$
 Ex. $(OX, Box) \in R$ & $(Box, Bird) \in R$
 but $(OX, Bird) \notin R$, so not transitive.

Q.11 (3)
 $S = \{(x, y) : y = x + 1 \text{ \& } 0 < x < 2\}$
 Here S is not reflexive as $(x, x) : x = x + 1$ is false.
 Hence S is not equivalence
 & $T = \{(x, y) : x - y \text{ is an integer}\}$
 Now $(x, x) : x - x = 0$ is an integer so reflexive.
 & $(xRy) \Rightarrow (yRx)$ so symmetric
 & $(xRy) \text{ \& } (yRz) \Rightarrow (xRz)$ is transitive.
 Hence R is equivalence.

Q.12 (4)
 Since $A \subseteq A \therefore$ Relation ' \subseteq ' is reflexive.
 Since $A \subseteq B, B \subseteq C \Rightarrow A \subseteq C$
 Relation ' \subseteq ' is transitive.

But $A \subseteq B, \Rightarrow B \subseteq A, \therefore$ Relation is not symmetric.

Q.13 (1)
 Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (1, 2)\}, S = \{(2, 2), (2, 3)\}$ be transitive relations on A .
 Then $R \cup S = \{(1, 1); (1, 2); (2, 2); (2, 3)\}$
 Obviously, $R \cup S$ is not transitive. Since $(1, 2) \in R \cup S$ and $(2, 3) \in R \cup S$ but $(1, 3) \notin R \cup S$.

Q.14 (4)
 On the set N of natural numbers,
 $R = \{(x, y) : x, y \in N, 2x + y = 41\}$. Since $(1, 1) \notin R$ as $2.1 + 1 = 3 \neq 41$. So, R is not reflexive.
 $(1, 39) \in R$ but $(39, 1) \notin R$. So R is not symmetric
 $(1, 39) \in R$. But $(20, 39) \notin R$, So R is not transitive.

Q.15 (2)
 For any $a \in R$, we have $a \geq a$, Therefore the relation R is reflexive but it is not symmetric as $(2, 1) \in R$ but $(1, 2) \notin R$. The relation R is transitive also, because $(a, b) \in R, (b, c) \in R$ imply that $a \geq b$ and $b \geq c$ which is turn imply that $a \geq c$.

Q.16 (1)
 $P : a \rho b$ iff $|a - b| \leq \frac{1}{2}$

Reflexive : $a \rho a : |0 - a| \leq \frac{1}{2}$ (True)

Symmetric : $a \rho b \Rightarrow b \rho a$

$|a - b| \leq \frac{1}{2} \Rightarrow |b - a| \leq \frac{1}{2}$ (True)

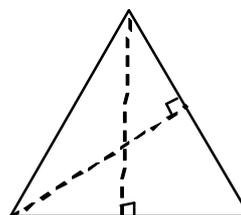
Transitive : $a \rho b : b \rho a \Rightarrow a \rho c$

$|a - b| \leq \frac{1}{2}; |b - c| \leq \frac{1}{2}$

$\Rightarrow |a - c| \leq \frac{1}{2}$

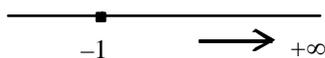
so not transitive

Q.17 (4)
 $f : A \rightarrow B$
 $f(\Delta) = \text{area of } \Delta, (\Delta \in A)$
 Area of Δ will be same for different sides combination



$\therefore f(x) = f(y)$ but $x \neq y$
 \therefore It is many one mapping
 It will also be onto mapping
 \therefore Given function is many-one onto mapping

Q.18 (2)
 We have to check for every option
For (1): $f(x) = |x + 1|; x \in [-1, \infty)$
 $|x + 1| = x + 1$



For one-one

$$f(x) = f(y) \Rightarrow x + 1 = y + 1$$

$$\Rightarrow x = y \text{ for } x \in [-1, \infty)$$

option (1) is Injective

For (2): $g(x) = x + \frac{1}{x}$; $x \in (0, \infty)$

If $g(x) = g(y)$

$$\Rightarrow x + \frac{1}{x} = y + \frac{1}{y}$$

$$\Rightarrow (x - y) + \frac{1}{x} - \frac{1}{y} = 0$$

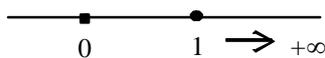
$$\Rightarrow (x - y) + \frac{y - x}{xy} = 0$$

$$\Rightarrow (x - y) \frac{[xy - 1]}{xy} = 0$$

$$\Rightarrow x = y \text{ and } xy - 1 \neq 0$$

$$\Rightarrow x^2 - 1 \neq 0 \Rightarrow x \neq \pm 1$$

But $x \in (0, \infty)$



Hence option (2) must-not be Injective

For (3): $h(x) = x^2 + 4x - 5$; $x \in (0, \infty)$

$$\text{If } h(x) = h(y) \Rightarrow x^2 + 4x - 5 = y^2 + 4y - 5$$

$$\Rightarrow (x - y)(x + y) + (x - y) \cdot 4 = 0$$

$$\Rightarrow (x - y)(x + y + 4) = 0$$

$$\Rightarrow x = y \text{ and } x + y + 4 \neq 0$$

$$\Rightarrow x \neq -2$$

But $x \in (0, \infty)$

\therefore Option (3) must be Injective

For (4): $k(x) = e^{-x}$; $x \in [0, \infty)$

$$\text{If } k(x) = k(y) \Rightarrow e^{-x} = e^{-y}$$

$$\Rightarrow -x = -y$$

$$\Rightarrow x = y \text{ for } x \in [0, \infty)$$

\therefore Option (4) is Injective

\therefore Correct option is (2)

Q.19

(2)

Since the function f is bijective, therefore f is onto. Therefore range of $f = B$.

$$\text{let } y = -x^2 + 6x - 8$$

$$\Rightarrow x^2 - 6x + (8 + y) = 0$$

$$\Rightarrow x = \frac{6 \pm \sqrt{36 - 4(8 + y)}}{2} = \frac{6 \pm \sqrt{4 - 4y}}{2}$$

For x to be real, $4 - 4y \geq 0 \Rightarrow y \leq 1$

$\therefore B = \text{range of } F = (-\infty, 1]$

Q.20

(4)

Q.21

(3)

Given $A = \mathbb{R} - \{2\}$
 $B = \mathbb{R} - \{1\}$

$$f: A \rightarrow B; f(x) = \frac{x - 3}{x - 2}$$

For one-one: $f(x) = f(y)$

$$\Rightarrow \frac{x - 3}{x - 2} = \frac{y - 3}{y - 2}$$

$$\Rightarrow (x - 3)(y - 2) = (y - 3)(x - 2)$$

$$\Rightarrow xy - 3y - 2x + 6 = xy - 3x - 2y + 6$$

$$\Rightarrow x = y$$

i.e. $f(x)$ is one-one mapping

$$\text{For onto: } y = \frac{x - 3}{x - 2} \Rightarrow x - 3 = yx - 2y$$

$$\Rightarrow 2y - 3 = x(y - 1)$$

$$\Rightarrow x = \frac{2y - 3}{y - 1}$$

$$f(x) = \frac{x - 3}{x - 2} = \frac{\frac{2y - 3}{y - 1} - 3}{\frac{2y - 3}{y - 1} - 2} = \frac{2y - 3 - 3y + 3}{2y - 3 - 2y + 2}$$

$$f(x) = y$$

i.e. $f(x)$ is onto mapping

\therefore Option (1), (2) and (3) are correct Answer.

Q.22

(1, 2)

$$f: [-1, 1] \rightarrow [0, 2]$$

$$f(x) = ax + b \rightarrow \text{onto}$$

$$\therefore \text{range} = \text{codomain}$$

checking options

Q.23

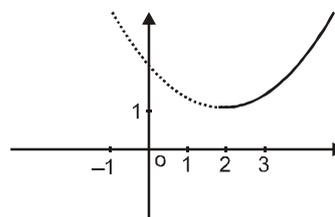
(2)

$$f: [2, \infty) \rightarrow Y$$

$$f(x) = x^2 - 4x + 5$$

$$f(x) = (x - 2)^2 + 1$$

For given domain by graph range is $[1, \infty)$



For function to be onto codomain $Y = [1, \infty)$

Q.24

(1)

$$f(x) = \frac{4a - 7}{3} x^3 + (a - 3)x^2 + x + 5$$

Case - 1 : $a = \frac{7}{4}$

$$f(x) = -\frac{5}{4}x^2 + x + 5$$

Which can't be one-one

Case - 2 : $a \neq \frac{7}{4}$

$$f(x) = (4a - 7)x^2 + 2(a - 3)x + 1$$

$D \leq 0$

$$\Rightarrow 4(a - 3)^2 - 4(4a - 7) \leq 0$$

$$\Rightarrow a^2 - 6a + 9 - 4a + 7 \leq 0$$

$$\Rightarrow a^2 - 10a + 16 \leq 0$$

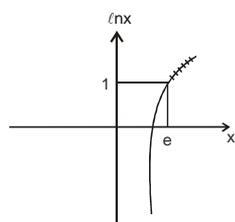
$$\Rightarrow (a - 8)(a - 2) \leq 0 \quad \therefore 2 \leq a \leq 8$$

Q.25

(3)

$$f : (e, \infty) \rightarrow \mathbb{R}$$

$$f(x) = \ln(\ln(\ln x))$$

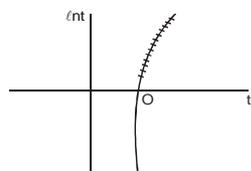


$$e < x < \infty$$

$$1 < \ln x < \infty$$

$$0 < \ln(\ln x) < \infty$$

$$-\infty < \ln(\ln(\ln x)) < \infty$$

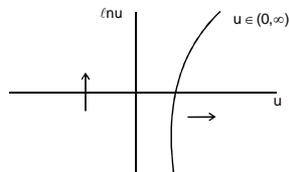


Range = Co-domain onto

Let $\ln t = u$

Let $\ln x = t$; $t \in (1, \infty)$

$\ln t \in (0, \infty)$



one-one

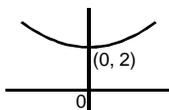
Hence, one-one onto

Q.26

(4)

$$f : \mathbb{R} \rightarrow \mathbb{R}; f(x) = 6^x + 6^{|x|}$$

$$x \geq 0 \quad f(x) = 2 \cdot 6^x$$



$$x < 0 \quad f(x) = 6^x + 6^{-x} = 6^x + \frac{1}{6^x} \geq 2$$

Many-one into

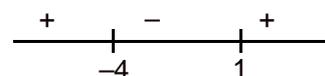
Q.27 (3)

$$f : \mathbb{R} \rightarrow \mathbb{R}; f(x) = \frac{x^2 - 4}{x^2 + 1} \Rightarrow f(-x) = \frac{x^2 - 4}{x^2 + 1} = f(x)$$

$f(x)$ is even that's why many-one.

$$y = \frac{x^2 - 4}{x^2 + 1} \Rightarrow yx^2 + y = x^2 - 4$$

$$\Rightarrow x^2 = \frac{y + 4}{1 - y} \geq 0$$



$$\Rightarrow \frac{y + 4}{y - 1} \leq 0 \quad \therefore y \in [-4, 1)$$

Range \neq Co-Domain \Rightarrow into

Q.28 (3)

We have to find value of $f(A)$.

Simply we can put $A = x$ at $x = \pi/6$

and $A = x$ at $\pi/3$ in $f(x)$

$$A = x \text{ at } x = \pi/6$$

$$f(A) = f(\pi/6) = \cos \pi/6 - \pi/6(1 + \pi/6)$$

$$= \frac{\sqrt{3}}{2} - \frac{\pi}{6} \left(1 + \frac{\pi}{6} \right)$$

$$A = x \text{ at } x = \pi/3$$

$$f(A) = f(\pi/3) = \cos \pi/3 - \pi/3(1 + \pi/3)$$

$$= \frac{1}{2} - \frac{\pi}{3} (1 + \pi/3)$$

Hence $f(A)$ in $\pi/6 \leq x \leq \pi/3$ is

$$\left[\frac{1}{2} - \frac{\pi}{3} (1 + \pi/3), \frac{\sqrt{3}}{2} - \frac{\pi}{6} (1 + \pi/6) \right]$$

\therefore Option (3) is correct Answer.

Q.29

Let $f(n) = 2^n$ for all positive integers n

Now, for $n = 1$

$$f(1) = 2 = 2!$$

\Rightarrow it is true for $n = 1$

Again let $f(k)$ is true

$\Rightarrow f(k) = 2^k$ for some $k \in \mathbb{N}$

Again $f(k + 1) = f(k) \cdot f(1)$ (by definition)

$$= 2^k \cdot 2 \text{ (from induction assumption)}$$

$$= 2^{k+1}$$

Therefore, the result is true for $n = k + 1$. Hence by principle of mathematical induction.

$$f(n) = 2^n \quad \square \quad n \in \mathbb{N}$$

$$\begin{aligned} \text{Now } \sum_{k=1}^n f(a+k) &= \sum_{k=1}^n f(a) f(k) = f(a) \sum_{k=1}^n 2^k \\ &= f(a) \cdot \frac{2(2^n - 1)}{2 - 1} \\ &= 2^a \cdot 2(2^n - 1) = 2^{a+1} (2^n - 1) \end{aligned}$$

$$\text{But } \sum_{k=1}^n f(a+k) = 16(2^n - 1) = 2^4 \cdot 2^n - 1$$

Therefore, $a + 1 = 4 \Rightarrow a = 3$

Q.30 (1)

$$f(x) = \frac{ax - c}{cx - a} = y$$

$$f(y) = \frac{ay - c}{cy - a}$$

$$= \frac{a\left(\frac{ax - c}{cx - a}\right) - c}{c\left(\frac{ax - c}{cx - a}\right) - a}$$

$$= \frac{a^2x - ac - c^2x + ac}{acx - c^2 - acx + a^2} = x$$

Q.31 (4)

$$f(x) = \cos \left[\frac{\pi^2}{2} \right] x + \sin \left[\frac{\pi^2}{2} \right] x; \frac{\pi^2}{2} \cong 4.9$$

$$f(x) = \cos 4x - \sin 5x$$

$$f(0) = 1$$

$$f\left(\frac{\pi}{3}\right) = \cos \frac{4\pi}{3} - \sin \frac{5\pi}{3} = \frac{\sqrt{3}-1}{2} = \frac{1}{\sqrt{3}+1}$$

$$f\left(\frac{\pi}{2}\right) = 0$$

$$f(\pi) = 1$$

Q.32 (4)

$$f(x) = |x - 1|$$

$$f(x^2) = |x^2 - 1| \text{ and } f^2(x) = |x - 1|^2$$

$$\Rightarrow f(x^2) \neq f^2(x)$$

$$f(x + y) = |x + y - 1| \text{ and } f(x) + f(y) = |x - 1| + |y - 1|$$

$$\Rightarrow f(x + y) \neq f(x) + f(y)$$

$$f(|x|) = ||x| - 1| \text{ and } |f(x)| = ||x - 1||$$

Q.33 (2)

(1) $f(x) = \sin^{-1}x + \cos^{-1}x, x \in [-1, 1]$ and $g(x) = \frac{\pi}{2}, x \in \mathbb{R}$

$f(x) = \frac{\pi}{2}, x \in [-1, 1]$ and $g(x) = \frac{\pi}{2}, x \in \mathbb{R}$ Non-identical functions

(2) $f(x) = \tan^{-1}x + \cot^{-1}x$ and $g(x) = \frac{\pi}{2}, x \in \mathbb{R}$

$f(x) = \frac{\pi}{2}, x \in \mathbb{R}$ and $g(x) = \frac{\pi}{2}, x \in \mathbb{R}$ Identical functions

(3) $f(x) = \sec^{-1}x + \operatorname{cosec}^{-1}x$ and $g(x) = \frac{\pi}{2}, x \in \mathbb{R}$

$f(x) = \frac{\pi}{2}, |x| \in [1, \infty)$ and $g(x) = \frac{\pi}{2}, x \in \mathbb{R}$ Non-identical functions

Q.34 (1)

$$\left[\frac{1}{2} + \frac{1999}{2000} \right] = [0.5 + 0.995] = 1 \text{ ; (first thousand terms)}$$

$$\left[\frac{1}{2} + \frac{1000}{2000} \right] = [0.5 + 0.5] = 1$$

will be equal to 0)

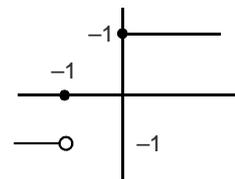
$$(1 + 1 + \dots + 1)_{1000 \text{ times}} = 1000$$

Q.35 (1)

$$f(x) = \operatorname{sgn} [x + 1]$$

$$= 1 \text{ if } [x + 1] > 0$$

$$\Rightarrow [x] > -1 \therefore x \geq 0$$



$$= 0 \text{ if } [x + 1] = 0$$

$$\Rightarrow [x] = -1 \therefore -1 \leq x < 0$$

$$= -1 \text{ if } [x + 1] < 0$$

$$\Rightarrow [x] < -1 \therefore x < -1$$

Q.36 (2)

$$f(x) = 2 \sin^2\theta + 4 \cos(x + \theta) \sin x \cdot \sin\theta + \cos(2x + 2\theta)$$

$$f(x) = \cos 2x$$

$$\Rightarrow f\left(\frac{\pi}{4} - x\right) = \cos\left(2\left(\frac{\pi}{4} - x\right)\right) = \sin 2x \Rightarrow f^2(x) + f^2\left(\frac{\pi}{4} - x\right) = 1$$

$$\left(\frac{\pi}{4} - x\right) = 1$$

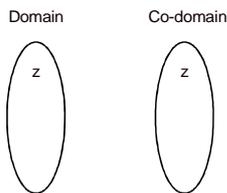
Q.37 (2)

Sol. $z \rightarrow \text{integer}$

$$f(x) = ax^2 + bx + c$$

$$f(0) = c \in I$$

$$f(1) = a + b + c \in I$$



$$f(1) - f(0) = a + b \in I$$

Q.38 (3)

(1) $e^{(\ln x)/2}$ and $\sqrt{x} \Rightarrow D_1 \in (0, \infty); D_2 \in [0, \infty)$

Domain are not same so not identical

(2) $\tan^{-1}(\tan x)$ and $\cot^{-1}(\cot x)$

Domain are not same so non identical

(3) $\cos^2 x + \sin^4 x$ and $\sin^2 x + \cos^4 x$

$\Rightarrow x \in \mathbb{R} \ \& \ x \in \mathbb{R}$ Identical

Q.39 (4)

$$f(x) = \cos(\ln x)$$

$$f(x) \cdot f(y) = \cos(\ln x) \cdot \cos(\ln y)$$

$$f\left(\frac{x}{y}\right) + f(xy) = \cos(\ln x - \ln y) + \cos(\ln x + \ln y)$$

$$\therefore f(x) \cdot f(y) - \frac{1}{2} \left(f\left(\frac{x}{y}\right) + f(xy) \right) = \cos(\ln x) \cos(\ln y) - \frac{1}{2} [2 \cos(\ln x) \cos(\ln y)] = 0$$

$$y - \frac{1}{2} [2 \cos(\ln x) \cos(\ln y)] = 0$$

Q.40 (4)

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

Replace $x + \frac{1}{x} = t$, where $|t| \geq 2$

$$\therefore f(t) = t^2 - 2, |t| \geq 2$$

Q.41 (3)

$$f(1) = 1 = 2 - 1$$

$$f(n+1) = 2f(n) + 1$$

$$\therefore f(2) = 2f(1) + 1 = 2 \cdot 1 + 1 = 3 = 2^2 - 1$$

$$f(3) = 7 = 2^3 - 1$$

$$f(4) = 15 = 2^4 - 1$$

Similarly $f(n) = 2^n - 1$

Q.42 (2)

Method 1 :

(usual but lengthy)

$$x^2 f(x) + f(1-x) = 2x - x^4 \quad \dots(1)$$

replace x by $(1-x)$ in equation (1)

$$(1-x)^2 f(1-x) + f(x) = 2(1-x) - (1-x)^4 \quad \dots(2)$$

eliminate $f(1-x)$ by equation (1) and (2)

we get

$$f(x) = 1 - x^2$$

Method 2 :

Since R.H.S. is polynomial of 4th degree and also by

options consider $f(x) = ax^2 + bx + c$

$$x^2 f(x) + f(1-x) = 2x - x^4$$

$$\Rightarrow x^2(ax^2 + bx + c) + a(1-x)^2 + b(1-x) + c = 2x - x^4$$

by comparing coefficients

$$a = -1$$

$$b = 0$$

$$c = 1$$

$$\therefore f(x) = -x^2 + 1$$

Q.43 (3)

$$y = 2[x] + 3 \text{ and } y = 3[x - 2]$$

$$2[x] + 3 = 3[x] - 6$$

$$\Rightarrow [x] = 9 \Rightarrow x \in [9, 10)$$

$$\therefore y = 21$$

$$\therefore [x + y] = 30$$

Q.44 (4)

$$y = f(x)$$

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \quad (x \neq 0)$$

$$f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^2 - 2 \Rightarrow f(x) = x^2 - 2$$

Q.45 (1)

$f(x)$ where $x \in [0, 1]$

$f(\ln(1-x^2))$ where $0 \leq \ln(1-x^2) \leq 1$

$$1 \leq 1 - x^2 \leq e$$

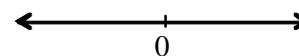
$$0 \leq -x^2 \leq e - 1$$

$$0 \geq x^2 \geq 1 - e$$

(i) $x = 0$

(ii) $x \in (-\infty, -\sqrt{1-e}] \cup [\sqrt{1-e}, \infty)$

Imaginary numbers as $1 < e$



$$\Rightarrow x \in \{0\}$$

Q.46 (3)

Domain of $f(g(x))$

Range of $g(x) \equiv$ Domain of $f(x)$

$$\Rightarrow -5 \leq |2x + 5| \leq 7$$

$$\Rightarrow 0 \leq |2x + 5| \leq 7$$

$$\Rightarrow -7 \leq 2x + 5 \leq 7$$

$$\Rightarrow -12 \leq 2x \leq 2$$

$$\Rightarrow -6 \leq x \leq 1$$

Q.47 (1)

$$f(x) = \frac{ax + b}{cx + d}$$

$$f \circ f(x) = \frac{a\left(\frac{ax+b}{cx+d}\right) + b}{c\left(\frac{ax+b}{cx+d}\right) + d}$$

$$f \circ f(x) = \frac{a^2x + ab + bcx + bd}{acx + bc + cdx + d^2}$$

$$f \circ f(x) = \frac{(a^2 + bc)x + (ab + bd)}{(ac + cd)x + (bc + d^2)} = x$$

on comparing coefficient of both side $(a^2 + bc)x + (ab + bd) = (ac + cd)x^2 + (bc + d^2)x$

$a^2 + bc = bc + d^2 \Rightarrow a = d$ or $a = -d$
 and $ab + bd = 0 \Rightarrow b = 0$ or $a = -d$
 and $ac + cd = 0 \Rightarrow c = 0$ or $a = -d$
 which can be simultaneously true for $a = -d$

Q.48 (1)

$$f(x) = \frac{ax+b}{cx+d}$$

$$f \circ f(x) = \frac{\left\{\frac{ax+b}{cx+d}\right\} + b}{c\left\{\frac{ax+b}{cx+d}\right\} + d} \Rightarrow \frac{a^2x + ab + bcx + bd}{acx + bc + cdx + d^2}$$

$$\Rightarrow (ac + dc)x^2 + (bc + d^2 - bc - a^2)x$$

$$-ab - bd = 0, \forall x \in R$$

$$\Rightarrow (a+d)c = 0, d^2 - a^2 = 0 \text{ and } (a+d)b = 0$$

$$\Rightarrow a + d = 0$$

Q.49 (1)

Q.50 (1)

$$f(g(x_1)) = f(g(x_2))$$

$$\Rightarrow g(x_1) = g(x_2)$$

as f is one - one function

$$\Rightarrow x_1 = x_2$$

as g is one - one function

$$\text{hence } f(g(x_1)) = f(g(x_2))$$

$$\Rightarrow x_1 = x_2$$

$$\Rightarrow f(g(x)) \text{ is one - one function}$$

Q.51 (3)

$$f(g(x)) = \cot^{-1}(2x - x^2)$$

$$-\infty < 2x - x^2 \leq 1$$

But domain of $f(x)$ is R^+

$$0 < 2x - x^2 \leq 1$$

$$\Rightarrow \frac{\pi}{2} > \cot^{-1}(2x - x^2) \geq \frac{\pi}{4}$$

$$\text{Range} \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$$

Q.52 (2)

$$g(x) = 1 + x - [x] f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

$$\Rightarrow f(x) = \text{sgn}x$$

$$f[g(x)] = f(1 + x - [x]) = \text{sgn}(1 + x - [x])$$

$$= \text{sgn}(1 + \{x\}) = 1$$

↓
positive

Q.53 (1)

$$f: [0, 1] \rightarrow [1, 2] \quad g: [1, 2] \rightarrow [0, 1]$$

$$f(x) = 1 + x$$

$$g(x) = 2 - x$$

$$\text{gof}(x) = g[f(x)] = g(1 + x) = 2 - (1 + x) = 1 - x$$

Linear polynomial that's why one-one onto.

Q.54 (4)

$$f(x) = \frac{x+|x|}{2} = \begin{cases} x & ; x \geq 0 \\ 0 & ; x < 0 \end{cases} \quad g(x) = \begin{cases} x^2, & x \geq 0 \\ x & x < 0 \end{cases}$$

$$\text{gof} = g[f(x)] = \begin{cases} g(x) & ; x \geq 0 \\ x^2 & ; x \geq 0 \\ g(0) & ; x < 0 \\ 0 & ; x < 0 \end{cases}$$

$$\Rightarrow \text{fog}(x) = f[g(x)] = \begin{cases} f(x^2) & ; x \geq 0 \\ x^2 & ; x \geq 0 \\ f(x) & ; x < 0 \\ 0 & ; x < 0 \end{cases}$$

$$\Rightarrow \text{fog}(x) = \text{gof}(x).$$

Q.55 (2)

$$f: [-20, 20] \rightarrow R$$

$$f(x) = \left[\frac{x^2}{a}\right] \sin x + \cos x$$

$$\text{since } f(-x) = f(x)$$

$$\left[\frac{x^2}{a}\right] (-\sin x) + \cos x = \left[\frac{x^2}{a}\right] \sin x + \cos x$$

$$\text{or } \left[\frac{x^2}{a}\right] \sin x = 0 \quad \left[\frac{x^2}{a}\right] \sin x = 0$$

$$\sin x = 0 \text{ But } \left[\frac{x^2}{a}\right] \neq 0 \text{ or } \left[\frac{x^2}{a}\right] = 0$$

$$\text{If } \left[\frac{x^2}{a}\right] = 0. \text{ It means } \frac{x^2}{a} \text{ must be +ve fractional}$$

number less than unity

i.e. $a \rightarrow$ value greater than x

If $\left[\frac{x^2}{a}\right] \neq 0$. It means, value of $\frac{x^2}{a}$ is 1

$\therefore a \in (400, \infty)$

\therefore Option (2) is correct Answer.

Q.56 (2)

$$f(x) = \log \left(\frac{1 + \sin x}{1 - \sin x} \right)$$

$$f(-x) = \log \left(\frac{1 - \sin x}{1 + \sin x} \right) = -\log \left(\frac{1 + \sin x}{1 - \sin x} \right) = -f(x)$$

odd function

Q.57 (2)

Let $f : R \rightarrow R$ be a function defined by

$$f(x) = \frac{x - m}{x - n}$$

For any $(x, y) \in R$

Let $f(x) = f(y)$

$$\Rightarrow \frac{x - m}{x - n} = \frac{y - m}{y - n} \Rightarrow x = y$$

$\therefore f$ is one - one Let $\alpha \in R$ such that $f(x) = \alpha$

$$\Rightarrow \alpha = \frac{x - m}{x - n} \Rightarrow (x - n)\alpha = x - m$$

$$\Rightarrow x\alpha - n\alpha = x - m$$

$$\Rightarrow x\alpha - x = n\alpha - m$$

$$\Rightarrow x(\alpha - 1) = n\alpha - m$$

$$\Rightarrow x = \frac{n\alpha - m}{\alpha - 1}, \text{ for } \alpha = 1, x \notin R$$

So, f is not onto.

Q.58 (2)

$$f(x) = [x] + \frac{1}{2}, x \notin I$$

$$f(-x) = [-x] + \frac{1}{2} = -[x] - 1 + \frac{1}{2} = -\left([x] + \frac{1}{2}\right) = -f(x)$$

odd function

Q.59 (4)

$$f(x) = \frac{xf(x^2)}{2 + \tan^2 x \cdot f(x^2)} \text{ given that } f(-x) = f(x) \dots(1)$$

$$f(-x) = \frac{-xf(x^2)}{2 + \tan^2 x + f(x^2)} \Rightarrow f(-x) = -f(x) \dots(2)$$

When both conditions are there only one possibility is there when $f(x) = 0 \Rightarrow f(10) = 0$

Q.60 (3)

$$g(x) = x^3 + \tan x + \frac{x^2 + 1}{P}$$

$$g(-x) = (-x)^3 + \tan(-x) + \frac{(-x)^2 + 1}{P}$$

$$g(-x) = -x^3 - \tan x + \frac{x^2 + 1}{P}$$

$g(x) + g(-x) = 0$ because $g(x)$ is a odd function

$$\therefore \left[x^3 + \tan x + \frac{x^2 + 1}{P} \right] + \left[-x^3 - \tan x + \frac{x^2 + 1}{P} \right] = 0$$

$$\Rightarrow \frac{2(x^2 + 1)}{P} = 0 \Rightarrow 0 \leq \frac{x^2 + 1}{P} < 1$$

because $x \in [-2, 2]$

$$\Rightarrow 0 \leq \frac{5}{P} < 1 \Rightarrow P > 5.$$

Q.61 (3)

$$f(x) = \sin \sqrt{[a] \cdot x}$$

$\sin \sqrt{[a] \cdot x}$ is Periodic with Period $\frac{2\pi}{\sqrt{[a]}}$

$$\therefore \frac{2\pi}{\sqrt{[a]}} = \pi \Rightarrow \sqrt{[a]} = 2 \Rightarrow [a] = 4 \Rightarrow a = 5$$

$a \in [4, 5]$

\therefore option (3) is correct Answer.

Q.62 (3)

$$f(x) = \sec(\sin x)$$

Since $\sin x$ is a periodic function with fundamental period 2π . $f(x)$ has a period 2π

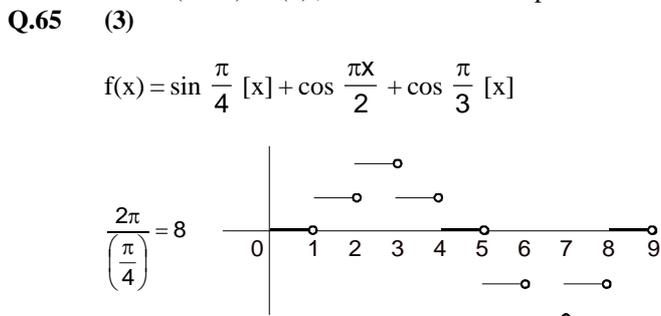
for fundamental period

$$f(x + \pi) = \sec(\sin(\pi + x)) = \sec(-\sin x) = \sec(\sin x) = f(x)$$

$$f\left(x + \frac{\pi}{2}\right) \neq f(x) \text{ hence fundamental period is } \pi$$

Q.63 (4)
 $f(x) = \sin(\sqrt{[a]} x)$
 Period = $\frac{2\pi}{\sqrt{[a]}} = \pi$
 $[a] = 4 \Rightarrow a \in [4, 5)$

Q.64 (1)
 $f(x) = x + a - [x + b] + \sin \pi x + \cos 2\pi x + \sin(3\pi x) + \cos(4\pi x) + \dots + \sin(2n-1)\pi + \cos(2n\pi)$
 $f(x) = \{x + b\} + a - b + \sin(\pi x) + \cos(2\pi x) + \sin(3\pi x) + \cos(4\pi x) + \dots + \sin(2n-1)\pi + \cos(2n\pi)$
 Period of $f(x) = \text{L.C.M}$
 $(1, 2, \frac{2}{3}, \frac{2}{4}, \dots, \frac{2}{2n-1}, \frac{2}{2n}) = 2$
 \therefore period of $f(x) = 2$
 since $f(1+x) \neq f(x)$, hence fundamental period is 2



If $0 \leq x < 1$ then $y = 0$
 If $1 \leq x < 2$ then $y = \frac{1}{\sqrt{2}}$
 If $2 \leq x < 3$ then $y = 1$
 If $3 \leq x < 4$ then $y = \frac{1}{\sqrt{2}}$
 If $4 \leq x < 5$ then $y = 0$
 Period $8/4/6 \Rightarrow \text{Lcm} = 24$

Q.66 (3)
 $f(x) = x(2-x); f(x+2) = f(x) \Rightarrow \text{period} = 2$

Q.67 (1)
 $f(x) = x + (-1)^{x-1}$
 $f(x) = \begin{cases} x+1 & ; x \in \text{odd natural number} \\ x-1 & ; x \in \text{even natural number} \end{cases}$
 $y = f(x)$
 $x = \begin{cases} y-1 & ; y \in \text{odd natural number} \\ y+1 & ; y \in \text{even natural number} \end{cases}$
 $f^{-1}(x) = \begin{cases} x-1 & ; x \in \text{odd natural number} \\ x+1 & ; x \in \text{even natural number} \end{cases}$
 $\therefore f(x)$ is inverse to itself

Q.68 (1)
 We have to check for every options as :
 For (1): $y = \frac{1-x}{1+x}$
 $\Rightarrow 1-x = y+yx$
 $\Rightarrow 1-y = x(1+y)$

$\Rightarrow x = \frac{1-y}{1+y}$
 which shows functions is inverse of itself.
 \therefore Option (1) is correct Answer.
 For (2): $y = 5^{\log x} \Rightarrow \log y = \log x \log 5$
 $\Rightarrow \log x = \log_5 y$
 $\Rightarrow x = e^{\log_5 y}$
 which is not inverse of itself.

Similarly for C; $y = 2^{x(x-1)}$
 $\log y = x(x-1) \log 2$
 $\log_2 y = x(x-1)$
 which certainly not show inverse of itself
 \therefore Only option (1) is correct Answer.

Q.69 (1)
Q.70 (1)
 $\frac{y}{1} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

By compendo and dividendo
 $\frac{1+y}{1-y} = \frac{2e^x}{2} \Rightarrow 2x = \ln \left(\frac{1+y}{1-y} \right)$
 $\Rightarrow x = \frac{1}{2} \ln \left(\frac{1+y}{1-y} \right)$
 $\therefore f^{-1}(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$

Q.71 (3)
 $f(x) = x - \left[\frac{x}{2} \right]$
 for the given domain (2, 4)
 $\left[\frac{x}{2} \right]$ will be equal to 1;
 so $y = f(x) = x - 1 \Rightarrow x = y + 1 \Rightarrow f^{-1}(x) = x + 1$

Q.72 (3)
 Greatest integer function defined as $[x] \leq x$ for all $x \in \mathbb{R}$. then
 $\left[\frac{1}{2} + \frac{1}{1000} \right] + \left[\frac{1}{2} + \frac{2}{1000} \right] + \dots$

$$+ \left[\frac{1}{2} + \frac{999}{1000} \right] \text{ (999 term)}$$

$$\text{i.e. } \left[\frac{1}{2} + \frac{1}{1000} \right] + \left[\frac{1}{2} + \frac{2}{1000} \right] + \dots$$

$$+ \left[\frac{1}{2} + \frac{499}{1000} \right] + \left[\frac{1}{2} + \frac{500}{1000} \right] + \dots$$

$$+ \left[\frac{1}{2} + \frac{999}{1000} \right]$$

First 499 terms, each will be zero and remaining 500 terms will be as follows

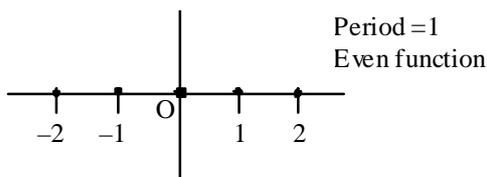
$$\left[\frac{1}{2} + \frac{500}{1000} \right] + \left[\frac{1}{2} + \frac{501}{1000} \right] + \dots + \left[\frac{1}{2} + \frac{999}{1000} \right]$$

$$= 1 + 1 + 1 \dots + 1 \text{ (500 terms)} = 500$$

∴ Option (3) is correct Answer.

Q.73 (2)

$$f(x) = \frac{0}{\{x\}} = \begin{cases} \text{not defined} & x \in I \\ 0 & x \notin I \end{cases}$$



$$g(x) = \underbrace{\text{sgn} \left(\frac{\text{signum} \left\{ \frac{x}{\sqrt{\{x\}}} \right\}}{\sqrt{\{x\}}} \right)}_1 - 1$$

= 0 → same domain & range

EXERCISE-III

Q.1 (1.75)
 $2f(x) - 3f(1/x) = x^2 \dots$ (i)
 replacing x by 1/x

$$2f\left(\frac{1}{x}\right) - 3f(x) = 1/x^2 \dots$$
 (ii)

solving (i) & (ii) we get
 $-5f(x) = 2x^2 + 3/x^2$
 $f(x) = -1/5 (2x^2 + 3/x^2)$
 ∴ $f(2) = -1/5 (8 + 3/4) = -7/4$

Q.2 (1000)
 $f(x) + f(x+4) = f(x+2) + f(x+6) \dots$ (i)

Replacing x by x+2 in equation (i), we get
 $f(x+2) + f(x+6) = f(x+4) + f(x+8) \dots$ (ii)
 Adding (i) & (ii) $f(x) = f(x+8)$
 $f(5) = f(13) = 10$
 $f(13) = f(21) = 10$

 $f(805) = 10$
 ∴ $\sum_{r=1}^{100} f(5+8r) = f(13) + f(21) + \dots + f(805)$
 $= 10 + 10 + \dots$ 100 times. $= 10 \times 100 = 1000$.

Q.3 (0001)

$$y = 1 + \alpha x \Rightarrow x = \frac{y-1}{\alpha}$$

$$f^{-1}(x) = \frac{x-1}{\alpha} = f(x) = 1 + \alpha x$$

$$\Rightarrow \frac{x-1}{\alpha} = 1 + \alpha x$$

$$\Rightarrow x - 1 = \alpha + \alpha^2 x$$

Equating the coefficient of x

$$\alpha^2 = 1 \text{ \& } \alpha = -1$$

$$\alpha = \pm 1$$

$$\alpha = -1.$$

Q.4 (0005)

$f\phi(x) = -f(x)$ and $f\phi(x) = g(x)$
 $\Rightarrow f'(x) \cdot f'(x) + f(x) \cdot f'(x) = 0$
 $\Rightarrow f(x)^2 + (f'(x))^2 = c$
 $\Rightarrow (f(x))^2 + (g(x))^2 = c$
 $\Rightarrow F(x) = c$
 $\Rightarrow F(10) = 5$.

Q.5 (0000)

$f(x) = 0 \forall x \in \mathbb{R}$
 $\Rightarrow f(3) - f(2) = 0$.

Q.6 (0013)

$x = 0 \Rightarrow f(2) = 2f(0) - f(1) = 2 \times 2 - 3 = 1$
 $x = 1 \Rightarrow f(3) = 6 - 1 = 5$
 $x = 2 \Rightarrow f(4) = 2f(2) - f(3) = 2 \times 1 - 5 = -3$
 $x = 3 \Rightarrow f(5) = 2f(3) - f(4) = 2(5) - (-3) = 13$.

Q.7 (0.875)

$f(2a-x) = f(x) \Rightarrow f(2a+x) = -f(x) \because f$ is odd $\Rightarrow f(x+4a) = f(x)$
 $\Rightarrow f$ is periodic with period 4a $\Rightarrow f(1+4r) = f(1)$

$$\text{Now } \sum_{r=0}^{\infty} [f(1)]^r = 8$$

$$\Rightarrow \frac{1}{1-f(1)} = 8 \Rightarrow f(1) = 7/8$$

Q.8 (0002)

Putting $x = f(y) = 0$ then $f(0) = f(0) + 0 + f(0) - 1$
 $\Rightarrow f(0) = 1$

Putting $x = f(y)$

We get $f(0) = f(x) + x^2 + f(x) - 1 \Rightarrow f(x) = 1 - x^2/2$

$$f(16) = 1 - \frac{256}{2} = 1 - 128 = -127$$

$$|f(16)| = 127.$$

Q.9 (0004)

$$3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$$

$$\text{For } x = 7, 3f(7) + 2f(11) = 70 + 30 = 100$$

$$\text{For } x = 11, 3f(11) + 2f(7) = 140.$$

$$\frac{f(7)}{-20} = \frac{f(11)}{-220} = \frac{-1}{9-4} \text{ or } f(7) = 4$$

Q.10 (0004)

$$[x]^2 = x + 2\{x\}$$

$$\text{or } [x]^2 = [x] + 3\{x\}$$

$$\text{or } \{x\} = \frac{[x]^2 - [x]}{3}$$

$$\text{or } 0 \leq \frac{[x]^2 - [x]}{3} < 1$$

$$\text{or } 0 \leq [x]^2 - [x] < 3$$

$$\text{or } [x] \in \left(\frac{1-\sqrt{3}}{2}, 0\right] \cup \left[1, \frac{1+\sqrt{3}}{2}\right)$$

$$\text{or } [x] = -1, 0, 1, 2$$

$$\text{or } \{x\} = \frac{2}{3}, 0, 0, \frac{2}{3} \text{ (respectively)}$$

$$\text{or } x = -\frac{1}{3}, 0, 1, \frac{8}{3} \text{ Ans. (0005)}$$

PREVIOUS YEAR'S

MHT CET

Q.1 (2)

Q.2 (1)

Q.3 (2)

Q.4 (4)

Q.5 (1)

Q.6 (4)

Q.7 (4)

Q.8 (4)

$$\text{Let } y = \frac{2x-1}{x+5}$$

$$\Rightarrow x = \frac{5y+1}{2-y}$$

$$\therefore f^{-1}(x) = \frac{5x-1}{2-x}, x \neq 2$$

Q.9 (3)

$$f(x) = \frac{\alpha x}{x+1}, x \neq -1$$

$$\therefore f(f(x)) = f\left(\frac{\alpha x}{x+1}\right) = \frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\left(\frac{\alpha x}{x+1}\right)+1} = \frac{\alpha^2 x}{\alpha x + x + 1}$$

$$\Rightarrow \frac{\alpha^2 x}{\alpha x + x + 1} = x \quad [\text{given}]$$

$$\Rightarrow x[\alpha^2 - \alpha x - x - 1] = 0$$

$$\Rightarrow x(\alpha + 1)(\alpha - 1 - x) = 0$$

$$\Rightarrow x = 0 \text{ or } \alpha + 1 = 0$$

$$\text{or } \alpha = 1 + x \quad [\because \alpha - 1 - x \neq 0]$$

$$\Rightarrow \alpha = -1 \text{ or } \alpha = 1 + x$$

$$\therefore \alpha = -1$$

[$\because a = 1 + x$ gives value for particular x , not for all x]

Q.10 (1)

$$\text{Given } f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x$$

$$\cos\left(x + \frac{\pi}{3}\right)$$

$$= \sin^2 x + \left[\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3}\right]^3$$

$$+ \cos x \left[\cos x \cos \frac{\pi}{3} - \sin x \sin \frac{\pi}{3}\right]$$

$$= \sin^2 x + \left[\frac{\sin x}{2} + \cos x \cdot \frac{\sqrt{3}}{2}\right]^2$$

$$+ \cos x \left[\frac{\cos x}{2} - \sin x \cdot \frac{\sqrt{3}}{2}\right]$$

$$= \sin^2 x + \frac{\sin^2 x}{4} + \frac{3\cos^2 x}{4} + \sin x \cos x \cdot \frac{\sqrt{3}}{2}$$

$$+ \frac{\cos^2 x}{2} - \sin x \cos x \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{5\sin^2 x}{4} + \frac{5\cos^2 x}{4} = \frac{5}{4}$$

$$\therefore \text{gof}(x) = g[f(x)] = g\left(\frac{5}{4}\right) = 1$$

Q.11 (3)

$$\text{Let } y = \frac{x}{1+x^2} \Rightarrow x^2y - x + y = 0$$

For x to be real, $1 - 4y^2 \geq 0$ {Discriminant = $1 - 4y^2$ }

$$\begin{aligned} \Rightarrow 4y^2 - 1 &\leq 0 \\ \Rightarrow (2y - 1)(2y + 1) &\leq 0 \\ \Rightarrow -\frac{1}{2} &\leq y \leq \frac{1}{2} \end{aligned}$$

$$\therefore y = f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

Q.12 (4)

Here, ${}^7P_{x-3}$ is defined if $7 - x \geq 0$, $x - 3 \geq 0$ and $7 - x \geq -3$

$$\Rightarrow 3 \leq x \leq 7 \text{ and } x \in \mathbb{I}$$

$$\begin{aligned} \therefore x &= 3, 4, 5 \\ \therefore f(3) &= {}^{7-3}P_{3-3} = {}^4P_0 = 1 \\ \therefore f(4) &= {}^{7-4}P_{4-3} = {}^3P_1 = 3 \\ \therefore f(5) &= {}^{7-5}P_{5-3} = {}^2P_2 = 2 \end{aligned}$$

Hence, range of $f(x) = \{1, 2, 3\}$

Q.13 (3)

Here, $(f \circ g)(x) = f[g(x)] = f(|3x + 4|)$

Since, the domain of f is $[-3, 5]$.

$$\Rightarrow -3 \leq |3x + 4| \leq 5 \Rightarrow |3x + 4| \leq 5$$

$$\Rightarrow -5 \leq 3x + 4 \leq 5 \Rightarrow -9 \leq 3x \leq 1 \Rightarrow -3 \leq x \leq \frac{1}{3}$$

$$\therefore \text{Domain of } (f \circ g)(x) \text{ is } \left[-3, \frac{1}{3}\right]$$

Q.14 (2)

Here $f: [0, \infty) \rightarrow [0, \infty)$ i.e., domain is $[0, \infty)$ and codomain is $[0, \infty)$.

$$\text{For one-one, } f(x) = \frac{x}{1+x}$$

$$\Rightarrow f'(x) = \frac{1}{(1+x)^2} > 0, \forall x \in [0, \infty)$$

$\therefore f(x)$ is increasing in its domain. Thus, $f(x)$ is one-one in its domain.

For onto, we find range

$$f(x) = \frac{x}{1+x} \text{ i.e., } y = \frac{x}{1+x}$$

$$\Rightarrow y + yx = x \text{ } \& \text{ } x = \frac{y}{1-y}$$

$$\Rightarrow \frac{y}{1-y} \geq 0 \text{ as } x \geq 0$$

$$\therefore 0 \leq y < 1$$

i.e., Range = Codomain

$\therefore f(x)$ is one-one but not onto.

Q.15 (3)

$$\text{Given, } f(x) = \frac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin 3x \sin 4x}$$

$$\begin{aligned} &= \frac{2 \sin 8x \cos x - 2 \sin 6x \cos 3x}{2 \cos 2x \cos x - 2 \sin 3x \sin 4x} \\ &= \frac{(\sin 9x + \sin 7x) - (\sin 9x + \sin 3x)}{(\cos 3x + \cos x) + (\cos 7x - \cos x)} \\ &= \frac{\sin 7x - \sin 3x}{\cos 7x + \cos 3x} = \frac{2 \cos 5x \sin 2x}{2 \cos 2x \cos 5x} = \tan 2x \end{aligned}$$

$$\therefore \text{Period of } f(x) = \frac{\pi}{2}$$

Q.16 (1)

$$\text{Let } y = x + \frac{1}{x} \Rightarrow y = \frac{x^2 + 1}{x}$$

$$\begin{aligned} \Rightarrow xy &= x^2 + 1 \\ \Rightarrow x^2 - xy + 1 &= 0 \end{aligned}$$

$$\Rightarrow x = \frac{y \pm \sqrt{y^2 - 4}}{2} \Rightarrow f^{-1}(y) = \frac{y \pm \sqrt{y^2 - 4}}{2}$$

$$\therefore f^{-1}(x) = \frac{x \pm \sqrt{x^2 - 4}}{2}$$

since, the range of the inverse function is $[1, \infty)$, then we take

$$f^{-1}(x) = \frac{x + \sqrt{x^2 - 4}}{2}$$

$$\text{If we consider, } f^{-1}(x) = \frac{x + \sqrt{x^2 - 4}}{2}, \text{ then } f^{-1}(x) > 1.$$

This is possible only, when $(x-2)^2 > x^2 - 4$

$$\begin{aligned} \Rightarrow x^2 + 4 - 4x &> x^2 - 4 \text{ } \& \text{ } 8 > 4x \\ \Rightarrow x &< 2 \text{ when } x > 2 \end{aligned}$$

JEE-MAIN

Q.1 (2)

Number of possible values of $a = 60$, for $b = pq$

If $p = 3$, $q = 3, 5, 7, 11, 13, 17, 19$

If $p = 5$, $q = 5, 7, 11$

If $p = 7$, $q = 7$

Total cases = $60 \times 11 = 660$

Q.2 (4)

For R to be reflexive $\Rightarrow x R x$

$$\Rightarrow 3x + \alpha x = 7x \Rightarrow (3 + \alpha)x = 7x$$

$$\Rightarrow 3 + \alpha = 7\lambda \Rightarrow \alpha = 7\lambda - 3 \text{ } \& \text{ } 7\lambda + 4, k, \lambda, N \in \mathbb{I}$$

\therefore When α divided by 7, remainder is 4.

R to be symmetric $xRy \Rightarrow yRx$

$$3x + \alpha y = 7N_1, 3y + \alpha x = 7N_2$$

$$\Rightarrow (3 + \alpha)(x + y) = 7(N_1 + N_2) = 7N_3$$

Which holds when $3 + \alpha$ is multiple of 7

$\therefore \alpha = 7N + 4$ (as did earlier)
 R to be transitive
 $xRy \& yRz \Rightarrow xRz$
 $3x + \alpha y = 7N_1 \& 3y + \alpha z = 7N_2$ and $3x + \alpha z = 7N_3$
 $\therefore 3x + 7N_2 - 3y = 7N_3$
 $\therefore 7N_1 - \alpha y + 7N_2 - 3y = 7N_3$
 $\therefore 7(N_1 + N_2) - (3 + \alpha)y = 7N_3$
 $\therefore (3 + \alpha)y = 7N$

Which is true again when $3 + \alpha$ divisible by 7, i.e. when α divided by 7, remainder is 4.

Q.3

(4)
 $aR_1b \Leftrightarrow ab \geq 0, a, b \in R$

Reflexive :-

$a.a = a^2 \geq 0 \Rightarrow aR_1a \Rightarrow R_1$ is reflexive

Symmetric :-

Let $a R_1 b \Rightarrow ab \geq 0$
 $\Rightarrow ba \geq 0$
 $\Rightarrow b R_1 a$

$\therefore R_1$ is symmetric

Transitive:-

$\therefore (3,0) \in R_1$ and $(0,-2) \in R_1$

But $(3,-2) \notin R_1$ ($\because 3(-2) = -6 < 0$)

$\therefore R_1$ is not transitive

$\Rightarrow R_1$ is not an equivalence relation

Now $aR_2b \Leftrightarrow a - b \geq 0$

Reflexive: $a - a = 0 \Rightarrow aR_2a$

symmetric

$\therefore (3, 2) \in R_2$

But $(2,3) \notin R_2$ ($\because 2 - 3 < 0$)

$\Rightarrow R_2$ is not symmetric $\Rightarrow R_2$ is not an equivalence relation

Q.4

(B)

$R_1 = \{(a,b) \in N \times N : |a - b| \leq 13\}$

Let $(a,a) \in R_1$ then

$|a - a| = 0 \leq 13 \Rightarrow R_1$ is Reflexive

Let $(a,b) \in R_1 \Rightarrow |a - b| \leq 13$

$\Rightarrow |b - a| \leq 13$

$\Rightarrow (b, a) \in R_1$

$\Rightarrow R_1$ is symmetric

$(1, 7) \in R_1, (7, 17) \in R_1$ but $(1, 17) \notin R_1$

$\Rightarrow R_1$ is not transitive

Hence, R_1 is not an equivalence.

$R_2 = \{(a, b) \in N \times N : |a - b| \neq 13\}$

Let $(a, a) \in R_2$ then

$|a - a| = 0 \neq 13 \Rightarrow R_2$ is Reflexive

Let $(a,b) \in R_2$, then

$|a - b| \neq 13 \Rightarrow |b - a| \neq 13$

$\Rightarrow R_2$ is symmetric

$\rightarrow |14 - 10| \neq 13$ and $|10 - 1| \neq 13$ but $|14 - 1| = 13$

$\therefore R_2$ is not transitive

Hence, R_2 is not an equivalence.

Hence option (B) is correct

Q.5

(4)

$A = \{1, 2, 3\}$

$R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3,$

$3)\}$

Hence it is

Transitive

Reflexive

as well as

symmetric

\therefore It is an equivalence relation

(8)

Here, $p, p^n \in \{1, 2, \dots, 50\}$

Now p can take values

2, 3, 5, 7, 11, 13, 17, 23, 29, 31, 37, 41, 43 and 47

\therefore we can calculate no. of elements in R_1 , as

$(2, 2^0), (2, 2^1), \dots, (2, 2^5)$

$(3, 3^0), \dots, (3, 3^3)$

$(5, 5^0), \dots, (5, 5^2)$

$(7, 7^0), \dots, (7, 7^2)$

$(11, 11^0), \dots, (11, 11^1)$

And rest for all other two elements each

$n(R_1) = 6 + 4 + 3 + 3 + (2 \times 10) = 36$

Similarly for R_2

$(2, 2^0), (2, 2^1), (3, 3^0), (3, 3^1), \dots, (47, 47^0), (47, 47^1)$

$\therefore n(R_2) = 2 \times 14 = 28$

$\therefore n(R_1) - n(R_2) = 36 - 28 = 8$

Q.7

[37]

$(1,1), (1,4), (4,1), (2,4), (4,2), (3,4), (4,3), (4,4)$ - all have one choice for image.

$(2,1), (1,2), (2,2)$ - all have three choices for image

$(3,2), (2,3), (3,1), (1,3), (3,3)$ - all have two choices for image.

So the total functions = $3 \times 3 \times 2 \times 2 \times 2 = 72$

Case 1 : None of the pre-images have 3 as image

Total functions = $2 \times 2 \times 1 \times 1 \times 1 = 4$

Case 2 : None of the pre-images have 2 as image

Total functions = $2 \times 2 \times 2 \times 2 \times 2 = 32$

Case 3 : None of the pre-images have either 3 or 2 as image

Total functions = $1 \times 1 \times 1 \times 1 \times 1 = 1$

\therefore Total onto functions = $72 - 4 - 32 + 1 = 37$

[18]

$f(g(x)) = 8x^2 - 2x, g(f(x)) = 4x^2 + 6x + 1$

$g(x) = 2x - 1$ and $f(x) = 2x^2 + 3x + 1$

$g(2) = 2(2) - 1 = 3$

$f(2) = 2(2)^2 + 3(2) + 1 = 15$

$f(2) + g(2) = 15 + 3 = 18$

Q.9

[31]

$2f(a) - f(b) + 3f(c) + f(d) = 0$

we have to make cases

like $f(a)$ can take value 0, 1, 2, 3, 4, 5

$f(c)$ can take value 0, 1, 2, 3

$\therefore 2f(a) + 3f(c) + f(d) = f(b)$

So from fundamental principle of counting we get number of one-one functions as 31.

Q.10

(4)

$f(g(x)) = \frac{x^2}{x^2 - 1} - 1 = \frac{1}{x^2 - 1}$

since fog is even function

\Rightarrow many - one function

$$\begin{aligned} \text{let } y &= \frac{1}{x^2-1} \\ \Rightarrow x^2-1 &= \frac{1}{y} \Rightarrow x^2 = \frac{1+y}{y} \\ \Rightarrow \frac{1+y}{y} &\geq 0 \\ \Rightarrow y &\in (-\infty, -1] \cup (0, \infty) \\ \therefore \text{Range} &\neq \text{co-domain} \\ \Rightarrow \text{Into - function} \end{aligned}$$

Q.11

$$\begin{aligned} f(x)+f(1-x) &= \frac{2e^{2x}}{e^{2x}+e} + \frac{2e^{2-2x}}{e^{2-2x}+e} = 2 \left[\frac{e^{2x}}{e^{2x}+e} + \frac{e^2}{e^2+e^{2x+1}} \right] \\ &= 2 \left[\frac{e^{2x-1}}{e^{2x-1}+1} + \frac{1^2}{1+e^{2x-1}} \right] = 2 \\ f\left(\frac{1}{100}\right) &+ f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + \dots + f\left(\frac{99}{100}\right) \\ \left\{ f\left(\frac{1}{100}\right) + f\left(\frac{99}{100}\right) \right\} &+ \left\{ f\left(\frac{2}{100}\right) + f\left(\frac{98}{100}\right) \right\} \dots + \\ &f\left\{\frac{49}{100}\right\} + f\left\{\frac{51}{100}\right\} + f\left(\frac{1}{2}\right) \\ &= (2+2+2+\dots 49 \text{ times}) + \frac{2e}{e+e} \\ &= 98 + 1 = 99 \end{aligned}$$

Q.12

$$\begin{aligned} f^{-1}(n) &= \begin{cases} \frac{n}{2}; & n = 2, 4, 6, 8, 10 \\ \frac{n+11}{2}; & n = 1, 3, 5, 7, 9 \end{cases} \\ f(g(n)) &= \begin{cases} n+1; & n \in \text{odd} \\ n-1; & n \in \text{even} \end{cases} \\ \Rightarrow g(n) &= \begin{cases} f^{-1}(n+1); & n \in \text{odd} \\ f^{-1}(n-1); & n \in \text{even} \end{cases} \\ \therefore g(n) &= \begin{cases} \frac{n+1}{2}; & n \in \text{odd} \\ \frac{n+10}{2}; & n \in \text{even} \end{cases} \\ g(10) \cdot [g(1) + g(2) + g(3) + g(4) + g(5)] & \\ = 10 \cdot [1 + 6 + 2 + 7 + 3] &= 190 \end{aligned}$$

Q.13

$$[2] \quad f(x) = \left(2 \left(1 - \frac{x^{25}}{2} \right) (2 + x^{25}) \right)^{\frac{1}{50}}$$

$$f(1) = \left(2 \left(\frac{1}{2} \right) (3) \right)^{\frac{1}{50}} = 3^{\frac{1}{50}}$$

$$f(f(1)) = \left\{ 2 \left(1 - \frac{\sqrt{3}}{2} \right) (2 + \sqrt{3}) \right\}^{\frac{1}{50}} = 1$$

$$\therefore g(1) = f(1) + 1 = 3^{\frac{1}{50}} + 1$$

$$[g(1)] = 2$$

(2)

$$f(x) = \frac{x-1}{x+1}$$

$$\Rightarrow f^2(x) = f(f(x)) = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = \frac{-1}{x}$$

$$f^3(x) = f(f^2(x)) = f\left(-\frac{1}{x}\right) = \frac{x+1}{1-x}$$

$$\Rightarrow f^4(x) = f\left(\frac{x+1}{1-x}\right) = \frac{-1}{x}$$

$$\Rightarrow f^6(x) = -\frac{1}{x} \Rightarrow f^6(6) = -\frac{1}{8}$$

$$f^7(7) = \frac{8}{-6} = \frac{-4}{3}$$

$$\therefore -\frac{1}{6} + -\frac{4}{3} + = -\frac{3}{2}$$

Q.15

(4)

$$f(n) = \begin{cases} 4R & , \quad n = 2R \\ 4R - 2 & , \quad n = 4R - 1 \\ 2R - 1 & , \quad n = 4R - 3 \end{cases} \quad (R \in \mathbb{N})$$

Note that for any element, it will fall into exactly one of these sets

$$\{y : y = 4R; y \in \mathbb{N}\}$$

$$\{y : y = 4R - 2; y \in \mathbb{N}\}$$

$$\{y : y = 2R - 1; y \in \mathbb{N}\}$$

Corresponding to that y, we will get exactly one value of n. Thus, f is one-one and onto.

Q.16

(2)

$$3 + (n-1)6 = 99$$

$$(n-1)6 = 96$$

$$n-1 = 16$$

$n = 17$

As bijective function has to be considered
 \therefore Out of 50 odd numbers remove 17 numbers and rest 33 numbers can be mapped with any number

So, answer = ${}^{50}C_{33} \times 33!$
 $= {}^{50}P_{33}$

Q.17

(4)
 $f(0) + 3 + \lambda + 4 = 14$
 $\therefore f(0) = 7 - \lambda = c$
 $f(1) = a + b + c = 3 \quad \dots (i)$
 $f(3) = 9a + 3b + c = 4 \quad \dots (ii)$
 $f(-2) = 4a - 2b + c = \lambda \quad \dots (iii)$
 $(ii) - (iii)$

$a + b = \frac{4 - \lambda}{5}$ put in equation (i)

$\frac{4 - \lambda}{5} + 7 - \lambda = 3$

$6\lambda = 24; \lambda = 4$

Q.18

(2)
 (i) $f(1) \in \{1\} \rightarrow f(2) \in \{1,2,3,4,5\}, f(3)$ corresponding & $f(6)$ all 6 $\Rightarrow 1 \times 5 \times 1 \times 6 = 30$
 (ii) $f(1) \in \{2\} \rightarrow f(2) \in \{1,2,3,4\}, f(3)$ corresponding & $f(6)$ all 6 $\Rightarrow 1 \times 4 \times 1 \times 6 = 24$
 (iii) $f(1) \in \{3\} \rightarrow f(2) \in \{1,2,3\}, f(3)$ corresponding & $f(6)$ all 6 $\Rightarrow 1 \times 3 \times 1 \times 6 = 18$
 (iv) $f(1) \in \{4\} \rightarrow f(2) \in \{1\}, f(3)$ corresponding & $f(6)$ all 6 $\Rightarrow 1 \times 1 \times 1 \times 6 = 6$
 hence total function = $30 + 24 + 18 + 12 + 6 = 90$

Q.19

(4)
 $f, g: N - \{1\} \rightarrow N$
 defined by $f(a) = \alpha$, where α is the maximum of the powers of those primes P such that P^2 divides a and $g(a) = a + 1$
 New $h(a) = (f + g)(a) = f(a) + g(a)$
 $= \alpha + a + 1$

one - one :-

$h(5) = 1 + 5 + 1 = 7$

and $h(4) = 2 + 4 + 1 = 7$

$\Rightarrow h(5) = h(4) \quad \text{But } 5 \neq 4$

$\Rightarrow h$ is not 1 - 1

onto :- $a \in N - \{1\}$ s.t

$h(a) = 1$

if possible

$h(a) = 1$

then

$\alpha + a + 1 = 1$

$\Rightarrow \alpha + a = 0$

$\Rightarrow \alpha = -a$

$\Rightarrow \alpha$ is negative ($\because a \in N - \{1\}$) contradiction

$\Rightarrow h$ is not into
 correct ans is = D

Q.20

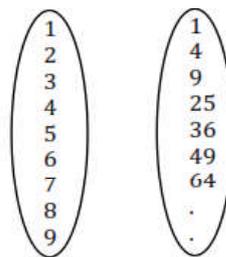
[1440]

$x^2 - 10x + 9 \leq 0$

$x \in [1, 9]$

$A = \{1, 2, 3, \dots, 9\}$

$B = \{1, 4, 9, 16, 25, \dots\}$



$f(1) \leq 5 \Rightarrow (1,4) \rightarrow 2$ values
 $f(2) \leq 2 \Rightarrow (1) \rightarrow 1$ values
 $f(3) \leq 5 \Rightarrow (1) \rightarrow 1$ values
 $f(4) \leq 2 \Rightarrow (1) \rightarrow 1$ values
 $f(5) \leq 5 \Rightarrow (1,4) \rightarrow 2$ values
 $f(6) \leq 10 \Rightarrow (1,4,9) \rightarrow 3$ values
 $f(7) \leq 17 \Rightarrow (1,4,9,16) \rightarrow 4$ values
 $f(8) \leq 26 \Rightarrow (1,4,9,16,25) \rightarrow 5$ values
 $f(9) \leq 37 \Rightarrow (1,4,9,16,25,36) \rightarrow 6$ values

no. of function = $2 \cdot 1 \cdot 1 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 1440$.
 [25]

Q.21

$f(x) = x^2 + bx + p \quad (\because f(0) = p)$

$f(1) = \frac{1}{3} \Rightarrow 1 + b + p = \frac{1}{3} \quad \dots (1)$

$p + b = \frac{-2}{3}$

Let α is common root

$f[f(f(\alpha))] = 0$

$f[f(f(0))] = 0$

$f[f(p)] = 0$

$f[p^2 + bp + p] = 0$

$f[p(p + b + 1)] = 0$

$f\left(\frac{p}{3}\right) = 0$

$\therefore \frac{p^2}{9} + \frac{bp}{3} + p = 0$

$\frac{p}{9} + \frac{b}{3} + 1 = 0 \quad (\because p \neq 0)$

$p + 3b + 9 = 0 \quad \dots (2)$

From (1) and (2)

$-\frac{2}{3} - b + 3b = -9$

$\therefore 2b = -9 + \frac{2}{3} = \frac{-25}{3}$

$b = \frac{-25}{6} \Rightarrow p = \frac{25 - 2}{6 \cdot 3} = \frac{21}{6}$

$\therefore f(x) = x^2 - \frac{25}{6}x + \frac{7}{2}$

$f(-3) = 9 + \frac{25}{2} + \frac{7}{2}$
 $= 9 + 16 = 25$

INVERSE TRIGONOMETRIC FUNCTIONS

EXERCISE-I (MHT CET LEVEL)

Q.1 (2)

$$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$

Q.2 (4)

Q.3 (2)

Q.4 (1)

Q.5 (2)

Q.6 (2)

Q.7 (2)

$$\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right]$$

$$\text{Let } \frac{1}{2}\cos^{-1}\frac{a}{b} = \theta \Rightarrow \cos 2\theta = \frac{a}{b}$$

$$\text{Thus, } \tan\left[\frac{\pi}{4} + \theta\right] + \tan\left[\frac{\pi}{4} - \theta\right]$$

$$= \frac{1 + \tan \theta}{1 - \tan \theta} + \frac{1 - \tan \theta}{1 + \tan \theta} = \frac{(1 + \tan \theta)^2 + (1 - \tan \theta)^2}{(1 - \tan^2 \theta)}$$

$$= \frac{1 + \tan^2 \theta + 2 \tan \theta + 1 + \tan^2 \theta - 2 \tan \theta}{(1 + \tan^2 \theta)}$$

$$= \frac{2(1 + \tan^2 \theta)}{1 - \tan^2 \theta} = 2 \sec 2\theta = \frac{2}{\cos 2\theta}$$

$$= \frac{2}{a/b} = \frac{2b}{a}$$

Q.8 (4)

$$\sin\left(4 \tan^{-1}\frac{1}{3}\right)$$

$$= 2 \sin\left(2 \tan^{-1}\frac{1}{3}\right) \cos\left(2 \tan^{-1}\frac{1}{3}\right)$$

$$= 2 \sin\left(\tan^{-1}\frac{3}{4}\right) = \cos\left(\tan^{-1}\frac{3}{4}\right)$$

$$= 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$$

$$\cos\left(2 \tan^{-1}\frac{1}{7}\right) = \cos\left(\tan^{-1}\frac{7}{24}\right) = \frac{24}{25}$$

The given expression = 0

Q.9 (3)

The given trigonometric ratio

$$= \cos\left[\frac{1}{2}\left(\cos\left(\cos^{-1}\frac{1}{8}\right)\right)\right]$$

$$= \cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$$

$$= \sqrt{\frac{1 + \cos\left(\cos^{-1}\frac{1}{8}\right)}{2}} = \frac{3}{4}$$

Q.10 (4)

Q.11 (4)

Q.12 (4)

$$\cot^{-1}\left[\frac{\sqrt{1 - \sin x} + \sqrt{1 + \sin x}}{\sqrt{1 - \sin x} - \sqrt{1 + \sin x}}\right]$$

$$= \cot^{-1}\left[\frac{(\sqrt{1 - \sin x} + \sqrt{1 + \sin x}) \cdot (\sqrt{1 - \sin x} + \sqrt{1 + \sin x})}{(\sqrt{1 - \sin x} - \sqrt{1 + \sin x}) \cdot (\sqrt{1 - \sin x} + \sqrt{1 + \sin x})}\right]$$

$$= \cot^{-1}\left[\frac{(1 - \sin x) + (1 + \sin x) + 2\sqrt{1 - \sin^2 x}}{(1 - \sin x) - (1 + \sin x)}\right]$$

$$= \cot^{-1}\left[\frac{2(1 + \cos x)}{-2 \sin x}\right] = \cot^{-1}\left[-\frac{2 \cos^2(x/2)}{2 \sin(x/2) \cos(x/2)}\right]$$

$$= \cot^{-1}\left(-\cot \frac{x}{2}\right) = \cot^{-1}\left[\cot\left(\pi - \frac{x}{2}\right)\right] = \pi - \frac{x}{2}$$

Trick : Put $x = \frac{\pi}{4}$, so that the expression becomes

$$\cot^{-1}\left[\frac{\sqrt{\sqrt{2}-1} + \sqrt{\sqrt{2}+1}}{\sqrt{\sqrt{2}-1} - \sqrt{\sqrt{2}+1}}\right]$$

$$= \cot^{-1} \left[\frac{\sqrt{2}-1+\sqrt{2}+1+2\sqrt{2-1}}{\sqrt{2}-1-\sqrt{2}-1} \right]$$

$$= \cot^{-1} \left[\frac{2\sqrt{2}+2}{-2} \right] = \cot^{-1}(-1-\sqrt{2}) = 157.5^\circ.$$

Q.13 (4)

The principal value of $\sin^{-1} \left[\sin \left(\pi - \frac{2\pi}{3} \right) \right]$

$$= \sin^{-1} \sin \left(\frac{\pi}{3} \right) = \frac{\pi}{3}$$

Q.14 (1)

$$\cos^{-1}(\cos 12) - \sin^{-1}(\sin 14) \Rightarrow 12 - 14 = -2.$$

Q.15 (4)

Given that $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$

$$\Rightarrow \cos^{-1}(x) + \cos^{-1}(y) + \cos^{-1}(z) = \cos^{-1}(-1)$$

$$\Rightarrow \cos^{-1}(x) + \cos^{-1}(y) = \cos^{-1}(-1) - \cos^{-1}(z)$$

$$\Rightarrow \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}) = \cos^{-1}\{(-1)(z)\}$$

$$\Rightarrow xy - \sqrt{(1-x^2)(1-y^2)} = -z$$

$$\Rightarrow (xy + z) = \sqrt{(1-x^2)(1-y^2)}$$

Squaring both sides we get $x^2 + y^2 + z^2 + 2xyz = 1$

Trick : Put $x = y = z = \frac{1}{2}$, so that

$$\cos^{-1} \frac{1}{2} + \cos^{-1} \frac{1}{2} + \cos^{-1} \frac{1}{2} = \pi$$

Obviously (4) holds for these values of x, y, z

Q.16 (2)

Obviously $x = \sin(\theta + \beta)$ and $y = \sin(\theta - \beta)$

$$\therefore 1 + xy = 1 + \sin(\theta + \beta)\sin(\theta - \beta)$$

$$= 1 + \sin^2 \theta - \sin^2 \beta = \sin^2 \theta + \cos^2 \beta$$

Q.17 (1)

$$\tan^{-1} \left[\frac{\cos x}{1 + \sin x} \right] = \tan^{-1} \left[\frac{\sin(\pi/2 - x)}{1 + \cos(\pi/2 - x)} \right]$$

$$= \tan^{-1} \left[\frac{2 \sin(\pi/4 - x/2) \cos(\pi/4 - x/2)}{2 \cos^2(\pi/4 - x/2)} \right]$$

$$= \tan^{-1} \tan \left(\frac{\pi}{4} - \frac{x}{2} \right) = \frac{\pi}{4} - \frac{x}{2}$$

Q.18 (3)

$$\tan^{-1} \frac{1}{\sqrt{x^2 - 1}} = \tan^{-1} \frac{1}{\sqrt{\operatorname{cosec}^2 \theta - 1}}$$

(Putting $x = \operatorname{cosec} \theta$)

$$= \tan^{-1} \frac{1}{\cot \theta} = \theta = \operatorname{cosec}^{-1} x.$$

Q.19 (3)

Let $\tan^{-1} 2 = \alpha \Rightarrow \tan \alpha = 2$

and $\cot^{-1} 3 = \beta \Rightarrow \cot \beta = 3$

$$\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3)$$

$$= \sec^2 \alpha + \operatorname{cosec}^2 \alpha = 1 + \tan^2 \alpha + 1 + \cot^2 \alpha$$

$$= 2 + (2)^2 + (3)^2 = 15$$

Q.20 (1)

$$\text{Given, } \tan^{-1} x = \sin^{-1} \left[\frac{3}{\sqrt{10}} \right]$$

$$\Rightarrow x = \tan \left\{ \sin^{-1} \left[\frac{3}{\sqrt{10}} \right] \right\} = \tan \{ \tan^{-1} 3 \}$$

$$\Rightarrow x = 3.$$

Q.21 (4)

$$\text{Given } \sin^{-1} C = \sin^{-1} \frac{3}{5} + \cos^{-1} \frac{12}{13}$$

$$\Rightarrow C = \sin \left(\sin^{-1} \frac{3}{5} + \cos^{-1} \frac{12}{13} \right)$$

using, $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$\Rightarrow C = \frac{3}{5} \times \frac{12}{13} + \sqrt{1 - \frac{9}{25}} \sqrt{1 - \frac{144}{169}} \Rightarrow C = \frac{56}{65}$$

Q.22 (1)

$$\cos^{-1} \left[\cos \frac{5\pi}{3} \right] + \sin^{-1} \left[\frac{\cos 5\pi}{3} \right] = \frac{\pi}{2}$$

$$(\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}).$$

Q.23 (4)

$$\sin^{-1}\left[\frac{\sqrt{3}}{2}\right] - \sin^{-1}\left[\frac{1}{2}\right] = 60^\circ - 30^\circ = 30^\circ.$$

Q.24 (a)

$$2 \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{2} + 2 \tan^{-1} \frac{1}{8}$$

$$= 2 \tan^{-1} \frac{\frac{1}{5} + \frac{1}{8}}{1 - \frac{1}{5} \times \frac{1}{8}} + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \times \frac{1}{7}}$$

$$= \tan^{-1} \frac{25}{25} = \tan^{-1} 1 = 45^\circ = \frac{\pi}{4}$$

Q.25 (d)

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left[\frac{1}{2}\right]$$

$$= \tan^{-1}\left[\frac{\frac{1}{4} + \frac{2}{9}}{1 - \frac{1}{4} \times \frac{2}{9}}\right] = \tan^{-1}\left[\frac{1}{2}\right]$$

Q.26 (d)

$$\sin^{-1}x + \cos^{-1}x + \tan^{-1}x = \frac{\pi}{2} + \tan^{-1}x$$

Since domain of the function $x \in [-1, 1]$

$$\therefore -\frac{\pi}{4} \leq \tan^{-1}x \leq \frac{\pi}{4}.$$

$$\text{Hence, } k = \frac{\pi}{4} \text{ and } K = \frac{3\pi}{4}$$

Q.27 (d)

We have $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18$

$$\therefore xy = \frac{1}{7} \cdot \frac{1}{8} < 1$$

$$\therefore \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{8} + \tan^{-1} \frac{1}{18}$$

$$= \tan^{-1} \left(\frac{\frac{1}{7} + \frac{1}{8}}{1 - \frac{1}{7} \times \frac{1}{8}} \right) + \tan^{-1} \frac{1}{18} =$$

$$\tan^{-1} \frac{15}{55} + \tan^{-1} \frac{1}{18}$$

$$\text{also; } \frac{3}{11} \cdot \frac{1}{18} < 1$$

$$\tan^{-1} \frac{3}{11} + \tan^{-1} \frac{1}{18} = \tan^{-1} \left(\frac{\frac{3}{11} + \frac{1}{18}}{1 - \frac{3}{11} \times \frac{1}{18}} \right)$$

$$= \tan^{-1} \frac{65}{192} = \tan^{-1} \frac{1}{3} = \cot^{-1} 3$$

Q.28 (b)

Q.29 (2)

Q.30 (4)

Q.31 (2)

Q.32 (2)

Q.33 (2)

Q.34 (2)

Q.35 (3)

$$\text{Given } \cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$$

$$\therefore 0 \leq \cos^{-1}x \leq \pi$$

$$\therefore 0 \leq \cos^{-1}y \leq \pi \text{ and } 0 \leq \cos^{-1}z \leq \pi$$

$$\text{Here } \cos^{-1}x = \cos^{-1}y = \cos^{-1}z = \pi$$

$$\Rightarrow x = y = z = \cos \pi = -1$$

$$\therefore xy + yz + zx = (-1)(-1) + (-1)(-1) + (-1)(-1) \\ = 1 + 1 + 1 = 3$$

Q.36 (4)

$$\tan \left[\tan^{-1} \frac{1}{2} - \tan^{-1} \frac{1}{3} \right] = \tan \left[\tan^{-1} \frac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{6}} \right]$$

$$= \tan \tan^{-1} \left(\frac{1}{6} \times \frac{6}{7} \right) = \frac{1}{7}.$$

Q.37 (2)

$$\cos^{-1} \sqrt{1-x} + \sin^{-1} \sqrt{1-x} = \sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x} = \frac{\pi}{2}$$

Q.38 (1)

$$\begin{aligned} & \sin^{-1} x + \sin^{-1} \frac{1}{x} + \cos^{-1} x + \cos^{-1} \frac{1}{x} \\ &= \{\sin^{-1}(x) + \cos^{-1}(x)\} + \left\{ \sin^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{x}\right) \right\} \\ &= \frac{\pi}{2} + \frac{\pi}{2} = \pi \end{aligned}$$

Q.39 (1)

Q.40 (3)

Q.41 (1)

Q.42 (1)

$$\sin^{-1} \frac{4}{5} = \tan^{-1} \frac{4}{3}, 2 \tan^{-1} \frac{1}{3} = \tan^{-1} \frac{3}{4} = \cot^{-1} \frac{4}{3} \text{ and}$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

Q.43 (1)

Q.44 (4)

Q.45 (4) $\sin[\cot^{-1}(\cos \tan^{-1} x)]$

$$\begin{aligned} &= \sin \left[\cot^{-1} \left(\cos \cos^{-1} \frac{1}{\sqrt{1+x^2}} \right) \right] \\ &= \sin \left[\cot^{-1} \frac{1}{\sqrt{1+x^2}} \right] = \sin \left[\sin^{-1} \sqrt{\frac{1+x^2}{2+x^2}} \right] = \sqrt{\frac{1+x^2}{2+x^2}} \end{aligned}$$

Q.46 (1)

Let $\cos^{-1} x = \theta$. Then $x = \cos \theta$

$$\tan \theta = \sqrt{\sec^2 \theta - 1} = \sqrt{\frac{1}{x^2} - 1} = \sqrt{\frac{1-x^2}{x}}$$

$$\therefore \tan(\cos^{-1} x) = \tan \theta = \frac{\sqrt{1-x^2}}{x}$$

Q.47 (4)

$$2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2} = \tan^{-1} \left(\frac{\frac{2}{3}}{1 - \frac{1}{9}} \right) + \tan^{-1} \left(\frac{1}{2} \right)$$

$$= \tan^{-1} \left(\frac{\frac{2}{3}}{\frac{8}{9}} \right) + \tan^{-1} \left(\frac{1}{2} \right) = \tan^{-1} \left(\frac{3}{4} \right) + \tan^{-1} \left(\frac{1}{2} \right)$$

$$= \tan^{-1} \left(\frac{\frac{1}{2} + \frac{3}{4}}{1 - \frac{1}{2} \times \frac{3}{4}} \right) = \tan^{-1}(2)$$

Q.48 (2)

Let $x = \tan^2 \theta \Rightarrow \theta = \tan^{-1} \sqrt{x}$

$$\text{Now, } \frac{1}{2} \cos^{-1} \left(\frac{1-x}{1+x} \right)$$

$$= \frac{1}{2} \cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right)$$

$$= \frac{1}{2} \cos^{-1} \cos 2\theta = \frac{2\theta}{2} = \theta = \tan^{-1} \sqrt{x}$$

Q.49 (2)

$$\sin \left(4 \tan^{-1} \frac{1}{3} \right) = \sin \left[2 \tan^{-1} \left(\frac{2/3}{1 - (1/9)} \right) \right]$$

$$= \sin \left[2 \tan^{-1} \frac{3}{4} \right] = \sin \sin^{-1} \left(\frac{2 \times (3/4)}{1 + (9/16)} \right)$$

$$= \frac{3}{2} \times \frac{16}{25} = \frac{24}{25}$$

$$\left(\because 2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2} \right)$$

Q.50 (2)

$$\sin \left[2 \tan^{-1} \left(\frac{1}{3} \right) \right] + \cos[\tan^{-1}(2\sqrt{2})]$$

$$= \sin \left[\tan^{-1} \frac{2/3}{1 - 1/9} \right] + \cos[\tan^{-1}(2\sqrt{2})]$$

$$= \sin[\tan^{-1} 3/4] + \cos[\tan^{-1} 2\sqrt{2}]$$

$$= \frac{3}{5} + \frac{1}{3} = \frac{14}{15}$$

Q.51 (4)

$$\sin^{-1} 2x = \sin^{-1} x - \sin^{-1} \frac{\sqrt{3}}{2}$$

$$\sin^{-1} 2x = \sin^{-1} \left(x \sqrt{\left(1 - \frac{3}{4}\right)} - \frac{\sqrt{3}}{2} \sqrt{1 - x^2} \right)$$

$$2x = \left(\frac{x}{2} - \frac{\sqrt{3}}{2} \sqrt{1 - x^2} \right)$$

$$\frac{\sqrt{3}}{2} \sqrt{1 - x^2} = \frac{x}{2} - 2x = \frac{-3x}{2}$$

$$\frac{3(1 - x^2)}{4} = \frac{9x^2}{4}$$

$$\Rightarrow 3 - 3x^2 = 9x^2 \Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm \frac{1}{2}.$$

Q.52 (1)

$$\begin{aligned} \sin \left[3 \sin^{-1} \frac{1}{5} \right] &= \sin \left[\sin^{-1} \left\{ 3 \left(\frac{1}{5} \right) - 4 \left(\frac{1}{5} \right)^3 \right\} \right] \\ &= \sin \left[\sin^{-1} \left\{ \frac{3}{5} - \frac{4}{125} \right\} \right] = \sin \left[\sin^{-1} \left(\frac{75 - 4}{125} \right) \right] \\ &= \sin \left[\sin^{-1} \frac{71}{125} \right] = \frac{71}{125} \end{aligned}$$

Q.53 (2)

$$\text{Given that } \tan \{ \cos^{-1}(x) \} = \sin \left(\cot^{-1} \frac{1}{2} \right)$$

$$\text{Let } \cot^{-1} \frac{1}{2} = \phi \Rightarrow \frac{1}{2} = \cot \phi$$

$$\Rightarrow \sin \phi = \frac{1}{\sqrt{1 + \cot^2 \phi}} = \frac{2}{\sqrt{5}}$$

$$\text{Let } \cos^{-1} x = \theta \Rightarrow \sec \theta = \frac{1}{x} \Rightarrow \tan \theta = \sqrt{\sec^2 \theta - 1}$$

$$\Rightarrow \tan \theta = \sqrt{\frac{1}{x^2} - 1} \Rightarrow \tan \theta = \frac{\sqrt{1 - x^2}}{x}$$

$$\text{So, } \tan \{ \cos^{-1}(x) \} = \sin \left(\cot^{-1} \frac{1}{2} \right)$$

$$\Rightarrow \tan \left(\tan^{-1} \frac{\sqrt{1 - x^2}}{x} \right) = \sin \left(\sin^{-1} \frac{2}{\sqrt{5}} \right)$$

$$\Rightarrow \frac{\sqrt{1 - x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow \sqrt{(1 - x^2)5} = 2x$$

$$\text{Squaring both sides, we get } x = \pm \frac{\sqrt{5}}{3}$$

Q.54 (4)

$$\text{Let } \alpha = \cos^{-1} \sqrt{p}; \beta = \cos^{-1} \sqrt{1 - p}$$

and

$$\gamma = \cos^{-1} \sqrt{1 - q} \text{ or } \cos \alpha = \sqrt{p}; \cos \beta = \sqrt{1 - p}$$

$$\text{and } \cos \gamma = \sqrt{1 - q}.$$

$$\text{Therefore } \sin \alpha = \sqrt{1 - p}, \sin \beta = \sqrt{p} \text{ and}$$

$$\sin \gamma = \sqrt{q}$$

The given equation may be written as

$$\alpha + \beta + \gamma = \frac{3\pi}{4} \text{ or } \alpha + \beta = \frac{3\pi}{4} - \gamma \text{ or}$$

$$\cos(\alpha + \beta) = \cos \left(\frac{3\pi}{4} - \gamma \right)$$

$$\Rightarrow \cos \alpha \cos \beta - \sin \alpha \sin \beta =$$

$$\cos \left\{ \pi - \left(\frac{\pi}{4} + \gamma \right) \right\} = -\cos \left(\frac{\pi}{4} + \gamma \right)$$

$$\Rightarrow \sqrt{p} \sqrt{1 - p} - \sqrt{1 - p} \sqrt{p} = - \left(\frac{1}{\sqrt{2}} \sqrt{1 - q} - \frac{1}{\sqrt{2}} \sqrt{q} \right)$$

$$\Rightarrow 0 = \sqrt{1 - q} - \sqrt{q} \Rightarrow 1 - q = q \Rightarrow q = \frac{1}{2}.$$

Q.55 (3)

$$\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}(1 + x) = \frac{\pi}{2} - \tan^{-1}(1 - x)$$

$$\Rightarrow \tan^{-1}(1 + x) = \cot^{-1}(1 - x)$$

$$\Rightarrow \tan^{-1}(1 + x) = \tan^{-1} \left(\frac{1}{1 - x} \right)$$

$$\Rightarrow 1 + x = \frac{1}{1 - x} \Rightarrow 1 - x^2 = 1 \Rightarrow x = 0$$

Q.56 (1)

From previous solution

$$\frac{\pi^3}{32} \leq (\sin^{-1} x)^3 + (\cos^{-1} x)^3 \leq \frac{7\pi}{8}$$

Here $a < \frac{1}{32}$. So, number of solution is zero.

Q.57 (1)

$$\begin{aligned}
 (\tan^{-1} x)^2 + (\cot^{-1} x)^2 &= \frac{5\pi^2}{8} \\
 \Rightarrow \\
 (\tan^{-1} x + \cot^{-1} x)^2 - 2 \tan^{-1} x \left(\frac{\pi}{2} - \tan^{-1} x \right) &= \frac{5\pi^2}{8} \\
 \Rightarrow \frac{\pi^2}{4} - 2 \times \frac{\pi}{2} \tan^{-1} x + 2(\tan^{-1} x)^2 &= \frac{5\pi^2}{8} \\
 \Rightarrow 2(\tan^{-1} x)^2 - \pi \tan^{-1} x - \frac{3\pi^2}{8} &= 0 \\
 \Rightarrow \tan^{-1} x = -\frac{\pi}{4}, \frac{3\pi}{4} \Rightarrow \tan^{-1} x = -\frac{\pi}{4} \Rightarrow x &= -1.
 \end{aligned}$$

Q.58 (3)

$$\begin{aligned}
 x + y &= \tan^{-1} 33 \\
 \Rightarrow y &= \tan^{-1} 33 - \tan^{-1} 3 \\
 &= \tan^{-1} \frac{33-3}{1+99} = \tan^{-1} \frac{30}{100} \Rightarrow y = \tan^{-1}(0.3)
 \end{aligned}$$

Q.59 (3)

$$\begin{aligned}
 \tan^{-1} \left(\frac{c_1 x - y}{c_1 y + x} \right) + \tan^{-1} \left(\frac{c_2 - c_1}{1 + c_2 c_1} \right) + \tan^{-1} \left(\frac{c_3 - c_2}{1 + c_3 c_2} \right) + \\
 \dots + \tan^{-1} \frac{1}{c_n} \\
 = \tan^{-1} \left(\frac{\frac{x}{y} - \frac{1}{c_1}}{1 + \frac{x}{y} \cdot \frac{1}{c_1}} \right) + \tan^{-1} \left(\frac{\frac{1}{c_1} - \frac{1}{c_2}}{1 + \frac{1}{c_1 c_2}} \right) \\
 + \tan^{-1} \left(\frac{\frac{1}{c_2} - \frac{1}{c_3}}{1 + \frac{1}{c_2 c_3}} \right) + \dots + \tan^{-1} \frac{1}{c_n} \\
 = \tan^{-1} \frac{x}{y} - \tan^{-1} \frac{1}{c_1} + \tan^{-1} \frac{1}{c_1} - \tan^{-1} \frac{1}{c_2} + \tan^{-1} \frac{1}{c_2} \\
 - \tan^{-1} \frac{1}{c_3} + \dots + \tan^{-1} \frac{1}{c_{n-1}} - \tan^{-1} \frac{1}{c_n} + \tan^{-1} \frac{1}{c_n} \\
 = \tan^{-1} \left(\frac{x}{y} \right)
 \end{aligned}$$

Q.60 (1)

$$\begin{aligned}
 \text{If } \cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} &= \theta \\
 \text{Then } \frac{x^2}{a^2} \cos^2 \theta + \frac{y^2}{b^2} &= \sin^2 \theta \\
 \text{Here } \cos^{-1} \frac{x}{1} + \cos^{-1} \frac{y}{2} &= \alpha; \\
 \therefore \frac{x^2}{1} - \frac{2xy}{2} \cos \alpha + \frac{y^2}{4} &= \sin^2 \alpha \\
 x^2 - xy \cos \alpha + \frac{y^2}{4} &= \sin^2 \alpha \\
 4x^2 - 4xy \cos \alpha + y^2 &= 4 \sin^2 \alpha
 \end{aligned}$$

Q.61 (3)

$$\begin{aligned}
 \cos^{-1} x + \cos^{-1}(2x) &= -\pi \\
 \Rightarrow \cos^{-1} 2x &= -\pi - \cos^{-1} x \\
 \Rightarrow 2x &= \cos(\pi + \cos^{-1} x) \\
 \Rightarrow 2x &= \cos \pi (\cos \cos^{-1} x) - \sin \pi \sin(\cos^{-1} x) \\
 2x &= -x \Rightarrow x = 0 \\
 \text{But } x = 0 &\text{ does not satisfy the given equation.} \\
 \text{No solution will exist.}
 \end{aligned}$$

Q.62 (a)

$$\begin{aligned}
 \text{We have } \sum_{m=1}^n \tan^{-1} \left(\frac{2m}{m^4 + m^2 + 2} \right) \\
 = \sum_{m=1}^n \tan^{-1} \left(\frac{2m}{1 + (m^2 + m + 1)(m^2 - m + 1)} \right) \\
 = \sum_{m=1}^n \tan^{-1} \left(\frac{(m^2 + m + 1) - (m^2 - m + 1)}{1 + (m^2 + m + 1)(m^2 - m + 1)} \right) \\
 = \sum_{m=1}^n [\tan^{-1}(m^2 + m + 1) - \tan^{-1}(m^2 - m + 1)] \\
 = (\tan^{-1} 3 - \tan^{-1} 1) + (\tan^{-1} 7 - \tan^{-1} 3) + \\
 (\tan^{-1} 13 - \tan^{-1} 7) + \dots + [\tan^{-1}(\frac{1}{n^2 + 1}) - \tan^{-1}(\frac{1}{(n-1)^2 + 1})]
 \end{aligned}$$

$$= \tan^{-1}(n^2 + n + 1) - \tan^{-1} 1 =$$

$$\tan^{-1}\left(\frac{n^2 + n}{2 + n^2 + n}\right).$$

Q.63 (3)

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2}$$

$\tan^{-1}\sqrt{x(x+1)}$ is defined when

$$x(x+1) \geq 0$$

.....(i)

$\sin^{-1}\sqrt{x^2 + x + 1}$ is defined when

$$0 \leq x(x+1) + 1 \leq 1 \text{ or } 0 \leq x(x+1) \leq 0$$

.....(ii)

From (i) and (ii), $x(x+1) = 0$

or $x = 0$ and -1 .

Hence number of solution is 2.

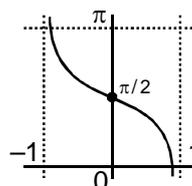
Q.8 (1)

$$\sum_{i=1}^n \cos^{-1} \alpha_i = 0$$

$$\cos^{-1} \alpha_1 + \cos^{-1} \alpha_2 + \dots + \cos^{-1} \alpha_n$$

so, $\cos^{-1} \alpha_1$ is always positive

So, in order to have their sum 0 all should be equal to 0



$$\cos^{-1} \alpha_1 = \cos^{-1} \alpha_2 = \dots = \cos^{-1} \alpha_n = 0$$

$$\Rightarrow \cos^{-1} \alpha_1 = 0 \Rightarrow \alpha_1 = 1$$

$$\cos^{-1} \alpha_2 = 0 \Rightarrow \alpha_2 = 1$$

\vdots

$$\cos^{-1} \alpha_n = 0 \Rightarrow \alpha_n = 1$$

\vdots

$$\therefore \sum_{i=1}^m \alpha_i = n$$

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (2)

$$\text{Given} = 60^\circ + 45^\circ = 105^\circ$$

Q.2 (4)

$$\therefore -1 \leq x \leq 1 \quad \dots(1)$$

$$x \in \mathbb{R} \quad \dots(2)$$

$$x \leq -1 \text{ or } x \geq 1 \quad \dots(3)$$

By (1) \cap (2) \cap (3)

$$\Rightarrow x \in \{-1, 1\}$$

Q.3 (3)

Domain of $f(x)$ is $x \in \{-1, 1\}$

$$f(-1) = -\frac{\pi}{2} - \frac{\pi}{4} + \pi = \frac{\pi}{4}$$

$$f(1) = \frac{\pi}{2} + \frac{\pi}{4} + 0 = \frac{3\pi}{4}$$

Q.4 (3)

Q.5 (4)

Q.6 (2)

Q.7 (2)

$$\cos [\tan^{-1} \{ \sin (\cot^{-1} \sqrt{3}) \}] = y$$

$$= \cos [\tan^{-1} (\sin \frac{\pi}{6})] = \cos \left[\tan^{-1} \frac{1}{2} \right] =$$

$$\cos \left[\cos^{-1} \frac{2}{\sqrt{5}} \right] = \frac{2}{\sqrt{5}}$$

Q.9 B

$$\cot^{-1} \left(\frac{n}{\pi} \right) > \frac{\pi}{6}, n \in \mathbb{N}$$

$$\frac{n}{\pi} < \cot \frac{\pi}{6} \Rightarrow n < \sqrt{3} - \pi$$

$$\Rightarrow n < \sqrt{3} \times 3.14 \Rightarrow n = 5$$

Q.10 (4)

$\operatorname{cosec}^{-1}(\cos x)$ is define if

$$\cos x \geq 1 \text{ or } \cos x \leq -1$$

$$\Rightarrow \cos x = \pm 1 \Rightarrow x = n\pi$$

Q.11 (c)

$$\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}(1+x) = \frac{\pi}{2} - \tan^{-1}(1-x)$$

$$\Rightarrow \tan^{-1}(1+x) = \cot^{-1}(1-x)$$

$$\Rightarrow \tan^{-1}(1+x) = \tan^{-1} \left(\frac{1}{1-x} \right)$$

$$\Rightarrow 1+x = \frac{1}{1-x}$$

$$\Rightarrow 1 - x^2 = 1 \Rightarrow x = 0$$

Q.12 (1)

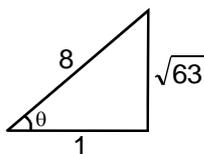
Q.13 (4)

Q.14 (4)

$$\pi \leq x \leq 2\pi \cos^{-1} \cos x = 2\pi - x$$

Q.15 (1)

$$y = \cos \left(\frac{1}{2} \cos^{-1} \left(\frac{1}{8} \right) \right)$$



$$\text{Let, } \cos^{-1} \frac{1}{8} = \theta \Rightarrow \cos \theta = \frac{1}{8}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \frac{1}{8}}{2}}$$

$$\Rightarrow \sqrt{\frac{9}{16}} = \frac{3}{4}$$

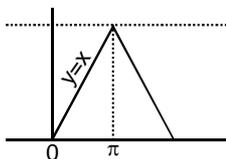
$$\& \cos \frac{\theta}{2} = \cos \left(\frac{\cos^{-1} \left(\frac{1}{8} \right)}{2} \right) = \frac{3}{4}$$

Q.16 (4)

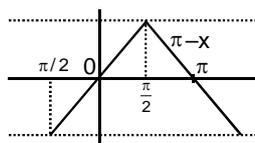
$$y = \sin^{-1} [\cos \{ \cos^{-1} (\cos x) + \sin^{-1} (\sin x) \}]$$

$$\text{given } x \in \left(\frac{\pi}{2}, \pi \right) \Rightarrow \frac{\pi}{2} < x < \pi$$

$$\text{Now } \cos^{-1} (\cos x) = x$$



$$\sin^{-1} (\sin x) = \pi - x$$



$$\text{so, } y = \sin^{-1} [\cos \{ x + \pi - x \}]$$

$$y = \sin^{-1} (\cos \pi) \Rightarrow \sin^{-1} (-1) \Rightarrow -\frac{\pi}{2}$$

Q.17 (4)

$$x \geq 0, \theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x$$

↓ ↓

$$[-1, 1] \quad [-1, 1] \quad x \in \mathbb{R}$$

but $x \geq 0$ so, $x \in [0, 1]$

$$\theta = \frac{\pi}{2} - \tan^{-1} x$$

$$R_\theta : \left[\theta \Big|_{x=1}, \theta \Big|_{x=0} \right] = \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$$

Q.18 (c)

$$\text{We have, } (\sin^{-1} x)^2 + (\cos^{-1} x)^2$$

$$= (\sin^{-1} x + \cos^{-1} x)^2 - 2 \sin^{-1} x \cdot \cos^{-1} x$$

$$= \frac{\pi^2}{4} - 2 \sin^{-1} x \left(\frac{\pi}{2} - \sin^{-1} x \right)$$

$$= \frac{\pi^2}{4} - \pi \sin^{-1} x + 2 (\sin^{-1} x)^2$$

$$= 2 \left[(\sin^{-1} x)^2 - \frac{\pi}{2} \sin^{-1} x + \frac{\pi^2}{8} \right]$$

$$= 2 \left[\left(\sin^{-1} x - \frac{\pi}{4} \right)^2 + \frac{\pi^2}{16} \right]$$

Thus, the least value is $2 \left(\frac{\pi^2}{16} \right)$ i.e. $\frac{\pi^2}{8}$

and the greatest value is $2 \left[\left(\frac{-\pi}{2} - \frac{\pi}{4} \right)^2 + \frac{\pi^2}{16} \right]$

$$\text{i.e. } \frac{5\pi^2}{4}$$

Q.19 (b)

$$2 \tan^{-1} \frac{1}{5} = \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{5}$$

$$= \tan^{-1} \frac{\frac{1}{5} + \frac{1}{5}}{1 - \frac{1}{5} \cdot \frac{1}{5}} = \tan^{-1} \frac{2/5}{24/25} = \tan^{-1} \frac{5}{12}$$

$$\begin{aligned}
 &= \tan\left(2 \tan^{-1} \frac{1}{5} - \frac{\pi}{4}\right) = \tan\left(\tan^{-1} \frac{5}{12} - \frac{\pi}{4}\right) \\
 &= \frac{\tan\left(\tan^{-1} \frac{5}{12}\right) - \tan \frac{\pi}{4}}{1 + \tan\left(\tan^{-1} \frac{5}{12}\right) \tan \frac{\pi}{4}} = \frac{-\frac{7}{12}}{\frac{17}{12}} = -\frac{1}{17}
 \end{aligned}$$

- Q.20** (3)
Q.21 (3)
Q.22 (1)
Q.23 (1)
Q.24 (2)

Let $\cos^{-1} x = y$
 $\Rightarrow x = \cos y,$

so that $\frac{1}{2} \leq x \leq 1$ or $0 \leq y \leq \frac{\pi}{3}$

and $\frac{x}{2} + \frac{1}{2} \sqrt{3-3x^2} = \frac{1}{2} \cos y + \frac{\sqrt{3}}{2} \sin y$
 $= \cos \frac{\pi}{3} \cos y + \sin \frac{\pi}{3} \sin y = \cos\left(\frac{\pi}{3} - y\right)$

$\Rightarrow \cos^{-1}\left(\frac{x}{2} + \frac{1}{2} \sqrt{3-3x^2}\right) = \frac{\pi}{3} - y$

\therefore the given expression is equal to

$y + \frac{\pi}{3} - y$, e.i., $\frac{\pi}{3}$

- Q.25** (c)

$\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$

$\Rightarrow \cos^{-1}\left(\frac{xy}{2} + \sqrt{(1-x^2)\left(1-\frac{y^2}{4}\right)}\right) = \alpha$

$\Rightarrow \cos^{-1}\left(\frac{xy + \sqrt{4-y^2-4x^2+x^2y^2}}{2}\right) = \alpha$

$\Rightarrow 4 - y^2 - 4x^2 + x^2y^2$
 $= 4 \cos^2 \alpha + x^2y^2 - 4xy \cos \alpha$
 $= 4x^2 + y^2 - 4xy \cos \alpha = 4 \sin^2 \alpha.$

- Q.26** (d)

Given,

$\sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\therefore 2 \tan^{-1} a - 2 \tan^{-1} b = 2 \tan^{-1} x$

$\Rightarrow \tan^{-1} a - \tan^{-1} b = \tan^{-1} x$

$\Rightarrow \tan^{-1}\left(\frac{a-b}{1+ab}\right) = \tan^{-1} x \quad \Rightarrow x = \frac{a-b}{1+ab}$

- Q.27** (3)

- Q.28** (1)

- Q.29** (2)

- Q.30** (3)

- Q.31** (2)

- Q.32** (2)

$\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2} - \cos^{-1} x - \cos^{-1} y + \frac{\pi}{2}$

$= \pi - (\cos^{-1} x + \cos^{-1} y) = \frac{2\pi}{3}$

$\Rightarrow \cos^{-1} x + \cos^{-1} y = \frac{\pi}{3}$

- Q.33** (2)

$\sin^{-1} x - \cos^{-1} x = \cos^{-1} \frac{\sqrt{3}}{2}$

$\frac{\pi}{2} - 2 \cos^{-1} x = \cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$

$\Rightarrow \cos^{-1} x = \frac{\pi}{6}$

$x = \frac{\sqrt{3}}{2}$

- Q.34** (2)

$\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1} x + \cos^{-1} \frac{1}{\sqrt{5}} = \frac{\pi}{2} \quad \therefore \sin^{-1} \theta + \cos^{-1}$

$\theta = \frac{\pi}{2}$

$\Rightarrow x = \frac{1}{\sqrt{5}}$

- Q.35** (3)

$$\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) - \frac{\pi}{6} = 0$$

$$\sin^{-1}(1) - \sin^{-1}\sqrt{\frac{3}{x}} = \frac{\pi}{6} \quad \Rightarrow$$

$$\sin^{-1}\sqrt{\frac{3}{x}} = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

$$\Rightarrow \sqrt{\frac{3}{x}} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = 4$$

Q.36 (2)

By property if $x < 0$, $\tan^{-1}\frac{1}{x} = \cot^{-1}x - \pi$

$$\therefore \tan^{-1}x + \tan^{-1}\frac{1}{x} = \tan^{-1}x + \cot^{-1}x - \pi = \frac{\pi}{2} - \pi$$

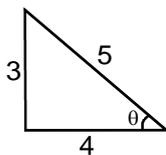
$$\Rightarrow \tan^{-1}x + \tan^{-1}\frac{1}{x} = -\frac{\pi}{2}$$

Q.37 (4)

$$y = \tan\left[\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$$

$$\text{Let } \theta = \sin^{-1}\frac{3}{5} \Rightarrow \sin\theta = \frac{3}{5}$$

$$\tan\theta = \frac{3}{4}$$



$$\text{so } \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\text{so } y = \tan\left[\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$$

$$y = \frac{\tan\left\{\tan^{-1}\left(\frac{3}{4}\right)\right\} + \tan\left\{\tan^{-1}\left(\frac{2}{3}\right)\right\}}{1 - \tan\left\{\tan^{-1}\left(\frac{3}{4}\right)\right\} \cdot \tan\left\{\tan^{-1}\left(\frac{2}{3}\right)\right\}}$$

$$y = \frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4} \cdot \frac{2}{3}} = \frac{9+8}{6} = \frac{17}{6}$$

Q.38 C

$$\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x\right) + \tan\left(\frac{\pi}{4} - \frac{\cos^{-1}x}{2}\right)$$

Let $\cos^{-1}x = \theta \Rightarrow x = \cos\theta$

$$\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

$$\Rightarrow \frac{\left(1 + \tan\frac{\theta}{2}\right)^2 + \left(1 - \tan\frac{\theta}{2}\right)^2}{1 - \tan^2\frac{\theta}{2}}$$

$$\Rightarrow \frac{2\left(\sec^2\frac{\theta}{2}\right)}{\left(1 - \tan^2\frac{\theta}{2}\right)} = \frac{2}{\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)} \Rightarrow \frac{2}{\cos\theta} =$$

$$\frac{2}{x}$$

Q.39 (3)

$$\tan^{-1}a + \tan^{-1}b = \pi + \tan^{-1}\left(\frac{a+b}{1-ab}\right)$$

if $ab > 1, a > 0, b > 0$

Q.40 (1)

$$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1}1 = \frac{\pi}{4}$$

Q.41 (4)

$$\tan^{-1}\left(\frac{\sqrt{x^2+1}-1}{x}\right) = \frac{\pi}{45^\circ}$$

$$\text{put } x = \tan\theta \Rightarrow \theta = \tan^{-1}x, \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\tan^{-1}\left(\frac{|\sec\theta|-1}{\tan\theta}\right) = \frac{\pi}{45^\circ}$$

(but $\sec\theta$ is +ve for $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$)

$$\tan^{-1} \left(\frac{1 - \cos \theta}{\sin \theta} \right) = \frac{\pi}{45^\circ}$$

$$\tan^{-1} \left(\tan \frac{\theta}{2} \right) = \frac{\pi}{45^\circ}$$

$$\text{Now, } -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$\left(-\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4} \right)$$

$$\frac{\theta}{2} = \frac{\pi}{45^\circ} \Rightarrow \tan^{-1} x = 2 \times \frac{\pi}{45^\circ} = 8^\circ$$

$$x = \tan 8^\circ$$

Q.42 (2)

$$y = \cot^{-1} \left(\frac{\sqrt{1 - \sin x} + \sqrt{1 + \sin x}}{\sqrt{1 - \sin x} - \sqrt{1 + \sin x}} \right)$$

$$\text{given } \frac{\pi}{2} < x < \pi$$

$$y = \cot^{-1} \left(\frac{\left| \sin \frac{x}{2} - \cos \frac{x}{2} \right| + \left| \sin \frac{x}{2} + \cos \frac{x}{2} \right|}{\left| \sin \frac{x}{2} - \cos \frac{x}{2} \right| - \left| \sin \frac{x}{2} + \cos \frac{x}{2} \right|} \right)$$

$$\Rightarrow \text{Now if } \frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Now $\sin \frac{x}{2} > \cos \frac{x}{2}$ so, modulus will open directly

$$y = \cot^{-1} \left(-\frac{2 \sin \frac{x}{2}}{2 \cos \frac{x}{2}} \right) = \cot^{-1} \left(-\tan \frac{x}{2} \right)$$

$$y = \pi - \cot^{-1} \left(\tan \frac{x}{2} \right) \Rightarrow \pi - \cot^{-1} \cot \left(\frac{\pi}{2} - \frac{x}{2} \right)$$

$$y = \pi - \frac{\pi}{2} + \frac{x}{2} = \frac{\pi}{2} + \frac{x}{2}$$

Q.43 (2)

$$y = \tan^{-1} \left(\frac{1-x}{1+x} \right), 0 \leq x \leq 1$$

$$\text{put } x = \cos \theta \Rightarrow \theta = \cos^{-1} x \theta \in [0, \pi]$$

$$y = \tan^{-1} \left(\frac{1 - \cos \theta}{1 + \cos \theta} \right) = \tan^{-1} \left(\tan \frac{\theta}{2} \right)$$

here given, $0 \leq x \leq 1$

$$0 \leq \cos \theta \leq 1$$

$$0 < \theta \leq \frac{\pi}{2}$$

comes in PVR of $\theta = \cos^{-1} x$

$$0 < \frac{\theta}{2} \leq \frac{\pi}{4}$$

$$\text{so, } y = \frac{\theta}{2} = \frac{\cos^{-1} x}{2}$$

$$\text{Now } y_{\min} = \frac{\theta}{2} \Big|_{\theta=0} = 0$$

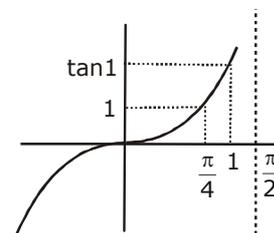
$$y_{\max} = \frac{\theta}{2} \Big|_{\theta=\frac{\pi}{2}} = \frac{\pi}{4}$$

$$\text{so, } \left(0, \frac{\pi}{4} \right)$$

Q.44 (1)

$$\tan 1 > 1$$

$$\tan 1 > \frac{\pi}{4}$$



$$\tan 1 > \tan^{-1} 1$$

Q.45 (3)

$$\tan^{-1} \left(\frac{1}{2x+1} \right) + \tan^{-1} \left(\frac{1}{4x+1} \right) = \tan^{-1} \left(\frac{2}{x^2} \right)$$

$$\tan^{-1} \left\{ \frac{\frac{1}{2x+1} + \frac{1}{4x+1}}{1 - \frac{1}{(2x+1)(4x+1)}} \right\} = \tan^{-1} \frac{2}{x^2}$$

$$\frac{4x+1+2x+1}{(2x+1)(4x+1)-1} = \frac{2}{x^2}$$

$$\frac{6x+2}{(8x^2+6x)} = \frac{2}{x^2} \Rightarrow 6x^3+2x^2=16x^2+12x$$

$$\Rightarrow 6x^3-14x^2-12x=0$$

$$\Rightarrow x(6x^2 - 14x - 12) = 0$$

$$x = 0, 6x^2 - 14x - 12 = 0$$

$$x = 0, x = 3, \frac{-3}{2}$$

check: $x = 0$

$$\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2} \quad (\text{Accepted})$$

$$x = 3$$

$$\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \tan^{-1}\left(\frac{2}{9}\right)$$

$$\tan^{-1} \frac{\frac{1}{7} + \frac{1}{13}}{1 - \frac{1}{91}}$$

$$\Rightarrow \tan^{-1}\left(\frac{13+7}{90}\right) \quad (\text{Accepted})$$

$$x = -3/2$$

$$\tan^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}\left(\frac{-1}{5}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{-6}{9}\right) \quad (\text{Rejected})$$

So, 2 solutions

Q.46 (2)

$$\cos^{-1} \frac{3}{5} + \cos^{-1} \frac{5}{13}$$

$$= \cos^{-1} \left(\frac{3}{5} \times \frac{5}{13} - \frac{4}{5} \times \frac{12}{13} \right)$$

$$= \cos^{-1} \left(-\frac{33}{65} \right)$$

Q.47 (2)

$$\frac{1}{2} \sin^{-1} \left(\frac{3 \sin 2\theta}{5 + 4 \cos 2\theta} \right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{3 \sin 2\theta}{5 + 4 \cos 2\theta} = 1 \Rightarrow \frac{6 \tan \theta}{\tan^2 \theta + 9} = 1$$

$$\Rightarrow \tan^2 \theta - 6 \tan \theta + 9 = 0 \Rightarrow (\tan \theta - 3)^2 = 0$$

$$\tan \theta = 3$$

Q.48 (4)

$$\sin^{-1} \left(2x \sqrt{1-x^2} \right) =$$

$$\begin{cases} 2 \sin^{-1} x & \text{if } |x| \leq \frac{1}{\sqrt{2}} \\ \pi - 2 \sin^{-1} x & \text{if } \frac{1}{\sqrt{2}} < x \leq 1 \\ -(\pi + 2 \sin^{-1} x) & \text{if } -1 \leq x < -\frac{1}{\sqrt{2}} \end{cases}$$

$$\therefore 2 \sin^{-1} x = \sin^{-1} (2x \sqrt{1-x^2}) \text{ is true for } |x| \leq \frac{1}{\sqrt{2}}$$

Q.49 (3)

$$\cos^{-1} \left\{ \frac{x^2}{2} + \sqrt{1-x^2} \sqrt{1-\frac{x^2}{4}} \right\} = \cos^{-1} \frac{x}{2} - \cos^{-1} x$$

The above holds iff

$$1 \geq x \geq 0 \text{ \& } 1 \geq \frac{x}{2} \geq 0$$

$$0 \leq x \leq 1 \text{ \& } 0 \leq x \leq 2$$

$$\underbrace{\hspace{10em}}_{\downarrow \cap}$$

$$0 \leq x \leq 1$$

Q.50 (3)

$$\cos^{-1} \sqrt{p} + \cos^{-1} \sqrt{1-p} + \cos^{-1} \sqrt{1-q} = \frac{3\pi}{4}$$

$$\cos^{-1} (\sqrt{p} \sqrt{1-p} - \sqrt{1-p} \sqrt{p}) + \cos^{-1} \sqrt{1-q} = \frac{3\pi}{4}$$

$$\cos^{-1} 0 + \cos^{-1} \sqrt{1-q} = \frac{3\pi}{4}$$

$$\cos^{-1} \sqrt{1-q} = \frac{\pi}{4} \Rightarrow 1-q = \frac{1}{2}$$

$$\Rightarrow q = \frac{1}{2} \text{ so, } 0 \leq \sqrt{p} \leq 1 \text{ \& } 0 \leq \sqrt{1-p} \leq 1$$

$$0 \leq p \leq 1 \text{ \& } 0 \leq 1-p \leq 1$$

$$-1 \leq -p \leq 0 \Rightarrow 0 \leq p \leq 1$$

Q.51 B

$$\sin^{-1} \left(\frac{x}{5} \right) + \operatorname{cosec}^{-1} \left(\frac{5}{4} \right) = \frac{\pi}{2}$$

$$\sin^{-1} \left(\frac{x}{5} \right) + \sin^{-1} \left(\frac{4}{5} \right) = \frac{\pi}{2}$$

$$\sin^{-1} \left\{ \frac{x}{5} \times \frac{3}{5} + \frac{4}{5} \times \frac{\sqrt{25-x^2}}{5} \right\} = \frac{\pi}{2}$$

$$\frac{3x}{25} + \frac{4\sqrt{25-x^2}}{25} = 1$$

$$x^2 - 6x + 9 = 0 \Rightarrow (x-3)^2 = 0 \Rightarrow x = 3$$

Q.52

(1)

$$\sin^{-1} x + \sin^{-1} (1-x) = \cos^{-1} x$$

$$(\sin^{-1} x + \sin^{-1} (1-x)) = \cos^{-1} x$$

$$\sin^{-1} (x\sqrt{1-(1-x)^2} + (1-x)\sqrt{1-x^2}) = \sin^{-1} \sqrt{1-x^2}$$

$$x\sqrt{2x-x^2} = \sqrt{1-x^2} (1-1+x)$$

$$x^2(2x-x^2) - (1-x^2)x^2 = 0$$

$$2x^3 - x^2 = 0$$

$$x^2(2x-1) = 0 \Rightarrow x = 0, \frac{1}{2}$$

Both accepted.

Q.53

(2)

$$\sin^{-1} (1-x) - 2 \sin^{-1} x = \frac{\pi}{2}$$

$$-2 \sin^{-1} x = \cos^{-1} (1-x)$$

$$1 - 2x^2 = 1 - x$$

$$2x^2 - x = 0 \Rightarrow x = 0, \frac{1}{2}$$

 check : $x = 0$

$$\text{L.H.S.} \Rightarrow \frac{\pi}{2} - 0 = \frac{\pi}{2} = \text{RHS}$$

$$x = \frac{1}{2} \Rightarrow \frac{\pi}{6} - \frac{\pi}{3} \Rightarrow -\frac{\pi}{6} \Rightarrow \text{so rejected}$$

EXERCISE-III

Q.1

0013

$$\text{Put } \sin^{-1} \frac{5}{x} = A \quad \text{or} \quad \frac{5}{x} = \sin A$$

$$\sin^{-1} \frac{12}{x} = B \quad \text{or} \quad \frac{12}{x} = \sin B$$

$$\Rightarrow A + B = \frac{\pi}{2}$$

$$\Rightarrow \sin A = \sin \left(\frac{\pi}{2} - B \right) = \cos B = \sqrt{1 - \sin^2 B}$$

$$\Rightarrow \frac{5}{x} = \sqrt{1 - \frac{144}{x^2}} \quad \text{or} \quad \frac{169}{x^2} = 1$$

$$\text{or } x^2 = 169 \quad \text{or } x = 13$$

 $[\because x = -13 \text{ does not satisfy the given equation}]$
Q.2

0003

$$\tan^{-1} \frac{1}{1+2x} + \tan^{-1} \frac{1}{1+4x} = \tan^{-1} \frac{2}{x^2}$$

$$\text{or } \tan^{-1} \left[\frac{\frac{1}{1+2x} + \frac{1}{1+4x}}{1 - \frac{1}{1+2x} \cdot \frac{1}{1+4x}} \right] = \tan^{-1} \frac{2}{x^2}$$

$$\text{or } \frac{2+6x}{6x+8x^2} = \frac{2}{x^2}$$

$$\text{or } 6x^3 - 14x^2 - 12x = 0$$

$$\text{or } x(x-3)(3x+2) = 0$$

$$\text{or } x = 3 \text{ or } x = -2/3$$

 But for $x = -2/3$, L.H.S. < 0 and R.H.S. > 0

 Hence, the only solution is $x = 3$.

Q.3

0003

$$\sin^{-1} \frac{x}{5} + \sin^{-1} \frac{4}{5} = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} \frac{x}{5} = \frac{\pi}{2} - \sin^{-1} \frac{4}{5}$$

$$\Rightarrow \sin^{-1} \frac{x}{5} = \cos^{-1} \frac{4}{5}$$

$$\Rightarrow \sin^{-1} \frac{x}{5} = \sin^{-1} \frac{3}{5}$$

$$\therefore x = 3$$

Q.4

2.83

$$\cos^{-1} \frac{4}{5} = \tan^{-1} \frac{3}{4}$$

$$\Rightarrow \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{2}{3}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4} \cdot \frac{2}{3}} \right) = \tan^{-1} \frac{17}{6}$$

Q.5

0003

$$\text{We have } \cot^{-1} x + \sin^{-1} \frac{1}{\sqrt{5}} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \frac{1}{x} + \tan^{-1} \frac{1/\sqrt{5}}{\sqrt{1-1/5}} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \frac{1}{x} + \tan^{-1} \frac{1}{2} = \tan^{-1} 1$$

$$\Rightarrow \tan^{-1} \frac{1}{x} = \tan^{-1} 1 - \tan^{-1} \frac{1}{2}$$

$$\Rightarrow \tan^{-1} \frac{1}{x} = \tan^{-1} \left(\frac{1 - \frac{1}{2}}{1 + 1 \cdot \frac{1}{2}} \right)$$

$$\Rightarrow \tan^{-1} \frac{1}{x} = \tan^{-1} \left(\frac{1}{3} \right) \Rightarrow x = 3$$

Q.6 0001

$$\text{Given } \sin^{-1} A + \cos^{-1} B = \frac{\pi}{2} \quad \dots (1)$$

$$\text{But } \sin^{-1} A + \cos^{-1} A = \frac{\pi}{2} \quad \dots$$

(2)

$$\therefore \text{ from (1) \& (2). } \quad A = B$$

$$\Rightarrow x - \frac{x^2}{2} + \frac{x^3}{4} - \dots \text{ to } \infty = x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \dots \text{ to } \infty$$

$$\Rightarrow \frac{x}{1 - \left(-\frac{x}{2} \right)} = \frac{x^2}{1 - \left(-\frac{x^2}{2} \right)}$$

$$\Rightarrow x(2 + x^2) = x^2(2 + x)$$

$$\Rightarrow 2x(x - 1) = 0$$

$$\therefore x = 0 \text{ or } 1$$

$$\text{but } x \neq 0$$

$$\therefore x = 1$$

Q.7 (0.93)

$$\sin \left(2 \tan^{-1} \frac{1}{3} \right) + \cos \left(\tan^{-1} 2\sqrt{2} \right)$$

$$= \sin \left(\tan^{-1} \frac{3}{4} \right) + \cos \left(\tan^{-1} 2\sqrt{2} \right)$$

$$= \sin \left(\sin^{-1} \frac{3}{5} \right) + \cos \left(\cos^{-1} \frac{1}{3} \right) = \frac{3}{5} + \frac{1}{3} = \frac{14}{15}$$

Q.8 (0010)

$$\tan \left\{ \pi - \cos^{-1} \left(\frac{2}{7} \right) - \frac{\pi}{2} \right\} = \tan \left\{ \frac{\pi}{2} - \cos^{-1} \left(\frac{2}{7} \right) \right\}$$

$$= \tan \left\{ \sin^{-1} \left(\frac{2}{7} \right) \right\} = \tan \tan^{-1} \left(\frac{2}{3\sqrt{5}} \right) = \frac{2}{3\sqrt{5}}$$

Q.9 (0002)

$$\cos^{-1} \left(\frac{1}{\sqrt{x^2 + x + 1}} \right) + \sin^{-1} \left(\sqrt{x^2 + x + 1} \right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{1}{\sqrt{x^2 + x + 1}} = \sqrt{x^2 + x + 1}$$

$$\Rightarrow x^2 + x + 1 = 1$$

$$\Rightarrow x^2 + x = 0$$

$$\Rightarrow x = -1, 0$$

Q.10 1

$$\left(\tan^{-1} x + \cot^{-1} x \right)^2 - 2 \tan^{-1} x \cot^{-1} x = \frac{5\pi^2}{8}$$

$$\Rightarrow \frac{\pi^2}{4} - 2 \tan^{-1} x \left(\frac{\pi}{2} - \tan^{-1} x \right) = \frac{5\pi^2}{8}$$

$$\Rightarrow \frac{\pi^2}{4} - \pi \tan^{-1} x + 2(\tan^{-1} x)^2 = \frac{5\pi^2}{8}$$

$$\Rightarrow 2(\tan^{-1} x)^2 - \pi \tan^{-1} x = \frac{5\pi^2}{8}$$

$$\Rightarrow 16(\tan^{-1} x)^2 - 8\pi \tan^{-1} x - 3\pi^2 = 0$$

$$\Rightarrow 16(\tan^{-1} x)^2 - 12\pi \tan^{-1} x + 4\pi \tan^{-1} x - 3\pi^2 = 0$$

$$\Rightarrow 4(\tan^{-1} x) [4 \tan^{-1} x - 3\pi] + \pi(4 \tan^{-1} x - 3\pi) = 0$$

$$\Rightarrow \tan^{-1} x = -\frac{\pi}{4}, \frac{3\pi}{4} \quad \text{but } \tan^{-1} x \neq \frac{3\pi}{4} \quad \text{as}$$

$$\tan^{-1} x \neq \frac{3\pi}{4} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$$

$$\therefore x = \tan \left(-\frac{\pi}{4} \right) = -1$$

PREVIOUS YEAR'S

MHT CET

Q.1 (4)

Q.2 (2)

Q.3 (2)

Q.4 (1)

Q.5 (3)

Q.6 (2)

Q.7 (3)

Q.8 (2)

Q.9 (3)

Q.10 (3)

Q.11 (3)

$$\cos^{-1} x > \sin^{-1} x \text{ [where } x \in [-1, 1]]$$

$$\therefore \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$\therefore \frac{\pi}{2} - \sin^{-1} x > \sin^{-1} x \Rightarrow 2 \sin^{-1} x < \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} x < \frac{\pi}{4} \Rightarrow x < \frac{1}{\sqrt{2}}$$

$$\therefore -1 \leq x \leq 1$$

$$-1 \leq x < \frac{1}{\sqrt{2}}$$

Q.12 (3)

Given, $f(x) = \sin^{-1} \left[\log_2 \left(\frac{x}{2} \right) \right]$

$$\Rightarrow -1 \leq \log_2 \left(\frac{x}{2} \right) \leq 1 \Rightarrow 2^{-1} \leq \frac{x}{2} \leq 2^1$$

$$\Rightarrow \frac{1}{2} \leq \frac{x}{2} \leq 2 \Rightarrow 1 \leq x \leq 4$$

Q.13 (3)
 $\cos^{-1} \alpha + \cos^{-1} \beta + \cos^{-1} \gamma = 3\pi$
 $\cos^{-1} \alpha = \cos^{-1} \gamma = \pi$
 $\cos \pi = \alpha = \beta = \gamma$
 $-1 = \alpha = \beta = \gamma$
 $\alpha = \beta = \gamma = -1$
 $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$
 $-1(-1-1) - 1(-1-1) - 1(-1-1)$
 $2 + 2 + 2 = 6$

Q.14 (3) Let $\tan^{-1} x = y$...(i)

$\tan y = x \Rightarrow \tan y = x/1$
 In ΔABC , $\tan y = AB/BC = x$
 By Pythagoras theorem,
 $AC^2 = AB^2 + BC^2$
 $\Rightarrow AC^2 = x^2 + 1^2 = x^2 + 1$

$\Rightarrow AC = \sqrt{x^2 + 1}$
 $\therefore \cos y = \frac{BC}{AC} = \frac{1}{\sqrt{x^2 + 1}}$...(ii)

$\Rightarrow y = \cos^{-1} \left(\frac{1}{\sqrt{x^2 + 1}} \right)$
 Now, let $\cot^{-1}(\cos(y)) = z$
 ...(iii)

$\cot z = \cos y$
 $\Rightarrow \cot z = \frac{1}{\sqrt{x^2 + 1}}$ [using Eq.(ii)]

$\Rightarrow z = \cot^{-1} \left(\frac{1}{\sqrt{x^2 + 1}} \right)$

In ΔPQR
 $\cot z = \frac{QR}{PQ} = \frac{1}{\sqrt{x^2 + 1}}$

By Pythagoras theorem,
 $PR^2 = PQ^2 + QR^2$

$\therefore PR = \sqrt{x^2 + 2}$

So, $\sin z = \frac{\sqrt{x^2 + 1}}{\sqrt{x^2 + 2}}$

$\therefore \sin [\cot^{-1}(\cos(\tan^{-1} x))] = \frac{\sqrt{x^2 + 1}}{\sqrt{x^2 + 2}}$
 [using Eqs. (i) and (iii)]

JEE-MAIN

Q.1 (4)

$$-1 \leq \frac{2 \sin^{-1} \left(\frac{1}{4x^2 - 1} \right)}{\pi} \leq 1$$

$$-\pi/2 \leq \sin^{-1} \frac{1}{4x^2 - 1} \leq \pi/2$$

Always $-1 \leq \frac{1}{4x^2 - 1} \leq 1$

$$x \in \left(-\infty, \frac{-1}{\sqrt{2}} \right] \cup \left[\frac{1}{\sqrt{2}}, \infty \right) \cup \{0\}$$

Q.2 (29)

$$50 \tan \left(3 \tan^{-1} \left(\frac{1}{2} \right) + 2 \cos^{-1} \left(\frac{1}{\sqrt{5}} \right) \right) + 4 \sqrt{2} \tan \left(\frac{1}{2} \tan^{-1} (2\sqrt{2}) \right)$$

$$50 \tan \left(\tan^{-1} \frac{1}{2} + 2 \left(\tan^{-1} \frac{1}{2} + \tan^{-1} 2 \right) \right) + 4 \sqrt{2} \tan \left(\frac{1}{2} \tan^{-1} (2\sqrt{2}) \right)$$

$$50 \tan \left(\tan^{-1} \frac{1}{2} + 2 \cdot \frac{\pi}{2} \right) + 4 \sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$= 50 \tan \left(\pi + \tan^{-1} \frac{1}{2} \right) + 4$$

$$\frac{50}{2} + 4$$

$$= 25 + 4 = 29$$

Q.3 (1)

Given $(\tan^{-1} x)^3 + (\cot^{-1} x)^3 = k\pi^3$, $x \in \mathbb{R}$
 using, $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
 $\therefore (\tan^{-1} x)^3 + (\cot^{-1} x)^3 = (\tan^{-1} x + \cot^{-1} x)^3 - 3 \tan^{-1} x \cdot \cot^{-1} x \cdot (\tan^{-1} x + \cot^{-1} x)$

$$(\tan^{-1} x)^3 + (\cot^{-1} x)^3 = \left(\frac{\pi}{2} \right)^3 - 3 \tan^{-1} x \cdot \cot^{-1} x \cdot \left(\frac{\pi}{2} \right)$$

$$= \frac{\pi}{2} \left[\frac{\pi^2}{4} - 3 \tan^{-1} \left(\frac{\pi}{2} - \tan^{-1} x \right) \right]$$

$$= \frac{\pi}{2} \left[\frac{\pi^2}{4} + 3 \left(\tan^{-1} x \right)^2 - \frac{3\pi}{2} \tan^{-1} x \right]$$

$$= \frac{\pi}{2} \left[3 \left(\tan^{-1} x - \frac{\pi}{4} \right)^2 + \frac{\pi^2}{16} \right]$$

$$\therefore \tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \Rightarrow \left(\tan^{-1} x - \frac{\pi}{4} \right) \in \left(-\frac{3\pi}{4}, \frac{\pi}{4} \right)$$

$$\left(\tan^{-1} x - \frac{\pi}{4} \right)^2 \in \left(0, \frac{9\pi^2}{16} \right)$$

$$\therefore (\tan^{-1} x)^3 + (\cot^{-1} x)^3 \in \left[\frac{\pi^3}{32}, \frac{7\pi^3}{8} \right]$$

$$\therefore K \in \left[\frac{1}{32}, \frac{7}{8} \right]$$

Q.4

(2)

$$\begin{aligned} x*y &= x^2y^3 \\ (x*1)*1 &= x*(1*1) \\ (x^2+1)*1 &= x*(2) \\ (x^2+1)^2+1 &= x^2+8 \\ x^2+1=t & \\ t^2+1 &= t-1+8 \\ t^2-t-6=0 & \\ t^2-3t+2t-6=0 & \\ t(t-3)+2(t-3)=0 & \\ (t+2)(t-3)=0 & \\ x^2+1=-2 \text{ (not possible)} & \\ x^2+1=3 & \\ x^2=2 & \end{aligned}$$

$$2\sin^{-1}\left(\frac{4+2-2}{4+2+2}\right)$$

$$2\sin^{-1}\left(\frac{4}{8}\right)$$

$$2\sin^{-1}\left(\frac{1}{2}\right)$$

$$2 \times \frac{\pi}{6} = \frac{\pi}{3}$$

Q.5

(2)

$$\tan^{-1} \left\{ \frac{\cos\left(3\pi + \frac{3\pi}{4}\right) - 1}{\sin\left(\frac{3\pi}{4}\right)} \right\}$$

$$-\tan^{-1} \left\{ \frac{1 - \left(-\cos\frac{3\pi}{4}\right)}{\sin\frac{3\pi}{4}} \right\}$$

$$-\tan^{-1} \left\{ \frac{2\cos^2\frac{3\pi}{8}}{2\sin\frac{3\pi}{8}\cos\frac{3\pi}{8}} \right\}$$

$$-\frac{\pi}{2} + \cot^{-1} \left(\cot\left(\frac{3\pi}{8}\right) \right)$$

$$-\frac{\pi}{2} + \frac{3\pi}{8} = \frac{-\pi}{8}$$

Q.6

(3)

$$\cos^{-1} \left(\frac{3}{10} \cos \left(\tan^{-1} \left(\frac{4}{3} \right) \right) + \frac{2}{5} \sin \left(\tan^{-1} \left(\frac{4}{3} \right) \right) \right)$$

$$= \cos^{-1} \left(\frac{3}{10} \cdot \frac{3}{5} + \frac{2}{5} \cdot \frac{4}{5} \right)$$

$$= \cos^{-1} \left(\frac{9}{50} + \frac{8}{25} \right) = \cos^{-1} \left(\frac{25}{50} \right)$$

$$= \cos^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{3}$$

Q.7

(1)

$$\sin^{-1} \left(\sin \frac{2\pi}{3} \right) + \cos^{-1} \left(\cos \frac{7\pi}{6} \right) + \tan^{-1} \tan \left(\tan \frac{3\pi}{4} \right)$$

Now,

$$\sin^{-1} \sin \left(\frac{2\pi}{3} \right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\cos^{-1} \left(\cos \frac{2\pi}{6} \right) = 2\pi - \frac{7\pi}{6} = \frac{5\pi}{6}$$

$$\tan^{-1} \tan \left(\frac{3\pi}{4} \right) = \frac{3\pi}{4} - \pi = \frac{-\pi}{4}$$

So,

$$\sin^{-1} \left(\sin \frac{2\pi}{3} \right) + \cos^{-1} \cos \frac{7\pi}{6} + \tan^{-1} \tan \frac{3\pi}{4}$$

$$= \frac{11\pi}{12}$$

Q.8

(1)

$$\tan^{-1} \left(\frac{1}{1+n+n^2} \right) = \tan^{-1} \left(\frac{(n+1)-n}{1+n(n+1)} \right)$$

$$= \tan^{-1}(n+1) - \tan^{-1}n$$

$$\text{So, } \sum_{n=1}^{50} (\tan^{-1}(n+1) - \tan^{-1}n)$$

$$= \tan^{-1}51 - \tan^{-1}1$$

$$\cot \left(\sum_{n=1}^{50} \left(\tan^{-1} \left(\frac{1}{1+n+n^2} \right) \right) \right)$$

$$= \cot(\tan^{-1}51 - \tan^{-1}1)$$

$$= \frac{1}{\tan(\tan^{-1}51 - \tan^{-1}1)}$$

$$= \frac{1+51 \times 1}{51-1} = \frac{52}{50} = \frac{26}{25}$$

Q.9

(3)

$$f(x) = \sin^{-1} \left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \right) \text{ Domain}$$

$$\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \geq -1 \text{ and } \frac{x^2 - 3x + 2}{x^2 + 2x + 7} \leq 1$$

$$2x^2 - x + 9 \geq 0 \text{ and } 5x \geq -5 \Rightarrow x \geq -1$$

$$x \in \mathbb{R}$$

Hence, Domain $x \in [-1, \infty)$

Q.10

(1)

$$\cos^{-1}x - 2\sin^{-1}x = \cos^{-1}2x$$

$$\cos^{-1}x - 2\left(\frac{\pi}{2} - \cos^{-1}x\right) = \cos^{-1}2x$$

$$\cos^{-1}x - \pi + 2\cos^{-1}x = \cos^{-1}2x$$

$$3\cos^{-1}x = \pi + \cos^{-1}2x \quad \dots(i)$$

$$\cos(3\cos^{-1}x) = \cos(\pi + \cos^{-1}2x)$$

$$4x^3 - 3x = -2x$$

$$4x^3 = x \Rightarrow x = 0, \pm \frac{1}{2}$$

All satisfy the original equation

$$\text{sum} = -\frac{1}{2} \text{ to } +\frac{1}{2} = 0$$

Q.11

[12]

$$\cos(\sin^{-1}x) = \cos(\cos^{-1}\sqrt{1-x^2}) = \sqrt{1-x^2}$$

$$\cot(\tan^{-1}\sqrt{1-x^2}) = \cot \cot^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right) = \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}} = K$$

$$\Rightarrow 1-2x^2 = k^2(1-x^2)$$

$$\Rightarrow (k^2-2)x^2 = k^2-1$$

$$x^2 = \frac{k^2-1}{k^2-2}$$

$$\alpha = \sqrt{\frac{k^2-1}{k^2-2}} \Rightarrow \alpha^2 = \frac{k^2-1}{k^2-2}$$

$$\beta = \sqrt{\frac{k^2-1}{k^2-2}} \Rightarrow \beta^2 = \frac{k^2-1}{k^2-2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 2\left(\frac{k^2-2}{k^2-1}\right) \& \frac{\alpha}{\beta} = -1$$

$$\text{Sum of roots} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{\alpha}{\beta} = b$$

$$\Rightarrow \frac{2(k^2-2)}{k^2-1} - 1 = b \quad \dots(1)$$

$$\text{Product of roots} = \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right) \frac{\alpha}{\beta} = -5$$

$$\Rightarrow \frac{2(k^2-2)}{k^2-1}(-1) = -5$$

$$\Rightarrow 2k^2 - 4 = 5k^2 - 5$$

$$\Rightarrow 3k^2 = 1 \Rightarrow k^2 = \frac{1}{3} \dots \text{Put in (1)}$$

$$\Rightarrow b = \frac{2(k^2-2)}{k^2-1} - 1 = 5 - 1 = 4$$

$$\frac{b}{k^2} = \frac{4}{\frac{1}{3}} = 12$$

Q.12

(2)

$$\frac{\sin^{-1}x}{x} = \frac{\cos^{-1}x}{\beta} = \lambda$$

$$\sin^{-1}x = \lambda x$$

$$\cos^{-1}x = \lambda \beta$$

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \lambda = \frac{\pi}{2(\alpha + \beta)}$$

$$\sin^{-1}\left(\frac{2\pi\alpha}{\alpha + \beta}\right) = \sin(4\sin^{-1}x)$$

$$= 2\sin(2\sin^{-1}x) \cos(2\sin^{-1}x)$$

$$= 4x\sqrt{1-x^2}(1-2x^2)$$

Q.13

(2)

Sol.

$$\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$$

$$= \tan\left(2\tan^{-1}\left(\frac{\frac{1}{5} + \frac{1}{8}}{1 - \frac{1}{40}}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right)$$

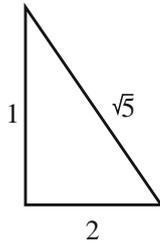
$$= \tan\left(2\tan^{-1}\left(\frac{13}{39}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{2\left(\frac{1}{3}\right)}{1 - \frac{1}{3}}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$= \tan \left(\tan^{-1} \left(\frac{2 \times 3}{8} \right) + \tan^{-1} \frac{1}{2} \right)$$

$$\therefore 16 \left[\frac{1}{8} + 8 \right] = 16 \times \frac{65}{8} = 130$$

$$= \tan \left(\tan^{-1} \left(\frac{\frac{3}{4} + \frac{1}{2}}{1 - \frac{3}{4} \cdot \frac{1}{2}} \right) \right)$$



$$= \tan \left(\tan^{-1} \left(\frac{5}{4} + \frac{8}{5} \right) \right)$$

$$= 2$$

Q.14 [130]

$$x = \frac{2\alpha}{1 + \alpha^2}$$

$$\text{Let } y = \sin \theta$$

$$2\theta = \tan^{-1} \frac{4}{3}$$

$$\tan 2\theta = \frac{4}{3}$$

$$\frac{2t}{1-t^2} = \frac{4}{3}$$

$$3t = 2 - 2t^2$$

$$2t^2 + 3t - 2 = 0$$

$$2t^2 + 4t - t - 2 = 0$$

$$(2t-1)(t+2) = 0$$

$$\tan \theta = \frac{1}{2}$$

$$\therefore \sin \theta = \frac{1}{\sqrt{5}}$$

$$y = \frac{1}{\sqrt{5}}$$

$$y^2 = 1 - x$$

$$\frac{1}{5} = 1 - \frac{2\alpha}{1 + \alpha^2}$$

$$\frac{1}{5} = \frac{1 + \alpha^2 - 2\alpha}{1 + \alpha^2}$$

$$1 + \alpha^2 = 5 + 5\alpha^2 - 10\alpha$$

$$4\alpha^2 - 10\alpha + 4 = 0$$

$$2\alpha^2 - 5\alpha + 2 = 0$$

$$(2\alpha - 1)(\alpha - 2) = 0$$

$$\alpha = \frac{1}{2}, \alpha = 2$$

MATRICES

EXERCISE-I (MHT CET LEVEL)

Q.1 (3)

$$2A + 2B = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}, \quad A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$$

On adding, we get $3A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

$$\Rightarrow A = \begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$$

Q.2 (c)

For two 2×2 matrices, A & B

$$(A-B) \times (A+B)$$

$$= A \times A + A \times B - B \times A - B \times B$$

$$= A^2 - B^2 + AB - BA$$

Hence, $(A-B)(A+B) = A^2 + AB - BA - B^2$

Q.3 (c)

$$\text{Let } D = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

$$\therefore A + B - C = \begin{bmatrix} 12 \\ 32 \\ 25 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-1-a & 3-2-b \\ 3+0-c & 2+5-d \\ 2+3-e & 5+1-f \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$-a = 0 \Rightarrow a = 0, \quad 1-b = 0 \Rightarrow b = 1,$$

$$3-c = 0 \Rightarrow c = 3, \quad 7-d = 0 \Rightarrow d = 7,$$

$$5-e = 0 \Rightarrow e = 5, \quad 6-f = 0 \Rightarrow f = 6,$$

$$\therefore D = \begin{bmatrix} 0 & 1 \\ 3 & 7 \\ 5 & 6 \end{bmatrix}$$

Q.4 (2)

It is obvious.

Q.5 (3)

Clearly, $AB = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix}$

$$= \begin{bmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{bmatrix} = BA \text{ (verify).}$$

Q.6 (a)

$$A^2 + 4A - 5I = A \times A + 4A - 5I$$

$$= \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + 4$$

$$\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9+4-5 & -4+8-0 \\ -8+16-0 & 17-12-5 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$$

$$= 4 \begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$$

Q.7 (a)

$$\text{As } A^2 = 0, A^k = 0 \quad \forall k \geq 2.$$

$$\text{Thus, } (A+I)^{50} = I + 50A \Rightarrow (A+I)^{50} - 50A = I$$

$$\therefore a = 1, b = 0, c = 0, d = 1$$

$$abc + abd + bcd + acd = 0$$

Q.8 (d)

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \sqrt{I_2}$$

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \alpha^2 + \beta\gamma = 1$$

Q.9 (b)

Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2A$

$A^3 = 2^2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, A^4 = 2^3 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^3 = 2^2 A,$

$A^4 = 2^3 A$

$\therefore A^n = 2^{n-1} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow A^{100} = 2^{100-1} A \therefore A^{100} = 2^{99} A$

Q.10 (a)

$R(s)R(t) = \begin{bmatrix} \cos s & \sin s \\ -\sin s & \cos s \end{bmatrix} \times \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$

$\begin{bmatrix} \cos \cos t - \sin s \sin t & \cos s \sin t + \sin s \cos t \\ -\sin s \cos t - \cos s \sin t & -\sin s \sin t + \cos s \cos t \end{bmatrix}$

$= \begin{bmatrix} \cos(s+t) & \sin(s+t) \\ -\sin(s+t) & \cos(s+t) \end{bmatrix} = R(s+t)$

Q.11 (1)

Q.12 (2)

Q.13 (1)

Q.14 (4)

Q.15 (4)

Q.16 (4)

Q.17 (1)

$A^2 = A.A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

Q.18 (4)

Since $AB = O$, even if $A \neq O$ and $B \neq O$.

Q.19 (1)

Since $(A+B)(A-B) = A^2 - B^2$

By matrix distribution law,

$\Rightarrow A^2 - AB + BA - B^2 = A^2 - B^2$

$\Rightarrow BA - AB = 0 \Rightarrow BA = AB$

Q.20 (2)

$A+B$ is defined $\Rightarrow A$ and B are of same order

Also AB is defined \Rightarrow Number of columns in A

= Number of rows in B

Obviously, both simultaneously mean that the matrices

A and B are square matrices of same order.

Q.21 (3)

We know that if all the elements below the diagonal in the matrix are zero, then it is an upper triangular matrix

Q.22 (4)

$A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & -1 \\ 3 & 1 & 2 \end{bmatrix} \Rightarrow A.A = A^2 = \begin{bmatrix} 6 & 11 & 7 \\ -11 & 4 & -11 \\ 7 & 11 & 12 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then, $A^2 + 9I = \begin{bmatrix} 15 & 11 & 7 \\ -11 & 13 & -11 \\ 7 & 11 & 21 \end{bmatrix}$

Q.23 (2)

$(A - 2I)(A - 3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Q.24 (4)

Given, Matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

We know that

$A^2 = A.A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Therefore

$A^{16} = (A^2)^8 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}^8 = \begin{bmatrix} (-1)^8 & 0 \\ 0 & (-1)^8 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q.25 (3)

Given $AB = A$, $\therefore B = I \Rightarrow BA = B$, $\therefore A = I$.

Hence, $A^2 = A$ and $B^2 = B$.

Q.26 (1)

$\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 35 \\ 40 \end{bmatrix}$;

$\therefore \begin{bmatrix} 35 \\ 40 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 43 \\ 44 \end{bmatrix}$

Q.27 (1)

Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

Check by options.

$$(i) A^2 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

$$(ii) (-1)I = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \neq A$$

(iii) $|A| = 1 \neq 0 \Rightarrow A^{-1}$ exists.

(iv) Clearly A , is not a zero matrix.

Q.28 (2)

$$A' = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -5 \\ -2 & 5 & 0 \end{bmatrix} = -A$$

Q.29 (1)

Q.30 (4)

Q.31 (4)

Q.32 (4)

$$(4) \text{ Matrix } \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & \lambda & 5 \end{bmatrix} \text{ be non singular,}$$

$$\text{only if } \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & \lambda & 5 \end{bmatrix} \neq 0$$

$$\Rightarrow 1(25 - 6\lambda) - 2(20 - 18) + 3(4\lambda - 15) \neq 0$$

$$\Rightarrow 25 - 6\lambda - 4 + 12\lambda - 45 \neq 0$$

$$\Rightarrow 6\lambda - 24 \neq 0 \Rightarrow \lambda \neq 4$$

Q.33 (3)

By inspection, A' is a matrix of order 3×3 and B' is a matrix of order 3×2 . Therefore multiplication of these matrices is defined.

Q.34 (3)

$$A' = [1 \ 2 \ 3]$$

$$\text{Therefore } AA' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [1 \ 2 \ 3] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

Q.35 (2)

It is a fundamental concept

Q.36

$$(3) \begin{aligned} &= (A - A^T)^T = A^T - (A^T)^T \\ &= A^T - A \quad [\because (A^T)^T = A] = -(A - A^T) \end{aligned}$$

So, $A - A^T$ is a skew symmetric matrix

Q.37

(2) In A^{-1} the element of 2nd row and 3rd column is the c_{32} element of the matrix (c_{ij}) of cofactors of element of A , (due to transposition) divided by $\Delta = |A| = -2$

$$\therefore \text{ Required element} = \frac{(-1)^{3+2} M_{32}}{-2} = \frac{-(-2)}{-2} = -1,$$

$$\text{where } M_{32} = \text{minor of } c_{32} \text{ in } A = \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = 0 - 2 = -2$$

Q.38 (1)

Q.39 (1)

Q.40 (4)

Q.41 (3)

Q.42 (4)

Q.43 (2)

Q.44 (4)

Q.45 (4)

All the given statements are true.

Q.46 (1)

Since A is symmetric, therefore $A^T = A$.

$$\text{Now } (A^n)^T = (A^T)^n = (A)^n$$

$\therefore A^n$ is also a symmetric matrix.

Q.47 (1) Every skew symmetric matrix of odd order is singular. So option (1) is incorrect.

Q.48 (1) In a skew-symmetric matrix $a_{ij} = -a_{ji} \forall i, j = 1, 2, 3$

$$\text{for } j = i, a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0$$

\Rightarrow each .

$$\text{Hence the matrix } \begin{bmatrix} 0 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & 0 \end{bmatrix} \text{ is skew-symmetric.}$$

Q.49 (3)

If $A' = A$, then order of A' will be same to order of A . So it is a square matrix

Q.50 (4)

$$\text{Sol. Let } A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |A| = 1$$

$$\text{adj}(A) = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 1 & 1 \\ 7 & -2 & 1 \end{bmatrix}^T$$

Hence, $A^{-1} = \frac{\text{adj}(A)}{|A|} \Rightarrow$

$$A^{-1} = \begin{bmatrix} 1 & -2 & 7 \\ 2 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

Hence, element $z = 3$.

Q.51 (2)

$$\begin{vmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{vmatrix} \neq 0 \Rightarrow \lambda \neq -17$$

Q.52 (3)

As $I_3 I_3 = I_3$, therefore $I_3^{-1} = I_3$.

Q.53 (2)

The given matrix is a skew-symmetric matrix

$$[\because A' = -A]$$

Q.54 (1)

$$\begin{aligned} 3A^3 + 2A^2 + 5A + I &= 0 \Rightarrow I = -3A^3 - 2A^2 - 5A \\ \Rightarrow IA^{-1} &= -3A^2 - 2A - 5I \\ \Rightarrow A^{-1} &= -(3A^2 + 2A + 5I) \end{aligned}$$

Q.55 (1)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |A| = -1(1+0) = -1$$

$$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$\Rightarrow A_{11} = 0, A_{12} = -1, A_{13} = 0$$

$$A_{21} = -1, A_{22} = 0, A_{23} = 0$$

$$A_{31} = 0, A_{32} = 0, A_{33} = -1$$

$$\Rightarrow A^{-1} = \frac{\text{adj}(A)}{|A|} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A$$

Q.56 (c)

$$[M(\alpha)M(\beta)]^{-1} = M(\beta)^{-1}M(\alpha)^{-1}$$

$$\text{Now } M(\alpha)^{-1} = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ -\sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} = M(-\alpha)$$

$$M(-\beta)^{-1} = \begin{bmatrix} \cos(-\beta) & 0 & -\sin \beta \\ 0 & 0 & 0 \\ -\sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix} = M(-\beta)$$

$$= \begin{bmatrix} \cos(-\beta) & 0 & -\sin \beta \\ 0 & 0 & 0 \\ -\sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix} = M(-\beta)$$

$$[M(\alpha)M(\beta)]^{-1} = M(-\beta)M(-\alpha)$$

Q.57 (1)

Q.58 (4)

Q.59 (2)

Q.60 (2)

It is obvious.

Q.61 (2)

$$\text{Let } A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

Then, $A_{11} = 1, A_{12} = -2, A_{13} = -2$

$$A_{21} = -1, A_{22} = 3, A_{23} = 3$$

$$A_{31} = 0, A_{32} = -4, A_{33} = -3$$

$$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

Q.62 (1)

$$K = [|A|]^{-1} = \frac{-1}{6}$$

Q.63 (1)

$$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{|A|} \cdot \text{adj}(A)$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = -1 ; |A| = 0 - 1(1-9) + 2(1-6) = 8 - 10$$

$$|A| = -2 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A_{11} = (-1)^{1+1}[(2)(1) - (3)(1)] = -1$$

$$A_{12} = 8, A_{13} = -5, A_{21} = 1, A_{22} = -6$$

$$A_{23} = 3, A_{31} = -1, A_{32} = 2, A_{33} = -1,$$

$$\therefore A^{-1} = \frac{1}{-2} \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

Q.64 (1) $A^{-1} = A^2$, because $A^3 = I$.

Q.65 (4)

Since $A \cdot A = I$, therefore $A^{-1} = A$.

Q.66 (4)

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{vmatrix} = I[3] + I[6] + I[-4] = 5$$

$$B = \text{adj } A = \begin{bmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{bmatrix}$$

$$\text{adj } B = \begin{bmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{bmatrix} = 5A \quad \text{and } C = 5A$$

$$C = \text{adj } B; |C| = |\text{adj } B|; \frac{|\text{adj } B|}{|C|} = 1.$$

Q.67 (3)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$$

Let c_{ij} be co-factor of a_{ij} in A .

Then co-factor of elements of A are given by

$$C_{11} = \begin{vmatrix} 4 & 9 \\ 8 & 27 \end{vmatrix} = 36, C_{21} = \begin{vmatrix} 2 & 3 \\ 8 & 27 \end{vmatrix} = -30,$$

$$C_{31} = \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 6$$

$$C_{12} = \begin{vmatrix} 1 & 9 \\ 1 & 27 \end{vmatrix} = -18, C_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 27 \end{vmatrix} = 24, C_{32} = \begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = -6$$

$$C_{13} = \begin{vmatrix} 1 & 4 \\ 1 & 8 \end{vmatrix} = 4, C_{23} = \begin{vmatrix} 1 & 2 \\ 1 & 8 \end{vmatrix} = -6, C_{33} = \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 2$$

$$\Rightarrow |A| = \begin{vmatrix} 2 & \lambda & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{vmatrix}$$

\Rightarrow

$$|\text{Adj}(A)| = 36(48 - 36) + 30(-36 + 24) + 6(108 - 96)$$

$$\Rightarrow |\text{Adj}(A)| = 144$$

Q.68 (3)

$$|A| \cdot |\text{dj}(A)| = |A|^3$$

Q.69 (4)

$$\text{We know } A \text{adj}(A) = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$$\therefore |A| \cdot |\text{adj}(A)| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$$\therefore |A| \cdot |\text{adj}(A)| = |A|^3$$

Now question gives $|A| = 8$

$$\therefore 8 \cdot |\text{adj}(A)| = 8^3 \text{ or } |\text{adj}(A)| = 8^2 = (2^3)^2 = 2^6$$

Q.70 (1)

Since $A^2 = O$ (Zero matrix) and 2 is the least +ve integer for which $A^2 = O$.

Thus, A is nilpotent of index 2.

Q.71 (3)

$$A^2 = A \cdot A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

\therefore Matrix A is nilpotent of order 2

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (3)

It is a 12 elements matrices. Possible orders are 1×12 , 12×1 , 2×6 , 6×2 , 3×4 and 4×3 .

\therefore Number of possible orders is 6.

Q.2 (1)

$$\begin{bmatrix} x^2+x & x \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -x+1 & x \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} x^2+x & x-1 \\ -x+4 & x+2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x^2+x & x-1 \\ -x+4 & x+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$$

On comparing

$$x^2+x=0 \Rightarrow x=0, -1x-1=-2 \Rightarrow x=-1$$

$$-x+4=5 \Rightarrow x=-1, x+2=1 \Rightarrow x=-1$$

Hence the value of x is -1.

Q.3 (2)

Q.4 (4)

Matrix A has order (3×1) and Matrix B has order (3×3) .

So multiplication AB is not possible.

Q.5 (1)

$$AB = \begin{bmatrix} 3ax^2 & 3bx^2 & 3cx^2 \\ a & b & c \\ 6ax & 6bx & 6cx \end{bmatrix}$$

Now, $\text{tr}(AB) = \text{tr}(C)$

$$\Rightarrow 3ax^2 + b + 6cx = (x+2)^2 + 2x + 5x^2$$

$$3ax^2 + 6c + b = 6x^2 + 6x + 4$$

$$\Rightarrow a=2, b=4, c=1$$

$$\Rightarrow a+b+c=7.$$

Q.6 (1)

Q.7 (1)

Q.8 (3)

Q.9 (3)

Q.10 (2)

$A = \text{diag}(2, -1, 3), B = \text{diag}(-1, 3, 2)$ then $A^2B = ?$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}; B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{bmatrix}; A^2B = \begin{bmatrix} -4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 18 \end{bmatrix} = \text{diag}(-4,$$

3, 18)

Q.11 (1)

$$A^2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2^2 & 0 \\ 0 & 2^2 \end{bmatrix}, A^6 = \begin{bmatrix} 2^3 & 0 \\ 0 & 2^3 \end{bmatrix},$$

$$A^8 = \begin{bmatrix} 2^4 & 0 \\ 0 & 2^4 \end{bmatrix}$$

$$\therefore (A^8 + A^6 + A^4 + A^2 + I)V = \begin{bmatrix} 31 \\ 62 \end{bmatrix} \Rightarrow \begin{bmatrix} 31 & 0 \\ 0 & 31 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 31 \\ 62 \end{bmatrix}$$

$$\Rightarrow x=1, y=2$$

$$\therefore xy=2.$$

Q.12 (3)

Given $A^2 = A$.

Now

$$(I+A)^3 - 7A$$

$$= I^3 + 3I^2A + 3IA^2 + A^3 - 7A = I + 3A + 3A + A - 7A$$

$$= I + O = I$$

Q.13 (2)

We have,

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \alpha^2 + \beta\gamma - 1 = 0$$

Q.14 (1)

$$A^{N+1} = (B+C)^{N+1}$$

We can expand $(B+C)^{N+1}$ like binomial expansion as $BC=CB$.

$$\therefore (B+C)^{N+1} = {}^{N+1}C_0 B^{N+1} + {}^{N+1}C_1 B^N C + {}^{N+1}C_2 B^{N-1} C^2 + \dots + C^{N+1}.$$

$$= B^{N+1} + (N+1)B^N C + 0 + 0 + \dots + 0 = B^N (B + (N+1)C).$$

Q.15 (1)

$$A^2 - 2A + I = 0$$

$$\Rightarrow (A-I)^2 = 0$$

$$A^n = (A-I+I)^n = {}^nC_0 (A-I)^n + \dots + {}^nC_{n-2} (A-I)^2 \cdot I^n$$

$$= 0 + 0 + \dots + 0 + n(A-I) + I = nA - (n-1)I$$

Q.16 (3)

$$AB = B$$

Premultiply both sides by B

$$BAB = B^2 \Rightarrow AB = B^2 \Rightarrow B = B^2$$

Similarly

$$BA = A \Rightarrow ABA = A^2 \Rightarrow BA = A^2 \Rightarrow A = A^2$$

Q.17 (3)

For upper triangle matrix, elements below diagonal are zero

$\therefore a_{ij} = 0$, where $i > j$

Q.18 (1)

Q.19 (2)

Q.20 (3)

Trace of $A = a_{11} + a_{22} + a_{33}$
 For skew symmetric matrix $a_{11} = a_{22} = a_{33} = 0$
 Trace of $A = 0$

Q.21 (b)

$|A| = 0$ as the matrix A is singular

$$\therefore |A| = \begin{vmatrix} 1 & 3 & \lambda + 2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{vmatrix} = 0$$

Apply $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - 3R_1$
 abd exoabd,

$$-2(4 - 3\lambda) + 4(4 - 2\lambda) = 0$$

$$\Rightarrow 8 - 2\lambda = 0 \Rightarrow \lambda = 4$$

For $\lambda = 4$, the second and the third column are proportional.

Q.22 (4)

Q.23 (4)

Q.24 (3)

Q.25 (4)

Q.26 (1)

$$p^2 - q^2 = r; \quad p = 3 \quad q = 2, r = 5$$

Q.27 (2)

$$|3AB| = |A| \cdot |3B|_{3 \times 3} = (-1) \cdot 3^3 |B| = -81$$

Q.28 (1)

Let $a = \cos^{-1}x$, $b = \cos^{-1}y$, $c = \cos^{-1}z$,
 $|A| = 0$

$$\Rightarrow \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a + b + c)$$

$$\left[\frac{1}{2} \{ (a-b)^2 + (b-c)^2 + (c-a)^2 \} \right] = 0$$

$$\Rightarrow a + b + c = 0 \Rightarrow x = y = z = 1$$

Q.29 (1)

From given data $|A| = 2^4$
 $\Rightarrow |\text{adj}(\text{adj} A)| = (2^4)^2 = 2^{36}$

$$\Rightarrow \left\{ \frac{dt(\text{adj}(\text{adj}A))}{7} \right\} = \left\{ \frac{2^{36}}{7} \right\} = \left\{ \frac{(7+1)^{12}}{7} \right\} = \frac{1}{7}$$

Q.30 (b)

$$|A| = \begin{vmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 2 & 2 \end{vmatrix} = 16 \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

Q.31

(b)

We know that, $M(\text{adj} M) = |M| I$
 Replacing M by $\text{adj} M$, we get $\text{adj} M [\text{adj}(\text{adj} M)] = \det(\text{adj} M) I$
 $= \det(M) M^{-1} [\text{adj} M] = \alpha^2 I$

$$\left[\because M^{-1} = \frac{1}{|M|} \text{adj}(M) \right]$$

$$\Rightarrow \alpha M^{-1} [\text{adj}(\text{adj} M)] = \alpha^2 I$$

$$\Rightarrow M^{-1} [\text{adj}(\text{adj} M)] = \alpha I$$

$$\text{But } M^{-1} [\text{adj}(\text{adj} M)] = KI$$

$$\text{Hence, } K = \alpha$$

Q.32

(c)

$$\text{If } A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix} \text{ and } A^{-1} = \frac{1}{k} \text{adj}(A) \dots (i)$$

$$\text{Also, we know } A^{-1} = \frac{\text{adj}(A)}{|A|} \dots (ii)$$

\therefore By comparing (i) and (ii) $|A| = k$

$$\Rightarrow |A| = \begin{vmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 3(2+1) + 2(1+0) + 4(1-0) = 9 + 2 + 4 = 15$$

Q.33 (3)

Q.34 (3)

Q.35 (2)

Q.36 (4)

Q.37 (3)

Q.38 (4)

Q.39 (1)

Q.40 (2)

Q.41 (2)

Statement-3 is false as system of equation solution rest an obvious

Q.42

(a)

As A is an orthogonal matrix, $AA^T = I$

$$\Rightarrow \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \cdot \frac{1}{3} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{9} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-ab \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\begin{aligned} \Rightarrow a+4+2b=0, 2a+2-2b=0 \text{ and } a^2+4+b^2=9 \\ \Rightarrow a+2b+4=0, a-b+1=0 \text{ and } a^2+b^2=5 \\ \Rightarrow a=-2, b=-1 \end{aligned}$$

EXERCISE-III

Q.1 (0001)

$$A^T = \begin{bmatrix} -0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix}$$

$$A^T A = I \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2}$$

$$3z^2 = 1 \Rightarrow z^2 = \frac{1}{3}$$

$$6y^2 = 1 \Rightarrow y^2 = \frac{1}{6}$$

$$x^2 + y^2 + z^2 = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{1}{2} + \frac{1}{2} = 1$$

Q.2 0001

$$2X = (2X + Y) - Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$X = \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

Q.3 0014

$$X + Y + X - Y = 2X$$

$$\therefore 2X = (X + Y) + (X - Y)$$

$$= \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\therefore X = \frac{1}{2} \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Again,

$$2Y = (X + Y) + (X - Y)$$

$$= \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7-3 & 0-0 \\ 2-0 & 5-3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\therefore Y = \frac{1}{2} \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$3x - 4y = 3 \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} - 4 \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ -1 & 8 \end{bmatrix}$$

$$\text{Sum of element of } 3x - 4y = 7 + 0 - 1 + 8 = 14$$

Q.4 0007

$$\text{Given, } A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$$\therefore A^2 = AA = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$$

$$\text{Again, } 8A + kI = 8 \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$$

$$\therefore A^2 = 8A + kI \Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$$

$$\Rightarrow 1 = 8 + k \text{ and } 56 + k = 49 \Rightarrow k = -7$$

Q.5 0044

$$A^2 = 4A - 5I$$

$$A^3 = 11A - 20I$$

$$A^6 = A^3 \cdot A^3 = 44A - 205I$$

Q.6 2009

$$\text{Let } X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

$$\therefore X^1 \cdot A \cdot X = 0$$

$$\Rightarrow a_{11}X_1^2 + a_{22}X_2^2 + a_{33}X_3^2 + (a_{12} + a_{21})X_1X_2$$

$$+ (a_{13} + a_{31})X_1X_3 + (a_{23} + a_{32})X_2X_3 = 0$$

$$\text{This is true } X_1$$

$$a_{11} = a_{22} = a_{33} = 0$$

$$a_{12} + a_{21} = 0$$

$$a_{13} + a_{31} = 0$$

$$a_{23} + a_{32} = 0$$

Q.7 (-4)

A is symmetric $\Rightarrow A^T = A$

$$\Rightarrow \begin{bmatrix} 3 & 2x+3 \\ x-1 & x+2 \end{bmatrix} = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$$

$$\Rightarrow x-1 = 2x+3 \Rightarrow x = -4$$

Q.8 5

$$10A^{-1} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & k \\ 1 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow 10A^{-1} \cdot A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & k \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow -5 + k = 0$$

$$\Rightarrow k = 5 \quad (\because \text{comparing } a_{21} \text{ element on both sides})$$

Q.9 25

$$|A(\text{adj } A)| = |A|^2 \quad (\text{order is } 2)$$

$$|A(\text{adj } A)| = 5^2$$

$$|A(\text{adj } A)| = 25$$

Q.10 0.5

$$\text{def}(\text{matrix}) = 0$$

$$\left(\sin \frac{\pi}{2}\right)(2k) - \left(\cos \frac{\pi}{3}\right)\left(2 \tan \frac{\pi}{4}\right) = 0$$

$$2k - \frac{1}{2}(2) = 0$$

$$2k = 1$$

$$k = \frac{1}{2}$$

PREVIOUS YEAR'S

MHT CET

Q.1 (3)

Q.2 (3)

Q.3 (1)

Q.4 (4)

Q.5 (1)

Q.6 (2)

Q.7 (1)

Q.8 (2)

Q.9 (2)

Q.10 (1)

Q.11 (3)

Q.12 (2)

Q.13 (3)

Q.14 (2)

Q.15 (4)

Q.16 (4)

Q.17 (1)

Q.18 (1)

Q.19 (1)

Q.20 (4)

Q.21 (3)

Q.22 (2)

Q.23 (3)

Q.24 (1)

Q.25 (2)

Q.26 (4)

Q.27 (3)

Q.28 (1)

$$\text{Given, } A(\text{adj } A) = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$$

$$20 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 20I$$

We know that

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\Rightarrow A(\text{adj } A) = |A|I$$

$$\Rightarrow 20I = |A|I$$

$$\Rightarrow |A| = 20$$

Q.29 (1)

$$\text{Given, } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 1 + (-1) \times 2 & 1 \times (-1) + (-1) \times 3 \\ 2 \times 1 + 3 \times 2 & 2 \times (-1) + 3 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1-3 \\ 2+6 & -2+9 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$$

Now, $A^2 - 4A + 5I$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - 4 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4+5 & -4+4+0 \\ 8-8+0 & 7-12+5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Q.30 (3)

We have, $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Now, $A^2 = A \cdot A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
 $= \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

and $A^3 = A^2 \cdot A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
 $= \begin{bmatrix} 2+2 & -2-2 \\ -2-2 & 2+2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$

$\therefore A^3 = 4A$

Q.31 (1)

$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \text{ adj } A = AA^T$

$\therefore \text{adj } A = \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix}$

Now, $AA^T = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$
 $= \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$

and $A \cdot \text{adj } A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix}$
 $= \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$

$\therefore A (\text{adj } A) = AA^T$ is given, so equating the two expressions, we get

$$\begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix} = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

we have, $10a + 3b = 13$ and $15a - 2b = 0$

On solving, we get

$$a = \frac{2}{5} \text{ and } b = 3$$

$$\Rightarrow 5a + b = 5 \times \frac{2}{5} + 3$$

$$\Rightarrow 5a + b = 2 + 3$$

$$\Rightarrow 5a + b = 5$$

Q.32 (2)

Given, $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

Also given, $A + A^T = \sqrt{2}I_2$

$$\therefore \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}^T$$

$$= \sqrt{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} \cos \alpha + \cos \alpha & \sin \alpha - \sin \alpha \\ -\sin \alpha + \sin \alpha & \cos \alpha + \cos \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$$

On equating the corresponding elements, we get

$$2 \cos \alpha = \sqrt{2}$$

$$\Rightarrow \cos \alpha = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{2}}$$

$\therefore \alpha = \frac{\pi}{4}$, which is satisfying $0 < \alpha < \frac{\pi}{2}$.

Q.33 (2)

Here, $(AB)(AB) = A(A^2B)B = A^3B^2$

$$\text{Now, } (AB)(AB)(AB) = (A^2B^2)AB = (A^3B^2AB)A^3B(BA)B = A^3B(A^2B)B = A^5BAB^2 = A^5 \cdot A^2B \cdot B^2 = A^7 \cdot B^3$$

So, $(AB)^n = A^{2^{n-1}} \cdot B^{10}$

$$\Rightarrow k = 2^{10} - 1 = 1023$$

Q.34 (4)

Here, $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

Now, transpose of A

$$A^t = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$A^t \cdot A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \cos \alpha \cdot \sin \alpha - \sin \alpha \cdot \cos \alpha \\ \cos \alpha \cdot \sin \alpha - \sin \alpha \cdot \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

So, A is an orthogonal matrix.

JEE-MAIN

Q.1 (1)

$$\Rightarrow A = \begin{bmatrix} 1 & 2 & 2^2 \\ 1/2 & 1 & 2 \\ 1/2^2 & 1/2 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 2 & 4 \\ 1/2 & 1 & 2 \\ 1/4 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 1/2 & 1 & 2 \\ 1/4 & 1/2 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 & 6 & 12 \\ 3/2 & 3 & 6 \\ 3/4 & 3/2 & 3 \end{bmatrix} = 3A$$

$$\begin{aligned} \Rightarrow A^3 &= 3^2 A \\ \Rightarrow A^2 + A^3 + A^4 + \dots + A^{10} \\ \Rightarrow 3A + 3^2 A + \dots + 3^9 A \end{aligned}$$

$$\Rightarrow 3 \left[\frac{3^9 - 1}{3 - 1} \right] A$$

$$\Rightarrow \left[\frac{3^{10} - 3}{2} \right] A$$

Q.2 (3)

$$A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix} \Rightarrow \text{adj}(A) = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$$

$$B = I^{-5} C_1 \text{adj}(A) + {}^5 C_2 (\text{adj}(A))^2 \dots {}^5 C_5 (\text{adj}(A))^5$$

$$= {}^5 C_0 (\text{adj}(A))^0 - {}^5 C_1 \text{adj}(A) + {}^5 C_2 (\text{adj}(A))^2 \dots {}^5 C_5 (\text{adj}(A))^5$$

$$= {}^5 C_0 (\text{adj}(A))^0 - {}^5 C_1 (\text{adj}(A))^1 \cdot I^4 + \dots - {}^5 C_5 (\text{adj}(A))^5 I$$

$$B = (I - \text{adj}(A))^5$$

$$I - \text{adj}(A) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix}$$

$$(I - \text{adj}(A))^2 = \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$(I - \text{adj}(A))^4 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$$

$$(I - \text{adj}(A))^5 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & -5 \\ 0 & -1 \end{pmatrix}$$

$$\therefore B = \begin{pmatrix} -1 & -5 \\ 0 & -1 \end{pmatrix}$$

Sum of elements of B = -1 - 5 + 0 - 1 = -7

Q.3

(1)

$$M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix} = \begin{bmatrix} -\alpha^2 & 0 \\ 0 & -\alpha^2 \end{bmatrix} = -\alpha^2 I$$

$$\Rightarrow (I - M^2) = (1 + \alpha^2)I$$

$$N = \sum_{k=1}^{49} M_2^k$$

$$= \sum_{k=1}^{49} (-\alpha^2 I)^k$$

$$= \sum_{k=1}^{49} (-\alpha^2)^k I$$

$$= -\alpha^2 \left(\frac{(-\alpha^2)^{49} - 1}{-\alpha^2 - 1} \right)$$

$$(1 + \alpha^2)N = (-\alpha^2(\alpha^{98} + 1))$$

$$\Rightarrow 2I = \alpha^2(\alpha^{98} + 1)I$$

$$\Rightarrow 2 = \alpha^2(\alpha^{98} + 1)$$

Only possible when $\alpha = 1$

Q.4

(2)

$$\begin{aligned} \det(\text{adj } A) &= |A|^{n-1} \\ &= |A|^2 \quad (n=3, \text{ order}) \end{aligned}$$

$$|A| = \begin{vmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{vmatrix} = (1 + a^2)$$

$$\text{Now } \sum_{a \in S} \det(\text{adj } A) = \sum (1 + a^2)^2$$

$$= 2^2 + 4^2 + 6^2 + 8^2 + 10^2 + \dots + 50^2 = 2^2[1^2 + 2^2 + 3^2 + \dots + 25^2]$$

$$= 4 \left(\frac{n(n+1)(2n+1)}{6} \right)$$

$$= \frac{2}{3} (25 \times 26 \times 51)$$

$$= 22100 = 100\lambda, \quad \lambda = 221$$

Q.5

(100)

$$A^2 = I$$

$$\begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix} \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -a+ab \\ 0 & b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$b^2 = 1 \Rightarrow b = \pm 1$$

$$-a + ab = 0$$

$$a(-1 + b) = 0$$

$$b = 1 \text{ and } a \text{ can take any value}$$

$$\therefore \text{Total} = 100 \text{ elements}$$

Q.6 [100]

$$X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$X^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix} + \begin{bmatrix} 0 & \beta & 0 \\ 0 & 0 & \beta \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \gamma \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}$$

$$YY^{-1} = I$$

$$\Rightarrow \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} \frac{1}{\alpha} & \frac{-\beta}{\alpha} & \frac{-\gamma}{\alpha} \\ 0 & \frac{1}{\alpha} & \frac{-\beta}{\alpha} \\ 0 & 0 & \frac{1}{\alpha} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\alpha}{5} & \frac{-2\alpha+\beta}{5} & \frac{\alpha-2\beta+\gamma}{5} \\ 0 & \frac{\alpha}{5} & \frac{-2\alpha+\beta}{5} \\ 0 & 0 & \frac{\alpha}{5} \end{bmatrix}$$

\Rightarrow Comparing every elements

$$\alpha = 5, \beta = 10, \gamma = 15$$

$$(\alpha - \beta + \gamma)^2 = (5 - 10 + 15)^2 = 100$$

Q.7 (180)

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; a, b, c, d \in \{0, 1, 2, 3, 4, 5\}$$

$$a + b + c + d = p, p \in \{3, 5, 7\}$$

Case-(i)

$$a + b + c + d = 3; a, b, c, d \in \{0, 1, 2, 3\}$$

$$\text{No. of ways} = {}^{3+4-1}C_{4-1} = 20 \quad \dots (1)$$

Case-(ii)

$$a + b + c + d = 5; a, b, c, d \in \{0, 1, 2, 3, 4, 5\}$$

$$\text{No. of ways} = {}^{5+4-1}C_{4-1} = {}^8C_3 = 56 \quad \dots (2)$$

Case-(iii)

$$a + b + c + d = 7$$

$$\text{No. of ways} = \text{total ways when } a, b, c, d \in \{0, 1, 2, 3, 4, 5, 6, 7\} - \text{total ways when } a, b, c, d \notin \{6, 7\}$$

$$\text{No. of ways} = {}^{7+4-1}C_{4-1} = \binom{10}{3} + \binom{10}{2}$$

$$= {}^{10}C_3 - 16 = 104 \quad \dots (3)$$

$$\text{Hence total number of ways} = 180$$

(2)

Q.8

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11} + a_{12} \\ a_{21} + a_{22} \\ a_{31} + a_{32} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} a_{11} + a_{12} = 1 \\ a_{21} + a_{22} = 1 \\ a_{31} + a_{32} = 0 \end{cases}$$

$$A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{cases} a_{11} + a_{13} = -1 \\ a_{21} + a_{23} = 0 \\ a_{31} + a_{33} = 1 \end{cases}$$

$$A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \Rightarrow \begin{cases} a_{13} = 1 \\ a_{23} = 1 \\ a_{33} = 2 \end{cases}$$

Solving all the equations

$$a_{11} = -2, a_{12} = 3$$

$$a_{31} = -1, a_{32} = 1$$

$$a_{21} = -1, a_{22} = 2$$

$$A = \begin{bmatrix} -2 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

$$A - 2I = \begin{bmatrix} -4 & 3 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$(A - 2I) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{cases} -4x_1 + 3x_2 + x_3 = 4 \\ -x_1 + x_3 = 1 \\ -x_1 + x_2 = 1 \end{cases}$$

$$\Delta = -1(3) - 1(-4 + 1) = 0$$

$$= -3 + 3 = 0$$

$$\Delta_1 = \begin{vmatrix} 4 & 3 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 1(3) - 1(4-1) = 0$$

$$\Delta_2 = \begin{vmatrix} -4 & 4 & 1 \\ -1 & 1 & 1 \\ -1 & 1 & 0 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} -4 & 3 & 4 \\ -1 & 0 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 0$$

∴ Infinite solutions Ans.

Q.9 (1)

$$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 2 & -4 \end{bmatrix} = -4I$$

$$A^3 = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ -8 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 0 & 8 \\ -8 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$$

$$\begin{aligned} \therefore M &= \sum_{k=1}^{10} A^{2k} = A^2 + A^4 + \dots + A^{20} \\ &= -4I + 16I - 2^6I + \dots \text{ up to 10 terms} \\ &= -4I \{ 1 - 4 + 4^2 + \dots \text{ up to 10 terms} \} \end{aligned}$$

$$= -4I \left\{ \frac{(-4)^{10} - 1}{-4 - 1} \right\} = \frac{4I}{5} \{ 4^{10} - 1 \} = \lambda I$$

$$M = \lambda I, \lambda \in \mathbb{R} \text{ v}$$

$$N = \sum_{k=1}^{10} A^{2k-1} = A + A^3 + \dots \text{ up to 10 terms} \\ = A - 4A + 16A + \dots \text{ up to 10 terms}$$

$$= A \left\{ \frac{(-4)^{10} - 1}{-5} \right\} = -\frac{A}{5} \{ 4^{10} - 1 \} = \mu A$$

$$N = \mu A$$

$$MN^2 = (kA^2) = -4kI \Rightarrow \text{symmetric matrix} \quad \text{Ans.}$$

Q.10 (3)

Given that $A^T = A, B^T = -B$

$$(1) C = A^4 - B^4$$

$$C^T = (A^4 - B^4)^T = (A^4)^T - (B^4)^T = A^4 - B^4 = C$$

$$(2) C = AB - BA$$

$$C^T = (AB - BA)^T = (AB)^T - (BA)^T$$

$$= B^T A^T - A^T B^T = -BA + AB = C$$

$$(3) C = B^5 - A^5$$

$$C^T = (B^5 - A^5)^T = (B^5)^T - (A^5)^T = -B^5 - A^5$$

$$(4) C = AB + BA$$

$$C^T = (AB + BA)^T = (AB)^T + (BA)^T$$

$$= -BA - AB = -C$$

∴ Option 3 is not true

Q.11 (3)

Point on plane is $(3, -4, 7)$

$$n_p = \vec{V}_{L1} \times (\vec{V}_{L2} \times \vec{V}_{L3})$$

$$= (9\hat{i} - \hat{j} - 5\hat{k}) \times ((2\hat{i} + 3\hat{j} + 5\hat{k}) \times (3\hat{i} + 7\hat{j} + 8\hat{k}))$$

$$= \langle 9, -1, -5 \rangle \times \langle -11, -1, 5 \rangle$$

$$\vec{n}_p = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 9 & -1 & -5 \\ -11 & -1 & 5 \end{vmatrix} = \langle -10, 10, -20 \rangle$$

$$\vec{n}_p = \langle 1, -1, 2 \rangle$$

$$\text{Eq. of Plane: } 1(x-3) - 1(y+4) + 2(z-7) = 0$$

$$P: x - y + 2z = 21$$

Now d of $(2, -5, 11)$ from plane

$$d = \left(\frac{2+5+22-21}{\sqrt{1+1+4}} \right)$$

$$d = \left(\frac{8}{\sqrt{6}} \right)$$

$$d = \frac{64}{6} = \frac{32}{3}$$

Q.12 (17)

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= B + I = \underbrace{\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}}_B + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_I$$

$$A^2 = (B + I)^2 = B^2 + 2B + I$$

$$B^2 = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

$$A^2 = B + I = A \quad B^2 = \begin{bmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} = -B$$

$$A^3 = B + I = A \quad B^3 = -B^2 = B$$

$$| \quad B^4 = -B$$

$$A^n = A \quad B^5 = B$$

$$\Rightarrow A^n + (WB)^n = A + W^n B^n = A + B$$

$$\Rightarrow n \text{ must be odd \& mult. of 3}$$

$$\Rightarrow n \in \{3, 9, 15, 21, \dots, 99\}$$

Total value of n = 17 Ans.

Q.13 (3)

$$A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

(1) $R_1 \rightarrow R_1 + R_2; \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ possible

(2) $R_1 \leftrightarrow R_2; \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ possible

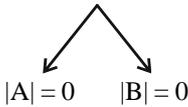
(3) Option is not possible

(4) $R_2 \rightarrow R_1 + 2R_1; \begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ possible

Q.14 (2)

$$AB = 0 \Rightarrow |AB| = 0$$

$$|A||B| = 0$$



If $|A| \neq 0, B = 0$ (not possible)

If $|B| \neq 0, A = 0$ (not possible)

Hence, $|A| = |B| = 0$

$\Rightarrow AX = 0$ has infinitely many solutions

Q.15 (282)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}; a_{ij} \in \{0, 1\}$$

$$\sum a_{ij} = 2, 3, 5, 7$$

$$\text{Total matrix} = {}^9C_2 + {}^9C_3 + {}^9C_5 + {}^9C_7 = 282$$

Q.16 (3)

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$C_1 \leftrightarrow C_3$$

$$- \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$B_0 = A^{49} + 2A^{98}$$

$$= A + 2I$$

$$B_n = \text{Adj.}(B_{n-1})$$

$$B_4 = \text{Adj}(\text{Adj}(\text{Adj}(B_0)))$$

$$= |B_0|^{(n-1)^4}$$

$$= |B_0|^{16}$$

$$B_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= [0 \ 2 \ 1]$$

$$= 2(4-0) - 1(0-1)$$

$$= 9$$

$$B_4(9)^{16} = (3)^{32}$$

Q.17 (2)

$$A + B = \begin{bmatrix} \beta+1 & 0 \\ 3 & \alpha \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} \beta+1 & 0 \\ 3 & \alpha \end{bmatrix} \begin{bmatrix} \beta+1 & 0 \\ 3 & \alpha \end{bmatrix}$$

$$= \begin{bmatrix} (\beta+1)^2 & 0 \\ 3(\beta+1) + 3\alpha & \alpha^2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1-\alpha \\ 2+2\alpha & \alpha^2-2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & -\alpha+1 \\ 2\alpha+4 & \alpha^2 \end{bmatrix} = \begin{bmatrix} (\beta+1)^2 & 0 \\ 3(\alpha+\beta+1) & \alpha^2 \end{bmatrix}$$

$$\alpha = 1 = \alpha_1$$

$$B^2 = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \beta^2+1 & \beta \\ \beta & 1 \end{bmatrix} = \begin{bmatrix} (\beta^2+1) & 0 \\ 3(\beta+1) + 3\alpha & \alpha^2 \end{bmatrix}$$

$$\therefore \beta = 0, \alpha = -1 = \alpha_2$$

$$|\alpha_1 - \alpha_2| = |1 - (-1)| = 2$$

Q.18 (4)

Given $A = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}$

Now $\alpha A^2 + \beta A = 2I$

$$\alpha \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} + \beta \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\alpha \begin{bmatrix} 1-4 & 2-10 \\ -2+10 & -4+25 \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -2\beta & -5\beta \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -3\alpha & -8\alpha \\ 8\alpha & 21\alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -2\beta & -5\beta \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$-3\alpha + \beta = 2 \dots\dots(1)$$

$$-8\alpha + 2\beta = 0$$

$$4\alpha = \beta \dots\dots(2)$$

(1) & (2)

$$-3\alpha + 4\alpha = 2$$

$$\alpha = 2$$

α in (2)

$$4 \times 2 = \beta$$

Now, $\alpha + \beta = 8 + 2$

$$\alpha + \beta = 10$$

Q.19 [5376]

$$A = [a_{ij}]_{3 \times 3}$$

$$\text{tr}(AA^T) = 6$$

$$a_{11}^2 + a_{12}^2 + a_{13}^2 + \dots + a_{33}^2 = 6$$

So out of 9 elements, 6 must be equal to 1 or -1 and rest elements must be 0 Possible cases

$$3(0\text{'s}) \& 6(1\text{'s}) \Rightarrow \text{Total case} = {}^9C_3$$

$$3(0\text{'s}) \& 6(-1\text{'s}) \Rightarrow \text{Total case} = {}^9C_3$$

$$3(0\text{'s}) \& 3(-1\text{'s}) \& 3(1\text{'s}) \Rightarrow \text{Total case} = {}^9C_3 \times {}^6C_3 \times {}^3C_3$$

$$3(0\text{'s}) \& 4(-1\text{'s}) \& 2(1\text{'s}) \Rightarrow \text{Total case} = {}^9C_3 \times {}^6C_4 \times {}^2C_2$$

$$3(0\text{'s}) \& 5(-1\text{'s}) \& 1(1\text{'s}) = {}^9C_3 \times {}^6C_5 \times {}^1C_1$$

$$3(0\text{'s}) \& 2(-1\text{'s}) \& 4(1\text{'s}) = {}^9C_3 \times {}^6C_2 \times {}^4C_4$$

$$3(0\text{'s}) \& 1(-1\text{'s}) \& 5(1\text{'s}) = {}^9C_3 \times {}^6C_1 \times {}^5C_5$$

$$\text{Total No. of matrices} = {}^9C_3 [1 + 1 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_2 + {}^6C_1]$$

$$= {}^9C_3 [2 + 20 + 15 + 6 + 15 + 6]$$

$$= 5376$$

Q.20 (4)

$$A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, B = \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$$

$$A'BA = [111] \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$[111] \begin{bmatrix} 81 & -100 & +121 \\ 144 & +169 & -196 \\ -225 & +256 & +289 \end{bmatrix} \Rightarrow [111] \begin{bmatrix} 102 \\ 117 \\ 320 \end{bmatrix}$$

$$\Rightarrow 102 + 117 + 320 \Rightarrow 539$$

Q.21 (3)

$$\beta = \lim_{x \rightarrow 0} \frac{ax - (e^{3x} - 1)}{ax(e^{3x} - 1)} \Rightarrow \lim_{x \rightarrow 0} \frac{ax - (e^{3x} - 1)}{3ax^2}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{ax + 1 - \left\{ 1 + 3x + \frac{(3x)^2}{2} \right\}}{3ax^2}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{x(\alpha - 3) - \frac{9x^2}{2}}{3\alpha x^2}$$

$$\Rightarrow \alpha - 3 = 0 \Rightarrow \alpha = 3$$

$$\& \beta = \lim_{x \rightarrow 0} \frac{-9x^2}{2(9x^2)} = -\frac{1}{2}$$

$$\Rightarrow \alpha + \beta = 3 - \frac{1}{2} = \frac{5}{2}$$

Q.22 [50]

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Given $A = A^{-1}$

$$\therefore A^2 = A \cdot A^{-1} = I$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore a^2 + bc = 1 \dots\dots(1)$$

$$ab + bd = 0 \dots\dots(2)$$

$$ac + cd = 0 \dots\dots(3)$$

$$bc + d^2 = 1 \dots\dots(4)$$

(1) - (4) gives

$$a^2 - d^2 = 0$$

$$\Rightarrow (a+d) = 0 \text{ or } a - d = 0$$

Case - I

$$a + d = 0 \Rightarrow (a,d) = (-1,1), (0,0), (1,-1)$$

(i) $(a,d) = (-1,1)$

\therefore from equation (1)

$$1 + bc = 1 \Rightarrow bc = 0$$

$b = 0$, $C = 12$ possibilities

$c = 0$, $b = 12$ possibilities

but $(0,0)$ is repeated

$$\therefore 2 \times 12 = 24$$

24 - 1 (repeated) = 23 pairs
 (ii) (a,d) = (1,-1) ⇒ bc = 0 → 23 pairs
 (iii) (a,d) = (0,0) ⇒ bc = 1
 ⇒ (b,c) = (1,1) & (-1, -1), 2 pairs

Case - II

a = d
 from (2) and (3)
 a ≠ 0 then b = c = 0
 a² = 1
 a = ±1 = d
 (a,d) = (1,1), (-1,-1) → 2 pairs
 ∴ Total = 23 + 23 + 2 + 2 = 50 pairs

Q.23

(2)
 |A| = -1
 |(A + I)(adj(A) + I)| = 4
 ⇒ |AadjA + A + adjA + I| = 4
 ∴ AadjA = |A|I ⇒ |-I + A + adjA + I| = 4

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = 4$$

$$\begin{bmatrix} a+d & 0 \\ 0 & a+d \end{bmatrix} = 4$$

(a + d) = ±2
 a + d = 2

Q.24

[2]
 $A = \begin{bmatrix} 4 & -2 \\ \alpha & \beta \end{bmatrix}$, |A| = 4β + 2α
 Characteristic equation |A - λI| = 0
 Or A² - (trace A)A + |A|I = 0
 ⇒ |A| = 18

Q.25

[42]
 $A = \begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ B+\gamma & \gamma+\alpha & \alpha+\beta \end{bmatrix}$ (∵ α-β, β-γ, γ-α)

$$A = (\alpha + \beta + \gamma)(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$$

$$\Rightarrow \frac{dt(\text{Adj}(\text{Adj}(\text{Adj}(a\text{Adj}A)))}{(\alpha - \beta)^{16}(\beta - \gamma)^{16}(\gamma - \alpha)^{16}} = 2^{32} \times 3^{16} \dots (1)$$

$$\Rightarrow \frac{|A|^{(n-1)^4}}{(\alpha - \beta)^{16}(\beta - \gamma)^{16}(\gamma - \alpha)^{16}} = \frac{|A|^{16}}{(\alpha - \beta)^{16}(\beta - \gamma)^{16}(\gamma - \alpha)^{16}} = 2^{32} \times 3^{16}$$

α + β + γ = 12
 Total number of solution = ¹²⁻¹C₃₋₁ = ¹¹C₂ = 55
 But α, β, γ has to be distinct
 Total = 55 - ((all are equal) + exactly equal)
 = 55 - (1 + 12) = 42
 Note : 2 are equal

1	1	10	=	$\frac{3!}{2!}$	=	3
2	2	6	=	3		
3	3	6	=	3		
5	5	2	=	<u>3</u>		

Q.26 [24]

Let $\begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = B$

So, $A = I_3 + B$ & $B^2 = \begin{bmatrix} 0 & 0 & ab \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

∴ B³ = 0 (Null Matrix)

$$A^n = (I + B)^n$$

$$= I + {}^nC_1 \cdot B + {}^nC_2 \cdot B^2$$

$$= I + n \cdot B + \frac{n(n-1)}{2} B^2$$

$$= \begin{bmatrix} 1 & na & na + \frac{n(n-1)}{2} ab \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{bmatrix}$$

∴ na = 48, nb = 96

$$\therefore \frac{na \cdot (nb - b)}{2} = 2160 - na$$

$$\therefore \frac{(48)(96 - b)}{2} = 2112$$

∴ 96 - b = 88

∴ b = 8, a = 4, n = 12

n + a + b = 24

Q.27 (25)

$$A = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix} \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

A⁴ⁿ⁺¹ = A

n = 1, 5, 9, , 97

⇒ total elements in the set is 25.

DETERMINANTS

EXERCISE-I (MHT CET LEVEL)

Q.1 (2)

$$\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix} = \begin{vmatrix} 0 & b-c & c-a \\ 0 & y-z & z-x \\ 0 & q-r & r-p \end{vmatrix} = 0$$

[by $C_1 \rightarrow C_1 + C_2 + C_3$]

Q.2 (3)

$$\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix} = 1 \begin{vmatrix} 3 & 6 \\ -7 & 9 \end{vmatrix} - 2 \begin{vmatrix} -4 & 6 \\ 2 & 9 \end{vmatrix} + 3 \begin{vmatrix} -4 & 3 \\ 2 & -7 \end{vmatrix}$$

$$= 1(3 \times 9 - 6(-7)) - 2(-4 \times 9 - 2 \times 6) + 3$$

$$((-4)(-7) - 3 \times 2)$$

$$= (27 + 42) - 2(-36 - 12)$$

$$+ 3(28 - 6) = 231$$

Q.3 (4)

$$\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$$

$$= 6i [3i^2 + 3] + 3i [4i + 20] + 1 [12 - 60i]$$

$$= 6i [-3 + 3] + 12i^2 + 60i + 12 - 60i$$

$$= -12 + 12 = 0 = x + iy$$

$$\therefore x = 0$$

Q.4 (4)

Q.5 (3)

Q.6 (3)

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}, \text{ by } \begin{cases} R_1 \rightarrow R_1 - R_2 \\ R_2 \rightarrow R_2 - R_3 \end{cases}$$

$$= (a-b)(b-c) \begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a-b)(b-c) \begin{vmatrix} 0 & 0 & a-c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix} \text{ by } R_1 \rightarrow R_1 - R_2$$

$$= (a-b)(b-c)(a-c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a-b)(b-c)(a-c) \cdot (-1) = (a-b)(b-c)(c-a)$$

Q.7 (2)

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix}$$

$$= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix} = 0$$

Q.8 (4)

by $C_1 \rightarrow C_1 + C_2 + C_3$

$$\text{we have } (9+x) \begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow (x+9) \begin{vmatrix} 0 & 1-x & 0 \\ 0 & -(1-x) & 1-x \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

Q.9 (4)

$$\begin{vmatrix} a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \end{vmatrix} = \begin{vmatrix} a+b & a+2b & a+3b \\ b & b & b \\ 2b & 2b & 2b \end{vmatrix} = 0$$

$$\left\{ \begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_2 \end{array} \right\}$$

Trick: Putting $a = 1 = b$. The determinant will be

$$\begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{vmatrix} = 0. \text{ Obviously answer is (d)}$$

Note : Students remember while taking the values of a, b, c,..... that for these values, the options (a), (b), (c) and (d) should not be identical.

Q.10

(2)

The cofactor of element 4, in the 2nd row and 3rd column is

$$= (-1)^{2+3} \begin{vmatrix} 1 & 3 & 1 \\ 8 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = -\{1(-2) - 3(8-0) + 1.16\} = 10.$$

Q.11

(2)

We know that

$$\Delta \Delta' = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$$

$$= \begin{vmatrix} \Sigma a_1 A_1 & 0 & 0 \\ 0 & \Sigma a_2 A_2 & 0 \\ 0 & 0 & \Sigma a_3 A_3 \end{vmatrix} = \begin{vmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{vmatrix} = \Delta^3$$

$$\Rightarrow \Delta' = \Delta^2$$

Q.12

(4)

It is a fundamental concept.

Q.13

(3)

$$C_{21} = (-1)^{2+1}(18+21) = -39$$

$$C_{22} = (-1)^{2+2}(15+12) = 27$$

$$C_{23} = (-1)^{2+3}(-35+24) = 11$$

Q.14

(2)

$$\text{Minor of } -4 = \begin{vmatrix} -2 & 3 \\ 8 & 9 \end{vmatrix} = -42, 9 = \begin{vmatrix} -1 & -2 \\ -4 & -5 \end{vmatrix} = -3$$

and cofactor of $-4 = (-1)^{2+1}(-42) = 42$,

$$\text{cofactor of } 9 = (-1)^{3+3}(-3) = -3.$$

Q.15

(1)

$$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ab \end{vmatrix} = \begin{vmatrix} 0 & a-b & (a-b)(a+b+c) \\ 0 & b-c & (b-c)(a+b+c) \\ 1 & c & c^2 - ab \end{vmatrix}$$

$$\text{by } \begin{cases} R_1 \rightarrow R_1 - R_2 \\ R_2 \rightarrow R_2 - R_3 \end{cases}$$

$$= (a-b)(b-c) \begin{vmatrix} 0 & 1 & a+b+c \\ 0 & 1 & a+b+c \\ 1 & c & c^2 - ab \end{vmatrix} = 0, \{ \because R_1 \equiv R_2 \}$$

Q.16

(d)

Applying $C_1 - C_2$ and $C_2 - C_3$, we get

$$\text{Det.} = \begin{vmatrix} 25 & 21 & 219 \\ 15 & 27 & 198 \\ 21 & 17 & 181 \end{vmatrix} = \begin{vmatrix} 4 & 21 & 9 \\ -12 & 27 & -72 \\ 4 & 17 & 11 \end{vmatrix}$$

(by $C_1 - C_2, C_3 - 10C_2$)

$$= \begin{vmatrix} 4 & 21 & 9 \\ 0 & 90 & -45 \\ 0 & -4 & 2 \end{vmatrix} \text{ [By } R_2 + 3R_1, R_3 - R_1]$$

$$= 4(180 - 180) = 0$$

Q.17

(2)

Q.18

(2)

Q.19

(3)

Q.20

(1)

Q.21

(3)

$$(3) \Delta = \begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}$$

$$= -2 \begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}, \text{ by } R_1 \rightarrow R_1 - (R_2 + R_3)$$

$$= -2 \begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & a^2 & 0 \\ c^2 & 0 & a^2 \end{vmatrix}, \text{ by } \begin{cases} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{cases}$$

$$= -2\{-c^2(b^2 a^2) + b^2(-c^2 a^2)\} = 4a^2 b^2 c^2.$$

Trick: Put $a = 1, b = 2, c = 3$ so that the option give different values.

Q.22

(4)

$$D_1 = \begin{vmatrix} 1 & 15 & 8 \\ 1 & 35 & 9 \\ 1 & 25 & 10 \end{vmatrix}, D_2 = \begin{vmatrix} 2 & 15 & 8 \\ 4 & 35 & 9 \\ 8 & 25 & 10 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} 3 & 15 & 8 \\ 9 & 35 & 9 \\ 27 & 25 & 10 \end{vmatrix}, D_4 = \begin{vmatrix} 4 & 15 & 8 \\ 16 & 35 & 9 \\ 64 & 25 & 10 \end{vmatrix}$$

$$D_5 = \begin{vmatrix} 5 & 15 & 8 \\ 25 & 35 & 9 \\ 125 & 25 & 10 \end{vmatrix}$$

$$\Rightarrow D_1 + D_2 + D_3 + D_4 + D_5 = \begin{vmatrix} 15 & 75 & 40 \\ 55 & 175 & 45 \\ 225 & 125 & 50 \end{vmatrix}$$

$$= 15(3125) - 75(-7375) + 40(-32500) \\ = 46875 + 553125 - 1300000 = -700000$$

Q.23 (2)

$$\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z) \begin{vmatrix} 2 & 1 & 1 \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

by $R_1 \rightarrow R_1 + R_2 + R_3$

$$= (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ x & z & x \\ x & y & z \end{vmatrix}; \text{ by } C_1 \rightarrow C_1 - C_2$$

$$= (x+y+z) \cdot \{(z^2 - xy) - (xz - x^2) + (xy - xz)\}$$

$$= (x+y+z)(x-z)^2 \Rightarrow k = 1$$

Trick : Put $x = 1, y = 2, z = 3$, then

$$\begin{vmatrix} 5 & 1 & 2 \\ 4 & 3 & 1 \\ 3 & 2 & 3 \end{vmatrix} = 5(7) - 1(12 - 3) + 2(8 - 9)$$

$$= 35 - 9 - 2 = 24$$

$$\text{and } (x+y+z)(x-z)^2 = (6)(-2)^2 = 24$$

$$\therefore k = \frac{24}{24} = 1$$

Q.24 (2)

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$$

Applying $C_2 \rightarrow C_2 - C_1$, and $C_3 \rightarrow C_3 - C_1$,

$$\begin{vmatrix} 1+a & -a & -a \\ 1 & b & 0 \\ 1 & 0 & c \end{vmatrix}$$

On expanding w.r.t. R_3 ,

$$ab + bc + ca + abc = \lambda$$

.....(i)

$$\text{Given, } a^{-1} + b^{-1} + c^{-1} = 0$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0 \Rightarrow ab + bc + ca = 0$$

$$\Rightarrow \lambda = abc \text{ , (From equation (i)).}$$

Q.25 (2)

Taking out 5 from R_2 makes $R_2 = R_1$.

Q.26 (2)

$$\Delta = \begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 - \omega^2 & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix} (C_1 \rightarrow C_1 + C_2 + C_3)$$

$$(\because 1 + \omega + \omega^2 = 0)$$

$$= 3[\omega \cdot \omega - \omega^4] = 3(\omega^2 - \omega) = 3\omega(\omega - 1)$$

Q.27 (1)

Put $x = 0$, which gives answer (a).

Q.28 (1)

On expanding, $-a(b-c) + 2b(b-c) + (a-b)(b-2c) = 0$

$$\Rightarrow -ab + ac + 2b^2 - 2bc + ab - 2ac - b^2 + 2bc = 0$$

$$\Rightarrow b^2 - ac = 0 \Rightarrow b^2 = ac$$

Q.29 (4)

$$\begin{vmatrix} 4+x^2 & -6 & -2 \\ -6 & 9+x^2 & 3 \\ -2 & 3 & 1+x^2 \end{vmatrix} = x^4(14+x^2)$$

$$= x \cdot x^3(14+x^2)$$

Hence, the determinant is divisible by x, x^3 and $(14+x^2)$, but not divisible by x^5 .

Q.30 (4)

$$\begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ z-y & z-x & x+y \end{vmatrix} = \begin{vmatrix} y+z & x-z & x-y \\ 2y & 2x & 0 \\ 2z & 0 & 2x \end{vmatrix}$$

$$R_2 \rightarrow R_2 + R_1 \text{ and } R_3 \rightarrow R_3 + R_1$$

$$= 4 \begin{vmatrix} y+z & x-z & x-y \\ y & x & 0 \\ z & 0 & x \end{vmatrix}$$

$$= 4[(y+z)(x^2) - (x-z)(xy) + (x-y)(-zx)]$$

$$= 4[x^2y + zx^2 - x^2y + xyz - zx^2 + xyz] = 8xyz$$

Hence $k = 8$

Q.31 (3)

If r is the common ratio, then $a_n = a_1 r^{n-1}$ for all

$$n \geq 1 \Rightarrow \log a_n = \log a_1 + (n-1) \log r = A + (n-1)R,$$

where $\log a_1 = A$ and $\log r = R$.

Thus in Δ , on applying $C_2 \rightarrow C_2 - C_1$ and

$C_3 \rightarrow C_3 - C_2$, we obtain C_2 and C_3 are identical.

Thus $\Delta = 0$.

Q.32 (4)

The system of equations has infinitely many (non-

$$\text{trivial) solution, if } \Delta = 0 \text{ i.e., if } \begin{vmatrix} 3 & -2 & 1 \\ \lambda & -14 & 15 \\ 1 & 2 & -3 \end{vmatrix} = 0$$

$$\Rightarrow 3(42 - 30) - \lambda(6 - 2) + 1(-30 + 14) = 0$$

$$\Rightarrow \lambda = 5$$

Q.33 (a)

Consider first two equations :

$$2x + 3y = -4 \quad \text{and} \quad 3x + 4y = -6$$

$$\text{We have } \Delta = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1 \neq 0$$

$$\Delta_x = \begin{vmatrix} -4 & 3 \\ -6 & 4 \end{vmatrix} = 2 \quad \text{and} \quad \Delta_y = \begin{vmatrix} 2 & -4 \\ 3 & -6 \end{vmatrix} = 0$$

$$\therefore x = -2 \text{ and } y = 0$$

Now this solution satisfies the third, so the equations are consistent with unique solution.

Q.34 (a)

Since the lines are concurrent, so

$$\begin{vmatrix} \ell & m & n \\ m & n & \ell \\ n & \ell & m \end{vmatrix} = 0 \Rightarrow 3\ell mn - \ell^3 - m^3 - n^3 = 0$$

$$\Rightarrow (\ell + m + n)(\ell^2 + m^2 + n^2 - \ell m - mn - n\ell) = 0$$

$$\Rightarrow \ell + m + n = 0 \left[\because \ell^2 + m^2 + n^2 > \ell m + mn + n\ell \right]$$

Q.35 (b)

For concurrency of 3 lines the determinant of coefficients of equations should be

$$\text{i.e. } \begin{vmatrix} 3 & -4 & -13 \\ 8 & -11 & -33 \\ 2 & -3 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 3(-11\lambda - 99) + 4(8\lambda + 66) - 13(-24 + 22) = 0$$

$$\Rightarrow -33\lambda - 297 + 32\lambda + 264 + 312 - 286 = 0$$

$$\Rightarrow -\lambda - 583 + 576 = 0 \Rightarrow \lambda = -7$$

Q.36 (2)

$$D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0; D_1 = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$$

\Rightarrow Given system, does not have any solution.

\Rightarrow No solution

Q.37 (3)

For the given set of equation, by Cramer's Rule

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 7 & 3 & -5 \\ 6 & 1 & 1 \\ 1 & -4 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & -5 \\ 1 & 1 & 1 \\ 3 & -4 & 2 \end{vmatrix}}$$

Q.38 (3)

It has a non-zero solution if

$$\begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0 \Rightarrow -6k + 6 = 0 \Rightarrow k = 1$$

Q.39 (2)

The given system of homogeneous equations has

$$\Delta = \begin{vmatrix} 1 & 4 & -1 \\ 3 & -4 & -1 \\ 1 & -3 & 1 \end{vmatrix} = 1(-4-3) - 4(3+1) - 1(-9+4)$$

$$= -7 - 16 + 5 \neq 0$$

There exists only one trivial solution.

Q.40 (4)

It is based on fundamental concept

Q.41 (1)

For the equation to be inconsistent $D = 0$

$$\therefore D = \begin{vmatrix} 1 & 2 & -3 \\ 0 & 0 & k+3 \\ 2k+1 & 0 & 1 \end{vmatrix} = 0 \Rightarrow k = -3 \quad \text{and}$$

$$D_1 = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \neq 0$$

So that system is inconsistent for $k = -3$.

Q.42 (1)

Given system of equation can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -18 \end{bmatrix}$$

On solving the above system we get the unique solution $x = -10, y = -4, z = 16$.

Q.43 (1)

$$\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0 \Rightarrow 1 + a(a^2) = 0 \Rightarrow a^3 = -1 \Rightarrow a = -1.$$

Q.44 (4)

Put the value $(x, y, z) = (1, 2, -1)$, which satisfies the equation. Hence, (d) is correct.

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (2)

$$\Delta_1 = \Delta_0^2$$

$$\Delta_2 = \Delta_1^2 = \Delta_0^4$$

$$\Delta_3 = \Delta_2^2 = \Delta_0^8$$

$$\Delta_4 = \Delta_3^2 = \Delta_0^{16}$$

and so on $\Delta_n = \Delta_0^{2^n}$ Ans.

Q.2 (d)

The given determinant vanishes, i.e.,

$$\begin{vmatrix} 1 & x-3 & (x-3)^2 \\ 1 & x-4 & (x-4)^2 \\ 1 & x-5 & (x-5)^2 \end{vmatrix} = 0$$

Expanding along C_1 , we get

$$\begin{aligned} & (x-4)(x-5)^2 - (x-5)(x-4)^2 - \{(x-3)(x-5)^2 \\ & - (x-5)(x-3)^2\} + (x-3)(x-4)^2 \\ & - (x-4)(x-3)^2 = 0 \\ \Rightarrow & (x-4)(x-5)(x-5-x+4) \\ & - (x-3)(x-5)(x-5-x+3) \\ & + (x-3)(x-4)(x-4-x+3) = 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow & -(x-4)(x-5) + 2(x-3)(x-5) - (x-3) \\ \Rightarrow & -x^2 + 9x - 20 + 2x^2 - 16x + 30 - x^2 + 7x - 12 = 0 \\ \Rightarrow & -32 + 30 = 0 \Rightarrow -2 = 0 \end{aligned}$$

Which is not possible, hence no value of x satisfies the given condition.

Q.3 (a)

Since, $-1 \leq x < 0$

$$\therefore [x] = -1$$

$$0 \leq z < 1 \therefore [y] = 1$$

$$1 \leq z < 2 \therefore [z] = 1$$

$$\therefore \text{Given determinant} = \begin{vmatrix} 0 & 2 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{vmatrix} = 1 = [z]$$

Q.4 (4)

Q.5 (4)

Q.6 (4)

Q.7 (Bonus)

Q.8 (1)

Q.9 (3)

Q.10 (1)

$$\begin{vmatrix} 3u^2 & 2u^3 & 1 \\ 3v^2 & 2v^3 & 1 \\ 3w^2 & 2w^3 & 1 \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 - R_2 \text{ and } R_2 \rightarrow R_2 - R_3$$

$$\Rightarrow \begin{vmatrix} u^2 - v^2 & u^3 - v^3 & 0 \\ v^2 - w^2 & v^3 - w^3 & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0 \Rightarrow$$

$$\begin{vmatrix} u+v & u^2 + v^2 + vu & 0 \\ v+w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 - R_2$$

$$\Rightarrow \begin{vmatrix} u-w & (u^2 - w^2) + v(u-w) & 0 \\ v+w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0 \Rightarrow$$

$$\begin{vmatrix} 1 & u+w+v & 0 \\ v+w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (v^2 + w^2 + vw) - (v+w)[(v+w) + u] = 0$$

$$\Rightarrow v^2 + w^2 + vw = (v + w)^2 + u(v + w)$$

$$\Rightarrow uv + vw + wu = 0 \text{ Ans.}$$

Q.11

(3)
Consider the det. B, using $R_1 \rightarrow R_1 + R_2 + R_3$

$$B = 2 \begin{vmatrix} a+p+x & b+q+y & c+r+z \\ a+x & b+y & c+z \\ a+p & b+q & c+r \end{vmatrix}$$

using $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$$= 2 \begin{vmatrix} a+p+x & b+q+y & c+r+z \\ -p & -q & -r \\ -x & -y & -z \end{vmatrix}$$

using $R_1 \rightarrow R_1 + R_2 + R_3$
 $B = 2 \det. A = 2 \cdot 6 = 12$

Q.12

(a)
let A be the 1st term and R the common ratio of G.P., then

$$a = T_p = AR^{p-1}$$

$$\therefore \log a = \log A + (p-1) \log R$$

Similarly, $\log b = \log A + (q-1) \log R$

And $\log c = \log A + (r-1) \log R$

$$\therefore \Delta = \begin{vmatrix} \log A + (p-1) \log R & p & 1 \\ \log A + (q-1) \log R & q & 1 \\ \log A + (r-1) \log R & r & 1 \end{vmatrix}$$

Split into two determinants and in the first
 Take $\log A$ common and in the second take
 $\log R$ common

$$\Delta = \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$$

Apply $C_1 \rightarrow C_2 \rightarrow C_3$ in the second

$$\Delta = 0 + \log R \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix} = 0$$

Q.13

(a)
 $R_3 \rightarrow 100R_1 + 10R_{10} + R_3$

$$\Rightarrow \begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix} = \begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ A & 8 & 8 \end{vmatrix} + \begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 6 & B & C \end{vmatrix} + \begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix}$$

Which is divisible by 72.

Q.14

(c)

$$\begin{vmatrix} p & q-y & r-z \\ p-x & q & r-z \\ p-x & q-y & r \end{vmatrix} = 0$$

Apply $R_1 \rightarrow R_1 - R_3$ and $R_2 \rightarrow R_2 - R_3$, we get

$$\begin{vmatrix} x & 0 & -z \\ 0 & y & -z \\ p-x & q-y & r \end{vmatrix} = 0$$

[Expansion along first row]

$$\Rightarrow xy r + zxq + yz p = 2xyz \Rightarrow \frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 2$$

Q.15

(1)

Q.16

(1)

Q.17

(4)

Q.18

(2)

Q.19

(3)

Q.20

(4)

Q.21

(3)

Q.22

(3)

Q.23

(1)

$$\begin{bmatrix} -1 & 2 & 1 \\ 3+2\sqrt{2} & 2+2\sqrt{2} & 1 \\ 3-2\sqrt{2} & 2-2\sqrt{2} & 1 \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ & $R_3 \rightarrow R_3 - R_1$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 4+2\sqrt{2} & 2\sqrt{2} & 0 \\ 4-2\sqrt{2} & -2\sqrt{2} & 0 \end{bmatrix} = 1(-8\sqrt{2} - 8 - 8\sqrt{2} + 8) = -16\sqrt{2}$$

So absolute value is $16\sqrt{2}$

Q.24

(4)

α, β, γ are roots of $x^3 + px + q = 0$

$$\therefore \alpha + \beta + \gamma = 0 \text{ Here } \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$(\alpha + \beta + \gamma) \begin{vmatrix} 1 & \beta & \gamma \\ 1 & \gamma & \alpha \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

Q.25 (4)

$$\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^y + b^{-y})^2 & (b^y - b^{-y})^2 & 1 \\ (c^z + c^{-z})^2 & (c^z - c^{-z})^2 & 1 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - C_2$

$$\begin{vmatrix} 4 & (a^x - a^{-x})^2 & 1 \\ 4 & (b^y - b^{-y})^2 & 1 \\ 4 & (c^z - c^{-z})^2 & 1 \end{vmatrix} = 0$$

Q.26 (2)

Taking two common, applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a_1 + b_1 + c_1) & c_1 + a_1 & a_1 + b_1 \\ 2(a_2 + b_2 + c_2) & c_2 + a_2 & a_2 + b_2 \\ 2(a_3 + b_3 + c_3) & c_3 + a_3 & a_3 + b_3 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ & $C_3 \rightarrow C_3 - C_1$

$$= 2 \begin{vmatrix} a_1 + b_1 + c_1 & -b_1 & -c_1 \\ a_2 + b_2 + c_2 & -b_2 & -c_2 \\ a_3 + b_3 + c_3 & -b_3 & -c_3 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$= 2 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Q.27 (3)

$$\Delta = \begin{vmatrix} x & x+y & x+y+z \\ 2x & 5x+2y & 7x+5y+2z \\ 3x & 7x+3y & 9x+7y+3z \end{vmatrix} = -16$$

Applying $R_2 \rightarrow R_2 - 2R_1$ & $R_3 \rightarrow R_3 - 3R_1$

$$\Delta = \begin{vmatrix} x & x+y & x+y+z \\ 0 & 3x & 5x+3y \\ 0 & 4x & 6x+4y \end{vmatrix} = -16$$

Applying $R_3 \rightarrow R_3 - R_2$

$$\begin{vmatrix} x & x+y & x+y+z \\ 0 & 3x & 5x+3y \\ 0 & x & x+y \end{vmatrix} = -16$$

Applying $R_2 \rightarrow R_2 - 3R_1$

$$\begin{vmatrix} x & x+y & x+y+z \\ 0 & 0 & 2x \\ 0 & x & x+y \end{vmatrix} = -16 \Rightarrow -2x(x^2 - 0) = -16$$

$$\Rightarrow x^3 = 8 \Rightarrow x = 2$$

Q.28 (2)

$$\begin{vmatrix} \cos(\theta + \phi) & -\sin(\theta + \phi) & \cos 2\phi \\ \sin \theta & \cos \theta & \sin \phi \\ -\cos \theta & \sin \theta & \cos \phi \end{vmatrix}$$

$$= \frac{1}{\sin \phi \cos \phi} \begin{vmatrix} \cos(\theta + \phi) & -\sin(\theta + \phi) & \cos 2\phi \\ \sin \theta \sin \phi & \sin \theta \cos \theta & \sin^2 \phi \\ -\cos \theta \cos \phi & \sin \theta \cos \phi & \cos^2 \phi \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$

$$= \frac{1}{\sin \phi \cos \phi} \begin{vmatrix} 0 & 0 & 2\cos^2 \phi \\ \sin \theta \sin \phi & \sin \phi \cos \theta & \sin^2 \phi \\ -\cos \theta \cos \phi & \sin \theta \cos \phi & \cos^2 \phi \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 2\cos^2 \phi \\ \sin \theta & \cos \theta & \sin \phi \\ -\cos \theta & \sin \theta & \cos \phi \end{vmatrix} = 2\cos^2 \phi (\sin^2 \theta + \cos^2 \theta)$$

$$= 2\cos^2 \phi$$

Q.29 (2)

$$\Delta = \begin{vmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \theta \sin \phi & \sin \theta \cos \phi & 0 \end{vmatrix}; \quad \Delta =$$

$$\sin^2 \theta \cos \theta \begin{vmatrix} \cos \phi & \sin \phi & \cot \theta \\ \cos \phi & \sin \phi & -\tan \theta \\ -\sin \phi & \cos \phi & 0 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_2$

$$\Delta = \sin^2 \theta \cos \theta \begin{vmatrix} 0 & 0 & \cot \theta + \tan \theta \\ \cos \phi & \sin \phi & -\tan \theta \\ -\sin \phi & \cos \phi & 0 \end{vmatrix}$$

$\Delta = \sin \theta$

Q.30 (2)

Expand the determinant using first row and use $x - y = A$, $y - z = B$ and $z - x = C$

$$\Rightarrow A + B + C = 0$$

Q.31 (4)

$$\text{For non trivial solution } \begin{vmatrix} \sin \theta & -\cos \theta & \lambda + 1 \\ \cos \theta & \sin \theta & -\lambda \\ \lambda & \lambda + 1 & \cos \theta \end{vmatrix} = 0$$

; this gives $2 \cos \theta (\lambda^2 + \lambda + 1) = 0$

Q.32 (4)

For non trivial solution

$$\begin{vmatrix} 1 & -\cos \theta & \cos 2\theta \\ -\cos \theta & 1 & -\cos \theta \\ \cos 2\theta & -\cos \theta & 1 \end{vmatrix} = 0$$

using $C_1 \rightarrow C_1 - C_3$

$$\begin{vmatrix} 2\sin^2\theta & -\cos\theta & \cos 2\theta \\ 0 & 1 & -\cos\theta \\ -2\sin^2\theta & -\cos\theta & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 2+\omega^2 & 1 & \omega^2 \\ 2+\omega & 1 & \omega \\ 0 & \omega & 1 \end{vmatrix} \quad (\text{as } \omega^3 = 1, \omega^4 = \omega)$$

$$2\sin^2\theta \begin{vmatrix} 1 & -\cos\theta & \cos 2\theta \\ 0 & 1 & -\cos\theta \\ -1 & -\cos\theta & 1 \end{vmatrix} = 0$$

$$= \begin{vmatrix} 2+\omega^2 & 1 & \omega^2 \\ \omega-\omega^2 & 0 & \omega-\omega^2 \\ 0 & \omega & 1 \end{vmatrix} \quad (R_2 \rightarrow R_2 - R_1)$$

$$\sin^2\theta = 0$$

$$\text{or } 1[1 - \cos^2\theta] - 1[\cos^2\theta - \cos 2\theta]$$

$$\sin^2\theta - [\cos^2\theta - (\cos^2\theta - \sin^2\theta)]$$

$$\sin^2\theta - \sin^2\theta = 0$$

hence $D = 0 \forall \theta \in \mathbb{R}$
 \Rightarrow (4)

Q.33

(b)

$$\Delta = \begin{vmatrix} -a & 1 & 1 \\ 1 & -b & 1 \\ 1 & 1 & -c \end{vmatrix} = 0 \text{ for non-zero solution}$$

$$\Rightarrow abc - a - b - c - 2 = 0$$

$$\Rightarrow abc = a + b + c + 2$$

Now, $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c}$

$$= \frac{3 + 2(a+b+c) + (ab+bc+ac)}{1 + (a+b+c) + (ab+bc+ac) + abc}$$

$$= \frac{3 + 2(a+b+c) + (ab+bc+ac)}{1 + 2(a+b+c) + 2ab + bc + ac} = 1$$

- Q.34** (2)
Q.35 (3)
Q.36 (4)
Q.37 (2)

$$D = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 3 & -3 \end{vmatrix} \text{ which vanishes}$$

hence for atleast one solution $D_1 = D_2 = D_3 = 0$

$$\therefore D_1 = \begin{vmatrix} a & -2 & 1 \\ b & 1 & -2 \\ c & 3 & -3 \end{vmatrix} = 0 \Rightarrow a - b + c = 0$$

Ans.

EXERCISE-III

Q.1 0003

$$\therefore \begin{vmatrix} 1 & \omega^3 & \omega^5 \\ \omega^3 & 1 & \omega^4 \\ \omega^5 & \omega^4 & 1 \end{vmatrix} = \begin{vmatrix} 1+\omega^3+\omega^5 & \omega^3 & \omega^5 \\ 1+\omega^3+\omega^4 & 1 & \omega^4 \\ 1+\omega^4+\omega^5 & \omega^4 & 1 \end{vmatrix}$$

$(C_1 \rightarrow C_1 + C_2 + C_3)$

Q.2

(0002)

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$$(\because a^2 + b^2 + c^2 + 2 = 0)$$

Operating $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix} = (1-x)^2$$

Hence degree of $f(x) = 2$

Q.3

(-2)

For no solution or infinitely many solutions

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0 \Rightarrow \alpha = 1, \alpha = -2$$

But for $\alpha = 1$, clearly there are infinitely many solutions and when we put $\alpha = -2$ in given system of equations and adding them together
 L.H.S. \neq R.H.S. \Rightarrow No solution

Q.4

(0000)

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$a_1 a_2 a_3 \dots$ are in G.P.

$$\text{Then } r = \frac{a_2}{a_1} = \frac{a_{n+1}}{a_n} = \frac{a_{n+2}}{a_{n+1}} = \dots$$

$$\log r = \log(a_{n+1}) - \log a_n = \log(a_{n+2}) - \log(a_{n+1})$$

Operating $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_2$

$$\Delta = \begin{vmatrix} \log a_n & \log r & \log r \\ \log a_{n+3} & \log r & \log r \\ \log a_{n+6} & \log r & \log r \end{vmatrix} = 0$$

Q.5 0001

$$\begin{vmatrix} \sin x + 2\cos x & \sin x + 2\cos x & \sin x + 2\cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} =$$

using $(R_1 \rightarrow R_1 + R_2 + R_3)$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ \cos x & \sin x - \cos x & 0 \\ \cos x & 0 & \sin x - \cos x \end{vmatrix} (\sin x + 2\cos x) = 0$$

using $(C_2 \rightarrow C_2 - C_3)$ and $C_3 \rightarrow C_3 - C_1$

$$= (\sin x - \cos x)^2 (\sin x + 2\cos x) = 0$$

$\Rightarrow \sin x = \cos x$, because $\sin x + 2\cos x \neq 0$;

\Rightarrow only one solution.

Q.6 (0000)

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-b & a(b-c) \end{vmatrix}$$

$$= 1 \cdot [a(b-c)(b-a) - c(a-b)(c-b)]$$

$$(b-c)(b-a)(a-c)$$

$$(a-b)(b-c)(c-a)$$

Again $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

So the given difference is 0.

Q.7 0001

Coefficient det = 0 $\Rightarrow \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0$

$$\Rightarrow (-4k + 6) - k(-12 + 4) + 3(9 - 2k) = 0$$

$$\Rightarrow -4k + 6 + 12k - 4k + 27 - 6k = 0$$

$$\Rightarrow -2k + 33 = 0$$

$$2K = 33$$

$$\frac{2K}{33} = 1$$

Q.8 0002

Let $\alpha = s - a, \beta = s - b, \gamma = s - c$

then determinant

$$= \begin{vmatrix} (\beta + \gamma)^2 & \alpha^2 & \alpha^2 \\ \beta^2 & (\gamma + \alpha)^2 & \beta^2 \\ \gamma^2 & \gamma^2 & (\alpha + \beta)^2 \end{vmatrix} = 2\alpha\beta\gamma(\alpha + \beta + \gamma)^2$$

(standard determinant)

$$= 2(s - a)(s - b)(s - c)s^3 \Rightarrow k = 2.$$

PREVIOUS YEAR'S

MHT CET

Q.1 (1)

Q.2 (4)

Q.3 (3)

Q.4 (3)

Q.5 (1)

Q.6 (4)

Q.7 (1)

Q.8 (1)

Given, $\begin{vmatrix} (\alpha^x + \alpha^{-x})^2 & (\alpha^x - \alpha^{-x})^2 & 1 \\ (\beta^x + \beta^{-x})^2 & (\beta^x - \beta^{-x})^2 & 1 \\ (\gamma^x + \gamma^{-x})^2 & (\gamma^x - \gamma^{-x})^2 & 1 \end{vmatrix}$

Applying $C_1 \rightarrow C_1 - C_2$,

$$\begin{vmatrix} 4 & (\alpha^x - \alpha^{-x})^2 & 1 \\ 4 & (\beta^x - \beta^{-x})^2 & 1 \\ 4 & (\gamma^x - \gamma^{-x})^2 & 1 \end{vmatrix} = 4 \begin{vmatrix} 1 & (\alpha^x - \alpha^{-x})^2 & 1 \\ 1 & (\beta^x - \beta^{-x})^2 & 1 \\ 1 & (\gamma^x - \gamma^{-x})^2 & 1 \end{vmatrix} = 0$$

Q.9 (1)

We have, $\begin{vmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{vmatrix}$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$,

$$\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & a \\ x+a+b+c & b & x+c \end{vmatrix} = 0$$

$$= (x+a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & x+b & a \\ 1 & b & x+c \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 - R_1$,

$$(x+a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & x & a-c \\ 0 & 0 & x \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c)(x^2 - 0) = 0$$

$$\Rightarrow x = 0 \text{ or } x = -(a+b+c)$$

Q.10 (4)

$$\text{Given, } \begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$$

$$\Rightarrow -3i \begin{vmatrix} 6i & 1 & 1 \\ 4 & -1 & -1 \\ 20 & i & i \end{vmatrix} = x + iy$$

$\Rightarrow x + iy = 0$ [$\because C_2$ and C_3 are identical]
 $\therefore x = 0, y = 0$

JEE-MAIN

Q.1 (58)
 $2x - 3y = \gamma + 5$
 $\alpha x + 5y = \beta + 1$
 For ∞ many solution,
 $\frac{2}{\alpha} = \frac{-3}{5} = \frac{\gamma + 5}{\beta + 1}$

$$\alpha = \frac{-10}{3} \quad -3\beta - 3 = 5\gamma + 25$$

$$9\alpha = -30 \quad 5\gamma + 3\beta = -28$$

$$\therefore |9\alpha + 3\beta + 5\gamma| = \left| 9 \times \frac{-10}{3} + (-28) \right| = |-30 - 28| = |-58| = 58$$

Q.2 (2)
 The plane represented by the equation's intersect each other a line
 $x + 4y + \delta z - k = p(2x + y - z - 7) + q(x - 3y + 2z - 1)$
 $x + 4y + \delta z - k = (2p + q)x + (p - 3q)y - (p + 2q)z - 7p - q$
 By comparing
 $2p + q = 1 \quad d = -p + 2q$
 $p - 3q = 4 \quad k = 7p + q$
 $p = 1$
 $q = -1$
 $\delta + k = 6p + 3q$
 $= 6 - 3 = 3$

Q.3 (2)
 For inconsistent system of equation value of determinant formed by coefficients must be equal to zero.

$$\det = 0 \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix} = 0$$

Solving $1(10\alpha - 9\alpha) - 1(5\alpha - 3) + 1(3\alpha^2 - 2\alpha) = 0$
 $\alpha - 5\alpha + 3 + 3\alpha^2 - 2\alpha = 0$
 $3\alpha^2 - 3\alpha - 3\alpha + 3 = 0$
 $3\alpha(\alpha - 1) - 3(\alpha - 1) = 0$

$(\alpha - 1)(3\alpha - 3) = 0$
 $3(\alpha - 1)^2 = 0 \quad \alpha = 1$
 But for inconsistent or no solution, at least one form Δ_x or Δ_y must be not equal to zero.

$$\Delta_x = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & 3 \\ 4 & 3 & 5 \end{vmatrix}$$

$\Delta_x = 1(10 - 9) - 1(-5 - 12) + 1(-3 - 8)$
 $\Delta_x = 1 + 17 - 11$
 $\therefore \Delta_x \neq 0$
 \therefore for $\alpha = 1$ system is inconsistent
 Hence ans is option (B)

Q.4

[8]
 $A \left(\frac{3}{\sqrt{a}}, \sqrt{a} \right), a > 0$

image of A w.r.t. y-axis is $B \left(\frac{-3}{\sqrt{a}}, \sqrt{a} \right)$

image of B wrt x-axis is $C \left(\frac{-3}{\sqrt{a}}, -\sqrt{a} \right)$

point D is $(3 \cos \theta, a \sin \theta)$ is in fourth quadrant

$$\Delta_{ACD} = \frac{1}{2} \begin{vmatrix} 3/\sqrt{a} & \sqrt{a} & 1 \\ -3/\sqrt{a} & -\sqrt{a} & 1 \\ 3 \cos \theta & a \sin \theta & 1 \end{vmatrix}$$

apply $R_1 \rightarrow R_1 + R_2$

$$\Delta_{ACD} = \frac{1}{2} \begin{vmatrix} 0 & 0 & 2 \\ -3/\sqrt{a} & -\sqrt{a} & 1 \\ 3 \cos \theta & a \sin \theta & 1 \end{vmatrix}$$

$$\Delta_{ACD} = |-3\sqrt{a} \sin \theta + 3\sqrt{a} \cos \theta|$$

$$= 3\sqrt{a} |\cos \theta - \sin \theta|$$

$$= \text{max. area} = 3\sqrt{2}\sqrt{a} = 3\sqrt{2}a$$

Given max. area = 12

$$\therefore 3\sqrt{2}\sqrt{a} = 12$$

$$\sqrt{2}\sqrt{a} = 4$$

$$\sqrt{a} = 2\sqrt{2} \Rightarrow a = 8$$

Q.5

(3)
 $\Delta = \begin{vmatrix} 1 & 1 & \alpha \\ 3 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix}$

$$= 1(2 - 0) - 1(6 - 1) + \alpha(0 - 1)$$

$$= 2 - 5 - \alpha$$

$$= -\alpha - 3$$

$$\Delta_x = \begin{vmatrix} 2 & 1 & \alpha \\ 4 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = 2(2-0) - 1(8-1) + \alpha[0-1]$$

$$= 4 - 7 - \alpha$$

$$= -3 - \alpha$$

$$\Delta_y = \begin{vmatrix} 1 & 2 & \alpha \\ 3 & 4 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 1(8-1) - 2(6-1) + \alpha(3-4)$$

$$= 7 - 10 - \alpha$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 1 & 4 \\ 1 & 0 & 1 \end{vmatrix} = -3 - \alpha$$

$$= 1(1-0) - 1(3-4) + 2(0-1)$$

$$= 1 + 1 - 2 = 0$$

$$\therefore x = \frac{\Delta_x}{\Delta} \quad y = \frac{\Delta_y}{\Delta} \quad z = \frac{\Delta_z}{\Delta}$$

$$x = 1 \quad y = 1 \quad z = 0$$

$$(x, y, z) = (1, 1, 0)$$

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$$\alpha(\alpha+1) - 1(0) + 1(-1-\alpha) = 0$$

$$\Rightarrow \alpha^2 + \alpha - 1 - \alpha = 0$$

$$\alpha = \pm 1$$

$$\therefore |1| + |-1| = 2$$

Q.6 (4)

$$\Delta = \begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} \neq 0$$

$$k^2 \neq 121 \Rightarrow k \neq -11, 11$$

Q.7 [1]

$$A^2 = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} = A$$

$$B^2 = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix} = B$$

$\Rightarrow A$ & B are idempotent
 Now, $nA^n + mB^m = nA + mB = I$
 which gives $m = n = 1$
 Only one set possible

Q.8 (3)

For infinitely many solutions

$$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$$

$$\Delta = 0$$

$$\Rightarrow \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \alpha(10-9) - 1(5-3) + 1(3-2) = 0 \quad \Rightarrow \alpha = 1$$

$$\Delta_1 = 0$$

$$\begin{vmatrix} 5 & 1 & 1 \\ 4 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow 5 - 1(20 - 3\beta) + (12 - 2\beta) = 0$$

$$\Rightarrow \beta = 3$$

Q.9 (4)

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ -2 & 1 & 2 \end{vmatrix}$$

$$= (6+1) - 2((2\alpha-\alpha) + 1(\alpha+3\alpha))$$

$$= 7 - 2\alpha + 4\alpha$$

$$= 7 + 2\alpha$$

$$\Delta = 0 \Rightarrow \alpha = -\frac{7}{2}$$

$$\Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix}$$

$$= 14 + 2\alpha$$

$$\alpha = -x_2 = 7$$

$$= 14 + 2\left(\frac{-7}{2}\right)$$

Q.10

$$\Delta_1 \neq 0$$

$$(14)$$

$$\text{Adj}(\text{Adj } A) = \begin{bmatrix} 14 & 18 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix}$$

$$|\text{Adj}(\text{Adj } A)| = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix} = 14 \times 14 \times 14$$

$$\begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$

$$(14)^3 [3 - 2(-5) - 1(-1)] = (14)^3 [14] = [14]^4$$

$$\{ \therefore |\text{Adj}(\text{Adj } A)| = |A|^{(n-1)^2} \}$$

$$|A|^4 = (14)^4 \Rightarrow |A| = 14$$

Q.11 (3)
 $AB = I$
 $|\text{adj}(\text{Badj}(2A))| = |\text{Badj}(2A)|^2$
 $= |B|^2 |\text{adj}(2A)|^2$
 $= |B|^2 (2A)^2 = |B|^2 (2^6 |A|^2)$

$$|A| = \frac{1}{8} \text{ and } |AB| = 1 \Rightarrow |A| |B| = 1$$

$$\Rightarrow \frac{1}{8} |B| = 1$$

$$\Rightarrow |B| = 8$$

$$\text{Required value} = 8^2 \times 2^{12} \cdot \left(\frac{1}{8}\right)^4 = 64$$

Q.12 (3)
 $|\text{adj}(24A)| = |\text{adj} 3(\text{adj} 2A)|$

$$\Rightarrow |24^2 \text{adj}(A)| = |\text{adj}(3 \times 2^2 (\text{adj}(A)))|$$

$$\Rightarrow (24)^6 |A|^2 = (12^2)^3 |A|^4$$

$$\Rightarrow |A|^2 = \left(\frac{24}{12}\right)^6 = (2)^6 = 64$$

Q.13 (3)

$$\begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0$$

$$3(-8a-9) + 2(5a-18) + 1(21) = 0$$

$$\Rightarrow a = -3$$

$$\text{Also, } \Delta_2 = \begin{vmatrix} 3 & -2 & b \\ 5 & 8 & 3 \\ 2 & 1 & -1 \end{vmatrix}^{1/3}$$

$$\text{If } b = \frac{1}{3}$$

$$\Delta_2 = 0$$

for no solution b must be equal to $-\frac{1}{3}$

Q.14 (2)

$$\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & |\lambda| \end{vmatrix} = 0$$

$$\Rightarrow |\lambda| = 7 \Rightarrow \lambda = \pm 7$$

System :

$$2x + 3y - z = -2 \quad \dots (1)$$

$$x + y + z = 4 \quad \dots (2)$$

$$x - y + |\lambda|z = 4\lambda - 4 \quad \dots (3)$$

Eliminating y from eq. (1) & (2) we get $x + 4z = 14$

$$(2) + (3) \Rightarrow x + \left(\frac{|\lambda|+1}{2}\right)z = 2\lambda$$

Clearly for $\lambda = -7$, system is inconsistent

Q.15 (1)
 $|(\det(A))\text{adj}(5\text{adj}(A^3))|$
 $= |2\text{adj}(5\text{adj}(A^3))|$
 $= 2^3 |\text{adj}(5\text{adj}(A^3))|$
 $= 2^3 \cdot |5\text{adj}(A^3)|^2$
 $= 2^3 (5^3 |\text{adj}(A^3)|)^2$
 $= 2^3 \cdot 5^6 \cdot |\text{adj}(A^3)|^2$
 $= 2^3 \cdot 5^6 \cdot (|A|^3)^2$
 $= 2^3 \cdot 5^6 \cdot 2^{12} = 2^{15} \times 5^6$
 $= 2^9 \times 10^6$
 $= 512 \times 10^6$

Q.16 (2)
 For no solution

$$\Delta = \begin{vmatrix} 3 \sin 3\theta & -1 & 1 \\ 3 \cos 2\theta & 4 & 3 \\ 6 & 7 & 7 \end{vmatrix} = 0$$

$$3 \sin 3\theta(7) + 1(21 \cos 2\theta - 18) + 1(21 \cos 2\theta - 24) = 0$$

$$21[\sin 3\theta + 2 \cos 2\theta] = 42$$

$$\sin 3\theta + 2 \cos 2\theta = 2$$

$$3 \sin \theta - 4 \sin^3 \theta + 2 - 4 \sin^2 \theta = 2$$

$$4 \sin^3 \theta + 4 \sin^2 \theta - 3 \sin \theta = 0$$

$$\sin \theta = 0 \text{ \& } 4 \sin^2 \theta + 4 \sin \theta + 1 = 4$$

$$\sin \theta = 0 \text{ \& } (2 \sin \theta + 1)^2 = 4$$

$$\theta \in \{\pi, 2\pi, 3\pi\} \quad 2 \sin \theta + 1 = 2 \text{ or } 2 \sin \theta + 1 = -2$$

$$\sin \theta = \frac{1}{2} \text{ or } \sin \theta = \frac{-3}{2}$$

$$\theta \in \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \right\} \text{ No sol.}$$

$$\text{and } \Delta_3 = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 4 & 3 \\ 9 & 7 & 7 \end{vmatrix} \Rightarrow \Delta_3 \neq 0$$

$$= 3(7) + 1(-6) + 1(-15) \neq 0$$

Hence total 7 values of θ are possible

Q.17 (3)
 $x + y + z = 6 \dots (I)$
 $2x + 5y + \alpha z = \beta \quad \dots (II)$
 $x + 2y + 3z = 14 \quad \dots (III)$
 $x + y = 6 - z$

$$x + 2y = 14 - 3z$$

On solving

$$x = z - 2 \Rightarrow y = 8 - 2z \text{ in (2)}$$

$$2(z - 2) + 5(8 - 2z) + \alpha z = \beta$$

$$(\alpha - 8)z = \beta - 36$$

For having infinite solutions

$$\alpha - 8 = 0 \text{ \& } \beta - 36 = 0$$

$$\alpha = 8, \beta = 36 (\alpha + \beta = 44)$$

$$2(3\alpha - 1) + 3(\alpha + 3) + 5(-1 - 9) = 0$$

$$6\alpha - 2 + 3\alpha + 9 - 50 = 0$$

$$9\alpha = 43$$

$$\therefore 9|\lambda|^2 - 9|\lambda| - 43 = 0 \Rightarrow |\lambda| = \frac{9 \pm \sqrt{81 + 1548}}{18}$$

$$\therefore |\lambda| = \frac{9 \pm 40.3}{18} \quad (-)\text{ve value rejected}$$

\therefore 2 values of λ

Q.18 (4)

$$\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$$

$$\Delta = p!(p+1)!(p+2)! \begin{vmatrix} 1 & 1 & 1 \\ p+1 & p+2 & p+3 \\ (p+2)(p+1) & (p+3)(p+2) & (p+4)(p+3) \end{vmatrix}$$

$$\Delta = 2p!(p+1)!(p+2)!$$

Which is divisible by P^α & $(P+2)^\beta$

$$\therefore \alpha = 3, \beta = 1$$

Q.19 (4)

$$8x + y + 4z = -2$$

$$x + y + z = 0$$

$$\lambda x - 3y + 0 \cdot z = \mu$$

$$\Delta_x = \begin{vmatrix} -2 & 1 & 4 \\ 0 & 1 & 1 \\ \mu & -3 & 0 \end{vmatrix} = 0$$

$$= (-2)(3) - 1(-\mu) + 4(-\mu) = 0$$

$$= -6 - 3\mu = 0 \Rightarrow \mu = -2$$

$$\Delta = 0$$

$$\begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix} = 0$$

$$= 8(3) - 1(0 - \lambda) + 4(-3 - \lambda) = 0$$

$$\Rightarrow 12 - 3\lambda = 0 = \lambda = 4$$

$$\left(4, -2, -\frac{1}{2} \right)$$

$$\text{Distance} = \left| \frac{32 - 2 - 2 + 2}{\sqrt{81}} \right| = \frac{30}{9} = \frac{10}{3}$$

Q.20 (3)

$$\text{Let } \alpha = \lambda^2 - |\lambda|$$

$$D = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 3 & -1 \\ 3 & -1 & \alpha \end{vmatrix} = 0$$